Coordinated Scheduling and Power Control in Cloud-Radio Access Networks

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Abstract—This paper addresses the joint coordinated scheduling and power control problem in cloud-enabled networks. Consider the downlink of a cloud-radio access network (CRAN), where the cloud is only responsible for the scheduling policy, power control, and synchronization of the transmit frames across the single-antenna base-stations (BS). The transmit frame consists of several time/frequency blocks, called power-zones (PZ). The paper considers the problem of scheduling users to PZs and determining their power levels (PL), by maximizing the weighted sum-rate under the practical constraints that each user cannot be served by more than one base-station, but can be served by one or more power-zones within each base-station frame. The paper solves the problem using a graph theoretical approach by introducing the joint scheduling and power control graph formed by several clusters, where each is formed by a set of vertices, representing the possible association of users, BSs, and PLs for one specific PZ. The problem is, then, formulated as a maximum-weight clique problem, in which the weight of each vertex is the sum of the benefits of the individual associations belonging to that vertex. Simulation results suggest that the proposed cross-layer scheme provides appreciable performance improvement as compared to schemes from recent literature.

Index Terms—Coordinated scheduling, power control, maximum-weight clique, joint scheduling and power control graph.

I. OVERVIEW

A. Introduction

The continuous increasing demand for high data rate services necessitates breakthroughs in network system architecture. With a progressive move towards full spectrum reuse and a positive trend in small-cell deployment, cloud-radio access networks (CRAN) become essential in large-scale interference management for next generation wireless systems (5G) [2]. Through its ability to allocate resources in a coordinated way across base-stations, cloud-enabled networks have the potential of mitigating inter-base-station interference through inter-base-station coordination.

Recent literature on CRAN assumes signal-level coordination and allows joint signal processing in the cloud [3]–[5].

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Such level of coordination allows joint beamforming, joint resource allocation, signal processing of data belonging to different users. However, signal-level coordination requires high-capacity links to share all data streams between all base-stations, and needs a substantial amount of backhaul communications. Furthermore, given the capacity limits of the backhaul links, substantial amount of backhaul/fronthaul communications may require sophisticated compression schemes. Designing the correlation matrices of the induced quantization noise becomes then a complicated process that is coupled with the determination of the beamforming vectors [3], [4]. This paper considers the CRAN problem from a different perspective, as it only considers *scheduling and power control coordination* at the cloud, which is more practical to implement, and at the same time allows base-stations to allocate the available radio resources efficiently. In the considered CRAN, base-stations are connected to a central processor (cloud) which is responsible for scheduling users to base-stations resource blocks and determining the power levels of each resource blocks. Hence, only the scheduling policy and the power levels are communicated to the different base-stations which can be achieved through low-rate backhaul links resulting in a cost-efficient network design.

Different coordination strategies have been studied in the literature as a means to mitigate interference, namely the joint scheduling and power control [6]. Such joint resource allocation problem is mathematically challenging and so a decoupling approach has been proposed to solve each of the problems independently (e.g., iterating between determining the power levels for a fixed scheduling and finding the schedule for a fixed powers). The goal of this paper is to show that jointly considering the coordinated scheduling and power control problem, in a cloud-radio access network, provides appreciable performance improvement as compared to the conventional iterative approach.

Consider the downlink of a cloud-radio access network comprising several single-antenna base-stations (BSs) connected to one central processor (the cloud), which is only responsible for the scheduling policy, the synchronization, and the power level determination. The frame structure of every base-station consists of several time/frequency resource blocks called power-zones. Across the network, users are multiplexed across the power-zones under the constraint that each user cannot be connected to more than one base-station since, otherwise, signal-level coordination between base-stations is needed. Each user, however, can be connected to several power-zones belonging to the frame of one base-station. Further, each power-zone, which can be in practice seen as a generic term to denote time/frequency resource block of every BS, serves one and only one user. The coordinated

scheduling problem becomes that of optimally scheduling users to base-stations and their power-zones and determining the optimal power levels used by each power-zone subject to the above practical constraints. A part of this paper considers the joint scheduling and power optimization problem with an objective of maximizing the weighted sum rate, where scheduling decisions are carried out by the cloud and relayed to the base-stations. Another part of the paper considers the coordinated scheduling problem alone (i.e., with fixed transmit power) with an objective to maximizing a generic utility function. Both parts of the paper solve the problems, i.e., the joint scheduling and power control, and the coordinated scheduling, using graph theory techniques.

B. Related Work

The problem of assigning users to base-stations is combinatorial in nature. Moreover, selecting the power levels for a fixed user schedule is a well-known non-convex problem. Therefore, the design of optimal solutions for the coordination problem is challenging, and a large amount of the literature considers user scheduling, and power allocation separately. The modular approach is more tractable and easier to implement.

In the past literature, scheduling is often performed on a per-base-station basis, given a pre-assigned association of users and base-stations, e.g., the classical proportionally fair scheduling [7], [8]. Unlike previous works where scheduling is performed with no inter-BS coordination, this paper considers the network-wide scheduling where coordination is carried by the cloud connecting the base-stations. The coordinated scheduling considered in this paper is particularly related to the concept developed in [9] in a soft-frequency reuse setup; however, the problem setup in [9] assumes an equal number of users and power-zones and boils down to a simple linear assignment problem, which can be solved using the classical auction methodology [10].

Inter-cell scheduling is specially investigated in [11]–[13]. While the authors in [11] consider maximizing the capacity of a multi-cell signal level coordinated network based on the complete co-channel gain information with a standard power control policy, the authors [12] consider a fully distributed scheme in which base-stations share the average channel state information only through low-rate backhaul communication. Through asymptotic analysis, the authors in [13] quantify the user diversity gain of their proposed inter-cell scheduling policy and derive bounds on the achievable ergodic capacity.

In the aforementioned works, the power levels used by each base-station is assumed to be fixed. To further account for the power optimization step, papers [14]–[16] consider updating the power in an outer step, i.e., for fixed scheduling. The authors in [17] present an alternative strategy which uses a dynamic interference avoidance scheme to mitigate excessive inter-cell interference. Reference [18] provides asymptotic rate-maximization properties of the power control and scheduling in multi-cell networks.

The joint scheduling and power control problem becomes an active research area of interest of late, e.g., [19]–[22]. However, such references only consider multi-cell OFDMA

networks with negligible cell-to-cell interference. Other works on the joint optimization problem only guarantee convergence to a stationary point of the overall optimization problem, e.g., the work in [23] which proposes a branch-and-bound algorithm to maximize the number of served users under rate constraints.

C. Contribution

This paper considers the joint scheduling and power control problem by maximizing the weighted sum rate subject to network connectivity constraints, i.e., the problem of scheduling users to power-zones (PZ) and determining their power levels (PL) in a coordinated fashion across the network. Unlike previous works that consider iterating between solving the scheduling problem for fixed transmit power and optimizing the power for a given schedule, this paper proposes solving the joint optimization problem. Scheduling is performed under the practical constraint that each user cannot be served by more than one base-station but can be served by one or more power-zone within each base-station frame. The paper main contribution is that it optimally solves the problem using a graph theoretical approach by introducing the *joint scheduling and power control graph* formed by several clusters, where every cluster is formed by a set of vertices, each representing the possible association of users, BSs, and PLs for one specific PZ. The problem is, then, formulated as a maximum-weight clique, in which the weight of each vertex is the sum of the benefits of the individual associations belonging to that vertex.

Further, the paper considers the coordinated scheduling problem only (i.e., under fixed transmit power) by maximizing a generic network-wide utility subject to the same connectivity constraint. The paper solves the problem by introducing the corresponding *scheduling graph* in which each vertex represents a single association of one user, one BS, and one PZ.

The above-considered problems are shown to be equivalent to maximum-weight clique problems, which can be globally solved using efficient algorithms [24], [25]. Simulation results suggest that the proposed cross-layer scheme outperforms iterative schemes from the literature and provide a significant gain, especially at high interference regime.

The rest of this paper is organized as follows: In Section II, the system model and the problem formulation are presented. Section III shows the optimal solution to the joint scheduling and power control problem. In Section IV, the optimal solution to the scheduling problem under fixed transmit power is investigated. Simulation results are presented in Section V and the paper concluded in Section VI.

