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Micro-mechanics based damage mechanics for 3D Orthogonal Woven Composites: Experiment and Numerical Modelling

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Abstract

Damage initiation and evolution of three-dimensional(3D) orthogonal woven carbon fibre composite (3DOWC) is investigated experimentally and numerically. Meso-scale homogenisation of the representative volume element (RVE) is utilised to predict the elastic properties, simulate damage initiation and evolution when loaded in tension. The effect of intra-yarn transverse cracking and shear diffused damage on the in-plane transverse modulus and shear modulus is investigated while one failure criterion is introduced to simulate the matrix damage. The proposed model is based on two major assumptions. First, the effect of the binder yarns, on the in-plane properties, is neglected, so the 3DOWC unit cell can be approximated as a (0°/90°) cross-ply laminate. Second, a micro-mechanics based damage approach is used at the meso-scale, so damage indicators can be correlated, explicitly, to the density of cracks within the material. Results from the simulated RVE are validated against experimental results along the warp (0° direction) and weft (90° direction). This approach paves the road for more predictive models as damage evolution laws are obtained from micro mechanical considerations and rely on few well-defined material parameters. This largely differs from classical damage mechanics approaches in which the evolution law is obtained by retrofitting experimental observations.

Keywords: Carbon fibre; 3-Dimensional reinforcement; Homogenisation; Micro-mechanics

1 Introduction

The thermo-mechanical behaviour of composites can be divided into three main regimes. First, the linear elastic regime, in which no damage is introduced. Then, damage initiation regime in which the stress state of the composite constituents exceeds the permissible stresses for the constituents. This regime also includes the damage evolution of the composite constituents beyond damage initiation. Finally, the failure regime occurs when the maximum degradation of material properties, due to damage propagation, is reached. Similar to laminated and 2D woven composites, simulating the damage evolution and strength prediction of 3D composites has been investigated by researchers using either analytical or numerical models. An analytical model to evaluate the failure strength of three-dimensional(3D) orthogonal woven carbon fibre composite (3DOWC) was developed by Tong *et al.*[1]. In this model, the RVE consists of warp, weft and binder tows. The macroscopic load is transferred to the RVE scale under iso-strain condition. Maximum stress failure criterion is chosen to determine the failure threshold. Longitudinal failure strength is in a good agreement with the experimental values while the transverse strength is not. This is justified by the fact that the model does not take into account the waviness of weft tows in the actual sample. Further development by Tong *et al.*[2] was done to investigate the tow waviness effect on the strength of 3DOWC. The model assumes that a single tow is represented by a curved beam supported by an elastic foundation. Maximum stress failure criterion is again chosen to determine the ultimate strength of the 3D composite. The predicted failure stresses in the fill direction match well the experimental obtained results. In addition parametric study is done to demonstrate the effect of the span length and waviness amplitude of the filler tow on the failure stress in that direction.

Bogdanovich proposed a progressive damage model based on the 3D Mosaic model at the RVE level for 3DOWC[3]. In this model the RVE is divided into blocks of either matrix or unidirectional composite lamina. Again the macro-load is transferred to the micro blocks under the assumption of either iso-strain or iso-stress. And, a maximum strain failure criterion for tension, compression and shear is used. Upon identification of a failure mode, a knock-down factor or in other words, a degradation factor of 0.1 is multiplied by the material property of the previous step. Incremental analysis is done till full reduction of the mechanical properties is reached. Once this occurs, the material is assumed to have reached its ultimate failure strength.

Moving on to some of the models that used CDM for 3DOWC, multi-scale finite element model is introduced by Jia *et al.*[4]. In this model, asymptotic expansion homogenisation is carried out on both the micro and meso-scale RVEs in order to evaluate first the elastic material properties. The RVE on the meso-scale is divided into two sub unit cells inner meso-RVE; and surface meso-RVE. A material constitutive behaviour is defined as a UMAT subroutine in ABAQUS/Standard to describe the damage initiation as well as damage evolution on the micro and meso scales. Modelling results demonstrated good agreement with the experiment.