D. Notations

Let $|\mathcal{X}|$ denotes the cardinality of a set $\mathcal{X}$ and $\mathcal{P}(\mathcal{X})$ its power set. The set denoted by $\mathcal{A} \times \mathcal{B}$ represents the Cartesian product of the two sets $\mathcal{A}$ and $\mathcal{B}$. Finally, let $\delta(\cdot)$ be the discrete Dirac function, i.e. $\delta(x)$ is 1 if $x = 0$, and $\delta(x)$ is 0 if $x \neq 0$. The notation $\{x\}$ denotes a set containing the single element x and (x, y) denotes the real interval between $x \leq y$.

Fig. 1. Cloud enabled network composed of 8 base-stations and 24 users. Only low-rate backhaul links are required to connect the cloud to the multiple base-stations.

Fig. 2. Frame structure of B base-stations each containing Z power-zones.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

Consider the downlink of a cloud radio wireless network of B single-antenna BSs connected to a central cloud through low-rate backhaul links and serving U users in total, as shown in Figure 1, which shows a CRAN formed by $B = 8$ BSs and $U = 25$ users. Let $\mathcal{B}$ be the set of all BSs in the system and $\mathcal{U}$ be the set of all users. The frame of each base-station consists of Z time/frequency resource blocks (called herein power-zones). Let $\mathcal{Z}$ be the set of PZs of the frame of a BS. As shown in Figure 2, the paper assumes orthogonal PZs, i.e., interference is only seen from power-zones with the same index z across the different base-stations. The total number of available PZs is $Z_{\text{tot}} = B \times Z$. The power level of each PZ z in BS b 's frame is denoted by P_{bz} , as shown in Figure 2. The value of P_{bz} is called the power levels (PLs) of BS b and PZ z . This paper assumes individual PZ constraint $P_{bz} \leq P_{bz}^{\max}$. Let $\mathbf{P}$ be a $B \times Z$ matrix containing the PLs of all PZs in the BSs. In other words, $\mathbf{P} = [P_{bz}]$.

The cloud, connecting the different BSs, guarantees that the transmission of the different frames is synchronized across all BSs. The cloud is further responsible for determining the scheduling and power control policies of the network. Let $h_{bz}^u \in \mathbb{C}$, $\forall (u, b, z) \in \mathcal{U} \times \mathcal{B} \times \mathcal{Z}$ be the channel from the b th BS to user u when user u is assigned to PZ z . The paper considers general channels without restrictions on their distributions. However, assumes that the values of the channel gains are perfectly known as the cloud. The corresponding

signal-to-interference plus noise-ratio (SINR) is:

$$\text{SINR}_{bz}^u(\mathbf{P}) = \frac{P_{bz}|h_{bz}^u|^2}{\Gamma(\sigma^2 + \sum_{b' \neq b} P_{b'z}|h_{b'z}^u|^2)}, \quad (1)$$

where σ^2 is the Gaussian noise variance and Γ denotes the SINR gap.

Remark 1. Note that the $\text{SINR}_{bz}^u(\mathbf{P})$ depends only on the scheduled users in PZ z and the corresponding PLs $P_{b'z}$ but does not depend on users scheduled in other PZ $z' \neq z$ or their corresponding PLs $P_{b'z'}$.

B. Problem Formulation

This paper considers the problem of assigning users to PZs in BSs under the following constraints:

- C1: Each user can connect at most to one BS but possibly to several PZs in that BS.
- C2: Each PZ is allocated to exactly one user.
- C3: Each PL is bounded by a nominal maximal value.

A part of this paper considers the weighted sum-rate maximization problem where the rate of user u when assigned to the z th PZ of the b th BS is weighted by a factor a_{ubz} , which is fixed throughout this paper. Such factor can typically be adjusted in an outer loop for load balancing and fairness purposes. However, this falls outside the scope of the current paper. Let X_{ubz} be a binary variable that is set to 1 if user u is mapped to the z th PZ of the b th BS, and zero otherwise. Further, let Y_{ub} be a binary variable that is set to 1 if user u is mapped to the b th BS, and zero otherwise. Next section considers the following network-wide optimization problem:

$$\max_{u, b, z} \sum a_{ubz} X_{ubz} \log_2 (1 + \text{SINR}_{bz}^u(\mathbf{P})) \quad (2a)$$

$$\text{s.t. } Y_{ub} = \min \left(\sum_z X_{ubz}, 1 \right), \forall (u, b) \in \mathcal{U} \times \mathcal{B}, \quad (2b)$$

$$\sum_b Y_{ub} \leq 1, \quad \forall u \in \mathcal{U}, \quad (2c)$$

$$\sum_u X_{ubz} = 1, \quad \forall (b, z) \in \mathcal{B} \times \mathcal{Z}, \quad (2d)$$

$$0 \leq P_{bz} \leq P_{bz}^{\max}, \forall (b, z) \in \mathcal{B} \times \mathcal{Z}, \quad (2e)$$

$$X_{ubz}, Y_{ub} \in \{0, 1\}, \forall (u, b, z) \in \mathcal{U} \times \mathcal{B} \times \mathcal{Z}, \quad (2f)$$

where the optimization is over the binary variables X_{ubz} , Y_{ub} , and the continuous variables P_{bz} , $\forall (u, b, z) \in \mathcal{U} \times \mathcal{B} \times \mathcal{Z}$. Here the constraints in (2b) and (2c) correspond to constraint C1, the equality constraint in (2d) corresponds to constraint C2, and (2e) corresponds to constraint C3. Finding the global optimum to the mixed discrete and continuous optimization problem (2) may involve searching over all possible user-to-power-zone assignments, and determining the PLs for every possible assignment. This is clearly infeasible for any reasonably sized network.

The next section solves this joint coordinated scheduling and power control optimization problem by introducing the joint scheduling and power control graph formed by several clusters (called herein local power control graphs), where every cluster is formed by a set of vertices, each representing the possible association of users, BSs, and PLs for one specific PZ. The problem is, then, formulated as a maximum-weight

clique problem, in which the weight of each vertex is the sum of the benefits of the individual associations belonging to that vertex. Further, the paper proposes a solution to the particular problem in which PZs are maintained at fixed PLs. The resulting coordinated scheduling solution is then obtained by introducing the scheduling graph in which each vertex represents an individual association of one user, one PZ, and one BS.

III. JOINT POWER CONTROL AND COORDINATED SCHEDULING

In this section, the joint coordinated scheduling and power control optimization problem is solved by showing that it is equivalent to a maximum-weight clique problem, which can be globally solved using existing efficient solvers, e.g., [24], [25]. Note that the proposed joint scheduling and power control method presented in this section is centralized. All computation are carried out at the cloud and relayed to the BSs.

The section relies on the fact that the optimal solution of (2) can be reached by solving the problem for an intelligently constructed discrete set of PLs, i.e., replacing constraint (2e) by the constraint $P_{bz} \in \mathcal{P}$, where $\mathcal{P}$ is a set simultaneously constructed with the joint scheduling and power control graph. Hence, the first part of the section illustrates the optimal solution to the following optimization problem:

$$\max \sum_{u,b,z} a_{ubz} X_{ubz} \log_2 (1 + \text{SINR}_{bz}^u(\mathbf{P})) \quad (3a)$$

$$\text{s.t. } Y_{ub} = \min \left(\sum_z X_{ubz}, 1 \right), \forall (u, b) \in \mathcal{U} \times \mathcal{B}, \quad (3b)$$

$$\sum_b Y_{ub} \leq 1, \quad \forall u \in \mathcal{U}, \quad (3c)$$

$$\sum_u X_{ubz} = 1, \quad \forall (b, z) \in \mathcal{B} \times \mathcal{Z}, \quad (3d)$$

$$P_{bz} \in \mathcal{P}, \forall (b, z) \in \mathcal{B} \times \mathcal{Z}, \quad (3e)$$

$$X_{ubz}, Y_{ub} \in \{0, 1\}, \forall (u, b, z) \in \mathcal{U} \times \mathcal{B} \times \mathcal{Z}, \quad (3f)$$

where the optimization is carried over the binary variables X_{ubz} and Y_{ub} and the discrete variable $P_{bz} \in \mathcal{P}$.

In the first subsection, the optimization problem (3) is rewritten in a more insightful form which allows it to be optimally solved, as shown in the second and third subsections. The philosophy of the proposed solution is to use clustering of vertices in a well-defined graph called *local power control graph* or cluster. Each cluster represents an index of a PZ in the network (across the multiple BSs). Inside each cluster, vertices are not connected. However, connections may exist between vertices belonging to different clusters, depending on the schedule feasibility. The graph containing the Z clusters is called the *joint scheduling and power control graph*. The problem is, then, shown to be a maximum-weight clique problem among the cliques of size Z .

In the fourth subsection, the solution of the original optimization problem (2) is shown to be achieved using the method developed for (3) for specific and well constructed set $\mathcal{P}$ of PLs. The last subsection characterizes the complexity of the optimal solution of (2).

A. Problem Reformulation

Let $\mathcal{A}$ be the set of all possible associations between users, base-stations, power-zones, and power levels i.e. $\mathcal{A} = \mathcal{U} \times \mathcal{B} \times \mathcal{Z} \times \mathcal{P}$. Define φ_u as the mapping function from the set $\mathcal{A}$ to the set of users $\mathcal{U}$, i.e., $\varphi_u(y) = u$, $\forall y = (u, b, z, p) \in \mathcal{A}$. In other words, for each association $y \in \mathcal{A}$, the function φ_u returns the index of the user considered in the association. In a similar way, we define $\varphi_b(y) = b$, $\varphi_z(y) = z$ and $\varphi_p(y) = p$, $\forall y = (u, b, z, p) \in \mathcal{A}$. In other words, for each association, the functions φ_b , φ_z and φ_p return the index of the BS, PZ, and PL, respectively, considered in the association.

The power-set $\mathcal{P}(\mathcal{A})$, representing all possible associations between users, base-stations, power-zones, and power levels is also the set of all schedules, regardless if the schedules satisfy the constraints C1, C2, and C3 or not. The set of all feasible schedules can then be characterized as a function of the individual schedules as outlined in the following lemma.

Lemma 1. $\mathcal{F}$, the set of schedules that satisfy constraints C1, C2, and C3, can mathematically be defined as follows:

$$\mathcal{F} = \{\mathbf{S} \in \mathcal{P}(\mathcal{A}) \text{ such that } \forall s \neq s' \in \mathbf{S}$$

$$\delta(\varphi_u(s) - \varphi_u(s'))\varphi_b(s) = \varphi_b(s')\delta(\varphi_u(s) - \varphi_u(s')), \quad (4a)$$

$$(\varphi_b(s), \varphi_z(s)) \neq (\varphi_b(s'), \varphi_z(s')), \quad (4b)$$

$$|\mathbf{S}| = Z_{\text{tot}}\}. \quad (4c)$$

Proof: The proof can be found in Appendix A. ■

Define the function g from $\mathcal{A} \times \mathcal{F}$ to $\mathbb{R}$ as the benefit of each individual association s and the schedule $\mathbf{S}$, i.e.: $g(s, \mathbf{S}) = \log_2 (1 + \text{SINR}_{bz}^u(\mathbf{P}))$, where (u, b, z) is the tuple corresponding to the association s , i.e. $(u, b, z) = (\varphi_u(s), \varphi_b(s), \varphi_z(s))$ and $\mathbf{P}$ is the PLs matrix corresponding to the schedule $\mathbf{S}$, i.e., $\mathbf{P} = [P_{b'z'}]$ such that $(b'z') = (\varphi_b(s'), \varphi_z(s'))$ and $P_{b'z'} = \varphi_p(s')$, $\forall s' \in \mathbf{S}$. Let $a(s)$ be the weight of the association s , i.e., $a(s) = a_{ubz}$ where $s = (u, b, z, p)$. Therefore, the optimization problem (3) can be written as:

$$\max \sum_{s \in \mathbf{S}} a(s)g(s, \mathbf{S}) \quad (5)$$

$$\text{s.t. } \mathbf{S} \in \mathcal{F},$$

B. Local Power Control Graph

In this section, we introduce the local graph called local power control graph. Note that this entire graph is seen as a single cluster in the joint scheduling and power control graph that is introduced in the next subsection.

For each index z of a PZ in the network, such graph is created. Note that throughout the paper, the words *local power control graph* and cluster are used interchangeably. We consider in this subsection, the generation of such graph for PZs indexed by z . Hence the associations that are considered in the generation of the graph are $s \in \mathcal{A}$ such that $\varphi_z(s) = z$.

Let $\mathcal{S}^z$ be the set of all schedules associated with PZ z (feasible or not) defined as follows:

$$\mathcal{S}^z = \{\mathcal{U}, \mathcal{B}, z, \mathcal{P}\}^B \quad (6)$$

From the feasibility condition C2 of a schedule, each PZ should be connected to exactly one user. Since there are B BSs in the network, then there are B PZs indexed by z . Hence, for

each index $z \in \mathcal{Z}$, there exist precisely B associations, which explains the existence of B associations in the definition of $\mathcal{S}^z$ (6). For simplicity of notations, we assume that the i th association in any element of $\mathcal{S}^z$, call it $\mathbf{S}$, is associated with the i th BS in the network. For example, the first association of the schedule $\mathbf{S} \in \mathcal{S}^z$ is $(u, 1, z, p)$.

Given that each user cannot connect to multiple BSs, the same user cannot appear more than once in a feasible schedule. Similarly, the same BS cannot appear more than one for the same reason. Let $\mathcal{F}^z$ be the set of the feasible associations for PZs indexed by z defined by:

$$\begin{aligned} \mathcal{F}^z &= \{\mathbf{S}^z \in \mathcal{S}^z \text{ such that} \\ &\varphi_u(s) \neq \varphi_u(s'), \forall s \neq s' \in \mathbf{S}^z \\ &\varphi_b(s) \neq \varphi_b(s'), \forall s \neq s' \in \mathbf{S}^z\} \end{aligned}$$

Let $\mathcal{G}^z(\mathcal{V}^z, \mathcal{E}^z)$ be the *local power control graph*. This graph is generated by creating a vertex $v^z \in \mathcal{V}^z$ for each possible association $\mathbf{S}^z \in \mathcal{F}^z$. This cluster does not contain any connection, i.e., $\mathcal{E}^z = \emptyset$. The weight of each vertex is the weighted sum rate of the represented association. The weight of vertex v^z associated with the schedule $\mathbf{S}^z$ is:

$$w(v^z) = \sum_{s \in \mathbf{S}^z} a(s)g(s, \mathbf{S}^z). \quad (7)$$

C. Construction of Joint Scheduling and Power Control Graph

To construct the joint scheduling and power control graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, we first generate all the clusters for PZs $z, z \in \mathcal{Z}$. The set of vertices of the joint scheduling and power control graph is the union of all the vertices in all local power control graphs generated in the previous subsection. In other words, we have $\mathcal{V} = \bigcup_{z \in \mathcal{Z}} \mathcal{V}^z$. Two vertices $v^{z_1} \in \mathcal{G}^{z_1}$ and $v^{z_2} \in \mathcal{G}^{z_2}$ belonging to distinct clusters $z_1 \neq z_2$ are connected if their combination results in a feasible schedule. In other words, they are connected if the users they represents are not connected to multiple BSs. In mathematical terms, let $v^{z_1} \in \mathcal{G}^{z_1}$ associated with the schedule $\mathbf{S}^{z_1}$ and $v^{z_2} \in \mathcal{G}^{z_2}$ associated with the schedule $\mathbf{S}^{z_2}$ with $z_1 \neq z_2$. The vertices v^{z_1} and v^{z_2} are connected if:

$$\begin{aligned} &\delta(\varphi_u(s^{z_1}) - \varphi_u(s^{z_2}))\delta(\varphi_b(s^{z_1}) - \varphi_b(s^{z_2})) \\ &= \delta(\varphi_u(s^{z_1}) - \varphi_u(s^{z_2})), \forall (s^{z_1}, s^{z_2}) \in \mathbf{S}^{z_1} \times \mathbf{S}^{z_2} \end{aligned} \quad (8)$$

Consider the graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ as constructed above. Then, define $\mathcal{C}$ as the set of all possible cliques with degree Z . The problem (3) can then be written as a maximum-weight clique problem, as highlighted in the following theorem.

Theorem 1. *The problem of associating users to power-zones and selecting the PL for each PZ (3) can be written as:*

$$\begin{aligned} \mathbf{S}^* &= \arg \max_{\mathbf{S} \in \mathcal{F}} \sum_{s \in \mathbf{S}} a(s)g(s, \mathbf{S}) \\ &= \arg \max_{\mathbf{C} \in \mathcal{C}} \sum_{v \in \mathbf{C}} w(v), \end{aligned} \quad (9)$$

where $\mathbf{C} \in \mathcal{C}$ is a clique in the joint scheduling and power control graph, $\mathcal{C}$ is the set of all possible cliques of degree Z , and $w(v)$ is the weight of each corresponding vertex defined in (7). In other words, the optimal solution of the coordinated scheduling problem (3) is the maximum-weight clique of degree Z .

Proof: The proof can be found in Appendix B ■
maximum-weight clique problems are NP-hard problems, in general. There exist, however, efficient algorithms to solve the problem; see [24], [25] and references therein.

D. Optimal Power Control and Assignment Solution

In this subsection, we provide an efficient method to reach the optimal solution of the original optimization problem (2), using the results developed for the optimization problem (3). We show that for a well constructed discrete set of PLs, we can achieve the optimal solution of the joint scheduling and power optimization problem. Define, for PZ z , the set of all (u, b, z) associations $\tilde{\mathcal{S}}^z$ and feasible schedules $\tilde{\mathcal{F}}^z$ for a cluster $\mathcal{G}^z$ by omitting the PLs as follows:

$$\begin{aligned} \tilde{\mathcal{S}}^z &= \{\mathcal{U}, \mathcal{B}, z\}^B \\ \tilde{\mathcal{F}}^z &= \{\tilde{\mathbf{S}} \in \tilde{\mathcal{S}}^z \text{ such that} \\ &\varphi_u(\tilde{s}) \neq \varphi_u(\tilde{s}'), \forall \tilde{s} \neq \tilde{s}' \in \tilde{\mathbf{S}} \\ &\varphi_b(\tilde{s}) \neq \varphi_b(\tilde{s}'), \forall \tilde{s} \neq \tilde{s}' \in \tilde{\mathbf{S}}\}. \end{aligned} \quad (10)$$

For a fixed schedule $\tilde{\mathbf{S}} = \{\tilde{s}_1, \dots, \tilde{s}_B\} \in \tilde{\mathcal{F}}^z$, the optimal PLs $(p_{1z}^*, \dots, p_{Bz}^*)$ that maximize the weighted sum rate are the solution to the following optimization problem:

$$\begin{aligned} &\max_{p_{bz}} \sum_{b \in \mathcal{B}} a(\tilde{s}_b)g(\tilde{s}_b, \tilde{\mathbf{S}}) \\ &\text{s.t. } 0 \leq p_{bz} \leq P_{bz}^{\max}, \forall b \in \mathcal{B}, \end{aligned} \quad (11)$$

where the optimization is over the power levels p_{bz} , $\forall b \in \mathcal{B}$. The optimization problem (11) is a well known non-convex problem [26], [27]. Despite the non-convexity of the problem, it can be solved efficiently. In [26], the authors propose an algorithm to achieve the optimal solution of (11). Other efficient algorithms (e.g., [27]–[30]) can be used but may only lead to a local optimal solution to the problem. Our scheme can make use of any of such power optimization algorithms to solve problem (11).

In order to solve the joint scheduling and power optimization problem, we first generate the joint scheduling and power control graph as follows: for each PZ $z \in \mathcal{Z}$, a schedule $\tilde{\mathbf{S}} \in \tilde{\mathcal{F}}^z$ is created for all possible associations. The optimal PLs of each association are, then, computed by solving the optimization problem (11). The vertex in the local power control graph $\mathcal{G}^z$ is generated by appending the calculated PLs to the schedule $\tilde{\mathbf{S}}$ whose PLs are omitted as shown in (10). The process is repeated for all PZs $z \in \mathcal{Z}$. The joint scheduling and power control graph $\mathcal{G}$ is generated afterwards by merging all the clusters and adding connections according to condition (8).

The optimal solution to the joint scheduling and power optimization problem is, therefore, obtained by solving the maximum-weight clique problem in $\mathcal{G}$ in which the weight of each vertex is computed using (7). The entire algorithm structure is provided in Algorithm 1. For illustration, the graph $\mathcal{G}$ resulting from Algorithm 1 for a network composed of 2 base-stations, 2 power-zones per base-station and 3 users is shown in Figure 3. The following theorem characterizes the solution reached by Algorithm 1:

Theorem 2. *The optimal solution to the joint scheduling and power optimization problem (2) is achieved using Algorithm 1.*

Algorithm 1 Optimal Scheduling and Power Control Algorithm

Require: $\mathcal{U}, \mathcal{B}, \mathcal{Z}$ and $h_{bz}^u, (u, b, z) \in \mathcal{U} \times \mathcal{B} \times \mathcal{Z}$.

for $z \in \mathcal{Z}$ **do**

Initialize $\mathcal{G}^z = \emptyset$.

for all $\tilde{\mathbf{S}} = \{\tilde{s}_1, \dots, \tilde{s}_B\} \in \tilde{\mathcal{F}}^z$ **do**

Solve (11) to obtain $\mathbf{p} = \{(p_{1z}^*, \dots, p_{Bz}^*)\}$.

Create $v = \{\{\tilde{s}_1, p_{1z}^*\}, \dots, \{\tilde{s}_B, p_{Bz}^*\}\}$.

Compute $w(v)$ using (7).

Set $\mathcal{G}^z = \mathcal{G}^z \cup \{v\}$.

end for

end for

Initialize $\mathcal{G} = \bigcup_{z \in \mathcal{Z}} \mathcal{G}^z$.

Create Connection of $\mathcal{G}$ using (8).

Solve maximum-weight clique in $\mathcal{G}$.

Fig. 3. Joint scheduling and power control graph for a network composed of 2 base-stations, 2 power-zones per base-station and 3 users.

Proof: The proof of can be found in Appendix C ■

E. Complexity Analysis

From Theorem 2, we clearly see that the joint scheduling and power optimization problem is equivalent to a maximum-weight clique, which is an NP-hard problem, and so is the problem (2). The following lemma characterizes the complexity of reaching the optimal solution of (2) using Algorithm 1.

Lemma 2. *The complexity of reaching the optimal solution of (2) using Algorithm 1 is $({}^U P_B) \cdot Z \cdot c(B) + f({}^U P_B) \cdot Z$, where ${}^n P_r = \frac{n!}{(n-r)!}$ is the number of permutations, $c(x)$ is the complexity of solving (11) with x variables and $f(x)$ is the complexity of solving the maximum-weight clique in a graph containing x vertices.*

Proof: The proof of can be found in Appendix D ■

IV. COORDINATED SCHEDULING

Most of the used algorithms in the literature to solve the scheduling and power control problem rely on the decoupling of the variables and iterating between the scheduling and the power control problems. In other words, for a fixed scheduling,

the PLs are determined then, for fixed PLs, the schedule is determined. Iteration between these two problems is carried out until convergence. For a specified user scheduling, the power control problem can be optimally solved as shown in (11). The optimal solution for the coordinated scheduling problem, i.e., for fixed PLs, is, however, not considered in the past literature. Towards that goal, this sections addresses the coordinated scheduling problem by maximizing a generic network utility and shows that the problem can be formulated as a maximum-weight clique problem which can be solved efficiently.

Let $P_{bz}, \forall (b, z) \in \mathcal{B} \times \mathcal{Z}$ be the PL of the b th BS and the z th PZ maintained at a fixed level. Further, let r_{ubz} be a generic network-wide utility of assigning user u to BS b and PZ z . The problem can be written as:

$$\max \sum_{u,b,z} r_{ubz} X_{ubz} \quad (12a)$$

$$\text{s.t. } Y_{ub} = \min \left(\sum_z X_{ubz}, 1 \right), \forall (u, b) \in \mathcal{U} \times \mathcal{B}, \quad (12b)$$

$$\sum_b Y_{ub} \leq 1, \quad \forall u \in \mathcal{U}, \quad (12c)$$

$$\sum_u X_{ubz} = 1, \quad \forall (b, z) \in \mathcal{B} \times \mathcal{Z}, \quad (12d)$$

$$X_{ubz}, Y_{ub} \in \{0, 1\}, \forall (u, b, z) \in \mathcal{U} \times \mathcal{B} \times \mathcal{Z}, \quad (12e)$$

where the optimization is over the binary variables $X_{ubz}, Y_{ub}, \forall (u, b, z) \in \mathcal{U} \times \mathcal{B} \times \mathcal{Z}$, where the constraints in (12b) and (12c) correspond to constraint C1, where the equality constraint in (12d) corresponds to constraint C2.

The solution hinges on the fact that the problem (12) can be reformulated as a maximum-weight clique problem. The section first shows how to build the corresponding scheduling graph, and then reformulates the problem. It also presents efficient heuristic to solve the scheduling problem. Note that, similar to the previous section, the proposed scheduling method presented in this section is centralized in nature. The scheduling solutions are carried out by the centralized processor at the cloud and coordinated to the base-stations.

A. Construction of Scheduling Graph

Let $\mathcal{A}$ be the set of all possible associations between users, base-stations, and power-zones, i.e. $\mathcal{A} = \mathcal{U} \times \mathcal{B} \times \mathcal{Z}$. The scheduling graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ can then be constructed as follows: Generate a vertex v for all possible associations $s \in \mathcal{A}$. Two distinct vertices v_1 and v_2 in $\mathcal{V}$ are connected by an edge in $\mathcal{E}$ if the following conditions hold:

- C1: if $\varphi_u(v_1) = \varphi_u(v_2)$ then $\varphi_b(v_1) = \varphi_b(v_2)$: this condition states that the same user cannot connect to multiple BSs.
- C2: $(\varphi_b(v_1), \varphi_z(v_1)) \neq (\varphi_b(v_2), \varphi_z(v_2))$: this constraint states that two different users cannot be connected to the same PZ.

Figure 4 shows an example of the scheduling graph in a system with $U = 2$ users, $B = 2$ BSs and $Z = 2$ PZs. In this example, each vertex is labeled ubz , where u, b and z represent the indices of users, BSs and PZs respectively. We clearly see

Fig. 4. Example of scheduling graph for 2 users, 2 BSs and 2 PZs.

that the only possible cliques of size $Z_{\text{tot}} = BZ = 4$ are $\{\{111, 112, 221, 222\}, \{121, 122, 211, 212\}\}$.

B. Optimal Assignment Solution

Consider the scheduling graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ associated with the constraints C1 and C2, as constructed in subsection IV-A. Then, define $\mathcal{C}$ as the set of all possible cliques with degree Z_{tot} . The problem (12) can then be written as a maximum-weight clique problem, as highlighted in the following theorem.

Theorem 3. *The scheduling problem of associating users to power-zones (12) can be written as:*

$$\begin{aligned} \mathbf{S}^* &= \arg \max_{\mathbf{S} \in \mathcal{F}} \sum_{s \in \mathbf{S}} r(s) \\ &= \arg \max_{\mathbf{C} \in \mathcal{C}} \sum_{v \in \mathbf{C}} w(v), \end{aligned} \quad (13)$$

where $\mathbf{C} \in \mathcal{C}$ is a clique in the scheduling graph, and $w(v)$ is the weight of each vertex v . In other words, the optimal solution of the scheduling problem (12) is the maximum-weight clique of degree Z_{tot} in the scheduling graph where the weight of each vertex $v \in \mathcal{V}$ associated with $s \in \mathcal{A}$ is defined as:

$$w(v) = r(s). \quad (14)$$

Proof: The proof can be found in Appendix E. ■

The complexity of reaching the optimal solution of (12) boils down to the complexity of solving the maximum-weight clique problem. In other words, the complexity of optimally solving the scheduling only problem is $f(U \cdot B \cdot Z)$, where $f(x)$ is the complexity of solving the maximum-weight clique in a graph containing x vertices.

C. Proposed Heuristic Algorithm

Maximum weight cliques problems are NP-hard problems. For a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, the complexity of solving the maximum weight clique problem is $\alpha^{|\mathcal{V}|}$ wherein $\mathcal{V}$ is the set of vertices and $1 < \alpha < 2$ is the complexity constant [24], [25], [31]. Even though such complexity improves tremendously over the $|\mathcal{V}|^{2|\mathcal{V}|}$ cost of generic binary program solvers, it is still prohibitive for moderate size networks. For example, using the globally convergent algorithm provided in [31], the complexity of the proposed solution is 1.21^{UBZ} .

To solve the maximum-weight clique problem in linear time with the problem size, a simple heuristic is proposed in this subsection. First, construct the graph $\mathcal{G}$. The idea here is to update the schedule $\mathbf{S}$ sequentially by adding the vertex with

Algorithm 2 Heuristic Coordinated Scheduling

Require: $\mathcal{U}, \mathcal{B}, \mathcal{Z}, P_{bz}$, and $h_{bz}^u, \forall (u, b, z) \in \mathcal{U} \times \mathcal{B} \times \mathcal{Z}$
Initialize $\mathbf{S} = \emptyset$.
Construct $\mathcal{G}$.
Compute $w(v), \forall v \in \mathcal{G}$.
while $\mathcal{G} \neq \emptyset$ **do**
 Select $v^* = \arg \max_{v \in \mathcal{G}} w(v)$.
 Set $\mathbf{S} = \mathbf{S} \cup \{v^*\}$
 Set $\mathcal{G} = \mathcal{G}(v^*)$ where $\mathcal{G}(v^*)$ is the sub-graph of $\mathcal{G}$ containing only the vertices adjacent to v^* .
end while
Output $\mathbf{S}$.

TABLE I
SYSTEM MODEL PARAMETERS

Channel Model	SUI-3 Terrain type B
Channel Estimation	Perfect
Background Noise Power	-168.60 dBm/Hz
SINR Gap Γ	0dB
Bandwidth	10 MHz

the highest weight at each step. Then, the graph is updated by removing all vertices not connected to the selected vertex, so as to guarantee that the constraints C1 and C2 are satisfied. The process is repeated until the graph becomes empty. The steps of the heuristic are summarized in Algorithm 2.

V. SIMULATION RESULTS

This section shows the performance of the proposed algorithms in the downlink of a cloud-radio access network, similar to Figure 1. Users are uniformly placed in the network. The number of users, numbers of base-stations, number of power-zones per BS frame, and the maximum power P^{max} vary in the simulations so as to study the methods performance for various scenarios. Additional simulations parameters are summarized in Table I. The scheduling and power allocation schemes considered in this part are:

- Maximum power transmission in which the PLs are set to their maximum value P^{max} .
- Iterative scheduling and power control in which the scheduling problem is solved for a fixed PLs. The PLs are then updated according to the new scheduling. Iteration between the two problems is carried until convergence.
- Optimal joint scheduling and power control using our proposed algorithm.

First, Figure 5 plots the sum-rate in bps/Hz versus the number of users for a CRAN composed of 3 base-stations and 4 power-zones per frame. The figure shows that our proposed solution outperform the iterative approach. It further shows that the optimal scheduling outperforms the heuristic solution for all policies, particularly for large number of users. The gain is due to the fact that as the number of users increases, interference becomes larger, and so the role of coordinated scheduling and power control as an interference mitigation technique becomes more pronounced.

Figure 6 plots the sum-rate in bps/Hz versus the number of power-zones per BS frame, for a CRAN composed of 3 base-stations and 5 users. The optimal joint scheduling and power

Fig. 5. Sum-rate in bps/Hz versus number of users U . Number of base-stations is 3, with 4 power-zones per BS's transmit frame, a maximum transmit power $P^{\max} = -42.60\text{dBm}$, and cell size $C = 500$. The dashed lines illustrate the optimal solution to the maximum-weight clique problem while the solid ones are obtained by solving the problem using Algorithm 2.

Fig. 6. Sum-rate in bps/Hz versus number of power-zones Z . Number of base-stations is 3. Number of users is 5. Maximum transmit power is $P^{\max} = -42.60\text{dBm/Hz}$, and cell size is $C = 500$. The dashed lines illustrate the optimal solution to the maximum-weight clique problem while the solid ones are obtained by solving the problem using Algorithm 2.

control outperforms iterative scheduling and power control scheme as the number of power-zones per frame increases, since the size of the search space becomes larger, which comes in the favour of our optimal algorithm.

Figure 7 and Figure 8 plot the sum-rate in bps/Hz versus the maximum power $P^{\max}$, for a CRAN composed of 3 base-stations, 5 power-zones and 5 users for a big cell network (cell size $C = 500\text{m}$) and a small cell network (cell size $C = 100\text{m}$), respectively. Both figures suggest that our joint optimization scheme outperforms all the other power allocation schemes particularly for large maximum power. The gain can be explained by the fact that as the maximum allowed power increases, the inter base-stations interference increases, which makes the role of joint scheduling and power control algorithm more vital. Similarly, when the cell size decreases, interference become larger. This clearly can be seen

Fig. 7. Sum-rate in bps/Hz versus maximum allowed power $P^{\max}$. Number of base-stations is 3, with 5 power-zones per BS's transmit frame and cell size $C = 500\text{m}$. Number of users is 10.

Fig. 8. Sum-rate in bps/Hz versus maximum allowed power $P^{\max}$. Number of base-stations is 3, with 5 power-zones per BS's transmit frame and cell size $C = 100\text{m}$. Number of users is 10.

from Figure 8 in which the gain of our joint optimization scheme reaches up to 50% in a small cell scenario as compared to a gain of 20% in a big cell network Figure 7.

Figure 9 plots the sum-rate in bps/Hz versus the cell size, for a CRAN composed of 3 base-stations, 5 power-zones, 5 users, and a maximum allowed power $P^{\max} = -26.98\text{dBm/Hz}$. The joint scheduling and power optimization scheme presents a noticeable gain compared to the iterative scheduling and power optimization algorithm particularly at the high interference level. For a small cell network ($50 \leq C \leq 200\text{m}$), our proposed solution achieves an appreciable performance improvement (between 45% for $C = 50\text{m}$ and 20% for $C = 200\text{m}$). For a big cell scenario, the proposed scheme achieves 10% performance improvement over the iterative algorithm and 15% improvement as compared to the maximum power allocation strategy.

Finally, to assess the performance of the proposed scheme in large CRAN, Figure 10 illustrates the sum-rate in bps/Hz versus the number of users for a CRAN composed of 18

Fig. 9. Sum-rate in bps/Hz versus number of cell size C . Number of base-stations is 3, with 5 power-zones per BS's transmit frame, and a maximum power $P^{\max} = -26.98\text{dBm/Hz}$. Number of users is 10.

Fig. 10. Sum-rate in bps/Hz versus number of users U for a large network. Number of base-stations is 18, with 4 power-zones per BS's transmit frame, a maximum transmit power $P^{\max} = -42.60\text{dBm}$, and cell size $C = 500$. The SINR gap is 1dbm.

base-stations and 4 power-zones per frame. The figures shows that the proposed joint scheduling and power optimization scheme outperforms both the iterative and the maximum power approaches, especially as the number of users increases. This can be explained by the fact that, as the number of users increases, the scheduling opportunities with power adaptation increase, which results in a noticeable performance gain.

VI. CONCLUSION

Optimization in cloud radio access networks is an emerging topic of interest of late. This paper considers the joint coordinated scheduling and power optimization problem in cloud-enabled networks. The paper solves the problem of maximizing the sum rate under the practical constraints that each user cannot be served by more than one base-station, but can be served by one or more power-zones within each base-station frame. A graph theoretical approach is used to solve the problem by introducing the joint scheduling and power

control graph formed by several clusters, where every cluster is formed by a set of vertices, each representing the possible association of users, BSs, and PLs for one specific PZ. The problem is, then, formulated as a maximum-weight clique, in which the weight of each vertex is the sum of the benefits of the individual associations belonging to that vertex. Simulation results suggest that the proposed cross-layer scheme provides appreciable performance improvement as compared to other schemes from the literature, especially at high interference level.

APPENDIX A PROOF OF LEMMA 1

Let $\mathbf{S} = \{s_1, \dots, s_{|\mathbf{S}|}\}$ be a schedule satisfying the second constraint C2. The mathematical formulation of this constraint is:

$$(\varphi_b(s_i), \varphi_z(s_i)) \neq (\varphi_b(s_j), \varphi_z(s_j)), \quad \forall 1 \leq i \neq j \leq |\mathbf{S}|$$

$$\left| \bigcup_{i=1}^{|\mathbf{S}|} \{(\varphi_b(s_i), \varphi_z(s_i))\} \right| = Z_{\text{tot}}. \quad (\text{A.1})$$

Since all the $\{(\varphi_b(s_i), \varphi_z(s_i))\}$ are disjoint, we can write:

$$Z_{\text{tot}} = \left| \bigcup_{i=1}^{|\mathbf{S}|} \{(\varphi_b(s_i), \varphi_z(s_i))\} \right| = \sum_{i=1}^{|\mathbf{S}|} \left| \{(\varphi_b(s_i), \varphi_z(s_i))\} \right|$$

$$= \sum_{i=1}^{|\mathbf{S}|} 1 = |\mathbf{S}|. \quad (\text{A.2})$$

Therefore, we can write the following:

$$(\varphi_b(s), \varphi_z(s)) \neq (\varphi_b(s'), \varphi_z(s')), \quad \forall s \neq s' \in \mathbf{S},$$

$$|\mathbf{S}| = Z_{\text{tot}}. \quad (\text{A.3})$$

If $\mathbf{S}$ further satisfies the first constraint C1, we can write:

$$\varphi_u(s_i) = \varphi_u(s_j) \Rightarrow \varphi_b(s_i) = \varphi_b(s_j), \quad \forall 1 \leq i \leq |\mathbf{S}|.$$

The above condition can simply be rewritten as:

$$\delta(\varphi_u(s_i) - \varphi_u(s_j))\varphi_b(s_i) = \varphi_b(s_j)\delta(\varphi_u(s_i) - \varphi_u(s_j)).$$

By definition of the set of associations $\mathcal{A}$, all the PLs are included in the set $\mathcal{P}$. Therefore, all schedules $\mathbf{S} \in \mathcal{P}(\mathcal{A})$ verify C3. Finally, $\mathcal{F}$, defined as the schedules satisfying C1, C2, and C3, can be written as:

$$\mathcal{F} = \{\mathbf{S} \in \mathcal{P}(\mathcal{A}) \text{ such that } \forall s \neq s' \in \mathbf{S}$$

$$\delta(\varphi_u(s) - \varphi_u(s'))\varphi_b(s) = \varphi_b(s')\delta(\varphi_u(s) - \varphi_u(s')),$$

$$(\varphi_b(s), \varphi_z(s)) \neq (\varphi_b(s'), \varphi_z(s')),$$

$$|\mathbf{S}| = Z_{\text{tot}}\}. \quad (\text{A.4})$$

APPENDIX B PROOF OF THEOREM 1

To prove this theorem, we first show that for any feasible schedule, there exists a unique clique in the joint scheduling and power control graph that represents the schedule. We then prove that for any clique of size Z , there exist a unique schedule that is feasible. Hence, we prove that there is a one to one mapping between the set of clique of size Z and the set of the feasible schedule. To conclude the proof we show that the weight of the clique is the objective function (3) to maximize.

Let $\mathbf{S} = \{s_1, \dots, s_{Z_{\text{tot}}}\}$ be a feasible schedule (i.e. $\mathbf{S} \in \mathcal{F}$). This schedule contains $Z_{\text{tot}} = Z \times B$ associations. To prove that $\mathbf{S}$ can be represented by a clique in the joint scheduling and power control graph, we prove that $\mathbf{S}$ can be decomposed into Z subset of associations each representing a vertex in a local power control graph. Finally, we prove that the vertices are connected in the joint scheduling and power control graph.

Let $S_z = \{s \in \mathbf{S} \mid \varphi_z(s) = z\}$, $z \in \mathcal{Z}$ be the set of associations which PZs is indexed by z . From the feasibility conditions of a schedule (4b), we have $|S_z| = B$. The schedule $\mathbf{S}$ is feasible and thus satisfy (4a), (4b) and (4c). Therefore, S_z satisfy (4b) and (4c) which concludes that $S_z \in \mathcal{F}^z$. This proves that S_z is represented by a vertices in the local power control graph $\mathcal{G}^z$.

We now prove that all the vertices S_z , $z \in \mathcal{Z}$ are connected in the joint scheduling and power control graph. given that the schedule satisfies (4a), we clearly see, from the connectivity condition (8) in the joint scheduling and power control graph, that all the vertices are pairwise connected. Therefore, $\mathbf{C} = \{S_1, \dots, S_Z\}$ is a clique of size Z in the joint scheduling and power control graph. The decomposition of $\mathbf{S}$ into clusters S_z , $z \in \mathcal{Z}$ is unique up to index permutation. Hence, the clique representing the schedule is unique. We now prove the converse.

Define $\mathcal{C}$ as the set of cliques of degree Z in the joint scheduling and power control graph $\mathcal{G} = \bigcup_{z \in \mathcal{Z}} \mathcal{G}^z$ where $\mathcal{G}^z$ is the local power control graph indexed by the PZ z . Let $\mathbf{C} = \{v_1, \dots, v_Z\} \in \mathcal{C}$ be a clique. Note that by definition of the local power control graph, each vertex $v_z \in \mathcal{G}^z$ represents B associations. Therefore, $\mathbf{C}$ satisfy condition (4c) since $|\mathbf{C}| = \sum_{z \in \mathcal{Z}} |v_z| = Z \times B = Z_{\text{tot}}$. By construction of the vertices $v_z \in \mathcal{G}^z$, we have $v_z \in \mathcal{F}^z$. Therefore, vertices in the clusters satisfy (4b). Each cluster represents a different PZ, then by extension $\mathbf{C}$ satisfy (4b). The following lemma characterize the vertices included in a clique $\mathbf{C}$ of size Z (i.e. $\mathbf{C} \in \mathcal{C}$).

Lemma 3. *Let $\mathbf{C} = \{v_1, \dots, v_Z\} \in \mathcal{C}$ be a clique of size Z in the joint scheduling and power control graph. There exist a one to one mapping between vertices v_z , $z \in \mathcal{Z}$ and the clusters $\mathcal{G}^z$, $z \in \mathcal{Z}$. In other words, two distinct vertices belongs to two different clusters.*

Proof: The proof can be found in Appendix F ■

Using result of Lemma 3 and the connectivity condition between two distinct clusters (8), clearly, $\mathbf{C}$ satisfy (4a). As a conclusion, $\mathbf{C}$ represents a feasible schedule. The uniqueness of the schedule follows from the unique nature of vertices in each cluster. Therefore, there exist a one to one mapping between the set of feasible schedules and the set of cliques in the graph. We now prove that the weight of the clique is the objective function in (3) to be maximized.

Let $\mathbf{C} = \{v_1, \dots, v_Z\} = \{s_1, \dots, s_{Z_{\text{tot}}}\} = \mathbf{S}$, $v_z \in \mathcal{G}^z$. Let v^z be associated with the schedule $\mathbf{S}^z = \{s_1^z, \dots, s_B^z\} \in \mathcal{F}^z$.

The weight of the clique can be expressed as:

$$\begin{aligned} w(\mathbf{C}) &= \sum_{z=1}^Z w(v_z) \\ &= \sum_{z=1}^Z \sum_{i=1}^B a(s_i^z) g(s_i^z, \mathbf{S}) = \sum_{i=1}^{Z \times B} a(s_i) g(s_i, \mathbf{S}) \\ &= \sum_{i=1}^{Z_{\text{tot}}} a(s_i) g(s_i, \mathbf{S}) = w(\mathbf{S}) \end{aligned} \quad (\text{B.1})$$

Therefore, the optimal solution of the optimization problem (3) can be written as:

$$\begin{aligned} \mathbf{S}^* &= \arg \max_{\mathbf{S} \in \mathcal{F}} \sum_{s \in \mathbf{S}} a(s) g(s, \mathbf{S}) \\ &= \arg \max_{\mathbf{C} \in \mathcal{C}} \sum_{v \in \mathbf{C}} w(v), \end{aligned} \quad (\text{B.2})$$

where $\mathbf{C} = \{v_1, \dots, v_Z\} \in \mathcal{C}$ is a clique in the joint scheduling and power control graph, $\mathcal{C}$ is the set of all possible cliques with degree Z , and $w(v)$ is the weight of each corresponding vertex. In other words, the optimal solution of the optimization problem (3) is the maximum-weight clique of degree Z where the weight of each vertex $v^z \in \mathcal{V}$ associated with $\mathbf{S}^z = \{s_1, \dots, s_B\} \in \mathcal{F}^z$ is:

$$w(v^z) = \sum_{i=1}^B a(s_i^z) g(s_i^z, \mathbf{S}^z) \quad (\text{B.3})$$

APPENDIX C PROOF OF THEOREM 2

First note that given the optimal PLs, the optimal solution can be achieved using Theorem 1. In other words, if $\mathbf{S}^* = \{s_1^*, \dots, s_Z^*\}$ is the optimal solution to the scheduling problem with the optimal power control and if $\{\varphi_p(s_1), \dots, \varphi_p(s_Z)\} \subseteq \mathcal{P}$, then the solution is given by the maximum-weight clique in the joint scheduling and power control graph. Therefore, to prove this theorem, we first show that for a particular PLs, only a subset of vertices in the clusters are needed to achieve the optimal solution. We, then prove that the optimal PLs are generated by Algorithm 1.

Consider a cluster $\mathcal{G}^z$ and define the equivalence relationship $\mathbf{R}$ between two vertices $v^z = \{v_1^z, \dots, v_B^z\}$ and $u^z = \{u_1^z, \dots, u_B^z\}$ in the same cluster as follows:

$$\begin{aligned} \{\varphi_u(v_i^z), \varphi_b(v_i^z), \varphi_z(v_i^z)\} &= \{\varphi_u(u_i^z), \varphi_b(u_i^z), \varphi_z(u_i^z)\} \\ &, 1 \leq i \leq B. \end{aligned} \quad (\text{C.1})$$

In other words, two vertices are associated with $\mathbf{R}$ if they represent the same associations up to a PL change. Clearly, $\mathbf{R}$ is reflexive ($v^z \mathbf{R} v^z$), symmetric ($v^z \mathbf{R} u^z \Rightarrow u^z \mathbf{R} v^z$) and transitive ($v^z \mathbf{R} u^z, u^z \mathbf{R} w^z \Rightarrow v^z \mathbf{R} w^z$). Hence, it is an equivalence relationship. Let $\mathbf{R}(v^z)$ be the equivalence class of v^z . The following Lemma characterizes the vertices of the maximum-weight clique:

Lemma 4. *For each equivalence class $\mathbf{R}(v^z)$, only the vertex with the highest weight is needed to achieve the optimal solution of the scheduling problem (2).*

Proof: The proof can be found in Appendix F ■

To conclude the proof, we show that, for each cluster and each equivalence class in it, Algorithm 1 produces the vertex

with the highest weight. Assume that $\mathbf{S}^* = \{s_1^*, \dots, s_Z^*\}$ is the optimal solution for the scheduling problem with optimal power control. Assume that $\exists j$ such that s_j^* is not generated by the algorithm. Let $\mathbf{R}(s_j^*)$ be the equivalence class of s_j^* . From Lemma 4, we have $w(s_j^*) \geq w(v)$, $\forall v \in \mathbf{R}(s_j^*)$. The vertex generated v^* by the algorithm is the optimal solution to (14), thus $w(v^*) \geq w(v)$, $\forall v \in \mathbf{R}(s_j^*)$. Therefore, we obtain $w(v^*) \geq w(s_j^*)$ but $\mathbf{S}^*$ is the optimal solution then $w(s_j^*) = w(v^*)$. From Lemma 4, we know that all vertices in the same clusters have the same connections, which conclude that the algorithm produces the same solution (if $s_j = v^*$) or a solution with the same weight ($w(s_j^*) = w(v^*)$ and $s_j^* \neq v^*$) in which case the optimal solution is not unique. As a conclusion, the optimal coordinated scheduling solution with optimal power control (2) is achieved using Algorithm 1.

APPENDIX D PROOF OF LEMMA 2

The complexity of reaching the optimal solution using Algorithm 1 can be decomposed in the complexity of generating the joint scheduling and power control graph and the complexity of solving the maximum-weight clique in that graph. We first look at the complexity of producing a single cluster. From Lemma 4, for an arbitrary cluster, the number of vertices is the number of distinct equivalence classes the cluster contains. For cluster z , this figure is equal to the total number of the feasible schedules is $|\tilde{\mathcal{F}}^z|$. Since the number of association each vertex contain is B , which is equal to the number of BSs, then the total number of possible vertices is the number of permutations of the users among these BSs. This number can be computed as $^U P_B$. For each vertex, the optimization problem (14) is run once to calculate the optimal power level. Hence, the complexity of creating a cluster is $^U P_B Z c(B)$. The whole graph contains $^U P_B Z$ which conclude that the total complexity for achieving the optimal solution is $^U P_B Z c(B) + f(^U P_B Z)$.

APPENDIX E PROOF OF THEOREM 3

To prove Theorem 3, we show that there is a one to one map between $\mathcal{F}$ (i.e., the set of feasible schedules) and $\mathcal{C}$ (i.e., the set of cliques of degree Z_{tot} in the scheduling graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$). We first prove that $\forall \mathbf{C} \in \mathcal{C}$, $\mathbf{C}$ satisfies the constraints (4a), (4b) and (4c). Then, we prove the converse: i.e. for each element in $\mathbf{S} \in \mathcal{F}$, there exists an associated clique $\mathbf{C} \in \mathcal{C}$. To conclude the proof, we show that the weight of the clique is equivalent to the merit function of the optimization problem defined in (12).

Let $\mathbf{C} = \{v_1, \dots, v_{|\mathbf{C}|}\}$ be a clique in the scheduling graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ (i.e., $\mathbf{C} \in \mathcal{C}$). Since $\mathbf{C}$ is a clique in $\mathcal{G}$, there exists an edge in $\mathcal{E}$ for every pair of vertices in $\mathcal{V}$. From the first condition C1 of creating an edge between two vertices, we have:

$$\begin{aligned} \varphi_u(v_i) &= \varphi_u(v_j) \text{ and } \varphi_b(v_i) = \varphi_b(v_j), \forall 1 \leq i \neq j \leq |\mathbf{C}|, \\ \delta(\varphi_u(v_i) - \varphi_u(v_j))\varphi_b(v_i) &= \varphi_b(v_j)\delta(\varphi_u(v_i) - \varphi_u(v_j)). \end{aligned}$$

Hence, the clique satisfies (4a). In a similar way, the second condition C2 of the connectivity gives the following:

$$(\varphi_b(v_i), \varphi_z(v_i)) \neq (\varphi_b(v_j), \varphi_z(v_j)), \forall 1 \leq i \neq j \leq |\mathbf{C}|.$$

Therefore, for any $\mathbf{C} = \{v_1, \dots, v_{|\mathbf{C}|}\} \in \mathcal{C}$, we can construct a schedule $\mathbf{S}$, such that $\mathbf{S} = \{s_1, \dots, s_{|\mathbf{C}|}\} \in \mathcal{P}(\mathcal{A})$, where v_i is the vertex associated with each association s_i , $\forall 1 \leq i \leq |\mathbf{C}|$, and where $\mathbf{S}$ satisfies the following constraints:

$$\begin{aligned} \mathbf{S} &\in \mathcal{P}(\mathcal{A}), \text{ and } \forall s \neq s' \in \mathbf{S} \\ \delta(\varphi_u(s) - \varphi_u(s'))\varphi_b(s) &= \varphi_b(s')\delta(\varphi_u(s) - \varphi_u(s')), \\ (\varphi_b(s), \varphi_z(s)) &\neq (\varphi_b(s'), \varphi_z(s')), \\ |\mathbf{S}| &= |\mathbf{C}| = Z_{\text{tot}}. \end{aligned} \quad (\text{E.1})$$

The conclusion in E.1 shows that $\mathbf{S} \in \mathcal{F}$.

In a similar manner, let $\mathbf{S} = \{s_1, \dots, s_{|\mathbf{S}|}\} \in \mathcal{F}$, and let $\mathbf{C} = \{v_1, \dots, v_{|\mathbf{C}|}\}$ where v_i is the vertex associated with s_i , $\forall 1 \leq i \leq |\mathbf{C}|$. Due to conditions (4a) and (4b), each pair of vertices are connected; thus, $\mathbf{C}$ is a clique. Further, (4c) guarantees that the size of the clique is Z_{tot} . This concludes the converse, i.e., $\mathbf{C} \in \mathcal{C}$.

Moreover, the weight of the clique $\mathbf{C} \in \mathcal{C}$ is simply given by:

$$w(\mathbf{C}) = \sum_{i=1}^{|\mathbf{C}|} w(v_i) = \sum_{i=1}^{|\mathbf{C}|} r(s_i), \quad (\text{E.2})$$

where s_i is the association corresponding to vertex v_i . This implies that the weight of the clique is equivalent to the merit function of the optimization problem defined in (12). Hence, the optimal scheduling is given by the maximum-weight clique of degree Z_{tot} in the scheduling graph.

APPENDIX F AUXILIARY LEMMAS

A. Proof of Lemma 3

To prove this Lemma, we first show that for a given schedule to satisfy condition (4b), the local power control graph for any power-zone should not contain any connection.

Assume there exists a connection between two distinct vertices v_i and v_j belonging to the same cluster $\mathcal{G}^z$. By construction of the cluster $\mathcal{G}^z$ each vertex represents B associations satisfying condition (4b) and (4c). These B associations have the same PZs indexed by z and satisfy the constraints. Given that exactly B PZs indexed by z across the entire network that need to be scheduled, all these B PZs indexed by z have been associated with one user. Vertices v_i and v_j are distinct vertices, there exist at least one association that is different and, therefore, there exist a BS whose z th PZ is associated with various users which violate condition (4b).

Let $\mathbf{C} = \{v_1, \dots, v_Z\} \in \mathcal{C}$ be a clique that represents a feasible schedule. Assume that two distinct vertices v_i and v_j , $1 \leq i \neq j \leq Z$ belong to the same cluster. Since these vertices are part of the clique $\mathbf{C}$, then they are connected by an edge. But as shown previously, such connection does not exist otherwise the schedule is not feasible. Therefore, two distinct vertices belongs to two different clusters. The number of vertices in the clique being equal to the number of clusters, there exists a one to one mapping between the vertices of $\mathbf{C} \in \mathcal{C}$ and the clusters $\mathcal{G}^z$, $z \in \mathcal{Z}$.

B. Proof of Lemma 4

Let v^z be a vertex in the cluster $\mathcal{G}^z$ and let $\mathbf{R}(v^z)$ be its equivalence class. To show that only the vertex with the highest weight is needed to achieve the optimal solution, we will show that all vertices belonging to $\mathbf{R}(v^z)$ are connected to the same vertices. Let v^z and u^z be two arbitrary vertices in $\mathbf{R}(v^z)$. From Lemma 3, vertices in the same cluster are not connected to each other. Hence, they have the same connections with vertices in the cluster $\mathcal{G}^z$. We now prove that they have the same connections with vertices in cluster $\mathcal{G}^m$, $m \in \mathcal{Z}$, $m \neq z$. Assume v^z , associated with the schedule $\mathbf{S}^z$, is connected to v^m , associated with the schedule $\mathbf{S}^m$, then from the connectivity condition between two different cluster (8), we have:

$$\begin{aligned} & \delta(\varphi_u(s^z) - \varphi_u(s^m))\delta(\varphi_b(s^z) - \varphi_b(s^m)) \\ &= \delta(\varphi_u(s^z) - \varphi_u(s^m)), \forall (s^z, s^m) \in \mathbf{S}^z \times \mathbf{S}^m \end{aligned} \quad (\text{F.1})$$

Since v^z and u^z are linked with the equivalence relationship $\mathbf{R}$, then

$$\begin{aligned} \{\varphi_u(v_i^z), \varphi_b(v_i^z), \varphi_z(v_i^z)\} &= \{\varphi_u(u_i^z), \varphi_b(u_i^z), \varphi_z(u_i^z)\} \\ &, 1 \leq i \leq B. \end{aligned} \quad (\text{F.2})$$

Substituting (F.2) in (F.1), we obtain:

$$\begin{aligned} & \delta(\varphi_u(s^z) - \varphi_u(t^z))\delta(\varphi_b(s^z) - \varphi_b(t^z)) \\ &= \delta(\varphi_u(s^z) - \varphi_u(t^z)), \forall (s^z, t^z) \in \mathbf{S}^z \times \mathbf{T}^z \end{aligned} \quad (\text{F.3})$$

where $u^z \in \mathbf{R}(v^z)$ associated with the schedule $\mathbf{T}^z$, the above equality concludes that u^z and v^m are connected. Since v^z and u^z play symmetric roles, then the vertices have the same connections with vertices in cluster $\mathcal{G}^m$, $m \in \mathcal{Z}$, $m \neq z$. Finally, we conclude that the two vertices have the same connection in the whole graph.

Assume $\mathbf{S}^* = \{s_1^*, \dots, s_Z^*\}$ is the optimal solution to the scheduling problem and assume that $\exists j$ such that s_j^* is not the vertex with the maximum weight in $\mathbf{R}(s_j^*)$. Let $s_j^\dagger$ be the vertex with highest weight in $\mathbf{R}(s_j^*)$. Since s_j^* and $s_j^\dagger$ have the same connections then $s_j^\dagger$ is connected to all s_i^* , $1 \leq i \neq j \leq Z$. Therefore, $\mathbf{S}^\dagger = (\mathbf{S}^* \setminus s_j^*) \cup s_j^\dagger$ is also a clique of size Z and with a higher weight than $\mathbf{S}^*$ which is in contradiction that $\mathbf{S}^*$ is the optimal solution and thus the maximum-weight clique of size Z . As a conclusion, only the vertices with the highest weight in their equivalence class are needed to reach the optimal solution.

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