In the model developed by Visrolia[5], the asymptotic expansion homogenisation is used to correlate the micro-scale stresses to the macro-scale stresses through the determination of the characteristic displacements for the unit cell. Then, stresses in the yarns and matrix are used to determine the damage state and the localised stiffness degradation if damage occurs. Damage growth is defined as a function of the energy release rate. The damage energy release rate is calculated as the derivative of the Gibbs free energy; and the damage indicator is defined as a linear function in the square root of the damage energy release rate. The threshold value for damage initiation is estimated by the authors due to the lack of information about this in literature. Finally, the unit cell constituents' properties are homogenised to get the macro-scale response of the composite. Three different loading conditions are applied “tension, compression and shear” to validate the proposed model. The stress strain response and the failure demonstrate good agreement with the experimental data. In addition, the model has the ability to locate the damage initiation and damage propagation regions in the RVE.

One recent study by Nasution[6] employed the asymptotic expansion homogenisation to calculate the thermo-mechanical properties of 3DOWC using periodical boundary conditions and an in-house FEM code developed in Fortran. Material properties obtained are validated against experimental measurements. Afterwards, localisation analysis is carried out to determine the micro-stress state of the individual constituents due to mechanical and thermal loads. Two mechanical loading states are applied to a cantilever beam model: uniaxial tension and bending. In both cases, thermal residual stresses are computed and superimposed on the mechanical stresses to determine the overall stress state for each constituent “warp, weft, binder and matrix”. The combined thermal and mechanical stress for each constituent is divided by the strength in the corresponding direction. Damage criteria are set according to the normalized stress criteria.

This study concludes that bending is more critical, for this type of composite architecture, than uniaxial load. In addition, incorporating thermal stresses may change the damage mechanisms as it may accelerate damage initiation in some cases. As geometrical distortions play a vital role in determining the mechanical properties of 3D woven composites after manufacturing, Green *et al.*[7] employed voxel method to generate a realistic unit cell model for 3DOWC and compare results obtained with idealised unit cell model. The proposed model accounted for the geometrical distortions due to the preform waviness and compaction during manufacturing. Continuum damage model was used to simulate damage evolution in 3DOWC loaded in tension. However, the damage evolution laws were defined by penalty functions that degrade the elastic properties with two empirical constants that define how fast damage propagates in a constituent. The study concluded that the idealised unit cell model overestimated the stress strain behaviour while the realistic model was in a better agreement with experimental data.

Most of the modelling strategies adopted in the literature take one of three approaches to describe damage evolution and material degradation. Once a failure criterion is met, either knock down factors are introduced [3,4,8–11] or penalty degradation function is defined [7,12]. In these two approaches, fitting parameters/ modelling variables are defined to fit the experimental results obtained. The last approach is the continuum damage modelling (CDM) approach. In this approach, damage is depicted by internal damage variable which has a continuous range from undamaged material ($d=0$) to a completely damaged material ($d=1$). The evolution constitutive relation requires experimental results to calibrate the damage evolution as function of the damage driving force [5]. Thus, all the previously listed models use approaches in which the evolution laws are obtained by retrofitting global experimental observations. However, the objective of this study is to propose a damage evolution model obtained from micro-mechanical considerations and relies on few well-defined material parameters and validate it against experimental results. In this paper, sections 2 and 3 detail the experimental campaign run to characterise mechanically the behaviour of the 3DOWC loaded in two orientations (0° & 90°). Tests are all conducted using Digital Image Correlation (DIC) to obtain a full-field strain distribution over the specimens' surfaces. X-ray computed tomography (X-CT) is implemented to visualise the 3DOWC unit cell after manufacturing. Section 4 is dedicated to detail the damage micromechanics model and its assumptions. Experimental and modelling results are

evaluated in (Section 5); and followed by discussion of the model efficiency and limitations. In Section 6, this discussion leads to the final concluding remarks and recommendations of this study which suggests some improvements for the proposed model. These recommendations will be the focus of future work by the authors.

2 Materials and experimental methods

2.1 Material system and processing

The carbon fibre used in this study is HexTow IM7-12K from Hexcel. The epoxy system used is a medium temperature MTM57 from Cytec. Binder yarns for the 3D orthogonal woven composite (3DOWC) are carbon fibre IM7-6K. The fabrication of the 3DOWC fabric was carried out by Sigmatax UK. The fabric Fig. 1a consists of 8 warp layers, 9 weft layers. The number of warp threads, weft threads and z-binder threads per unit length (centimetre) is specified in as 31.52ends/cm, 38picks/cm and 3.94binders/cm, respectively. In addition, the areal density of the dry 3D woven fabric is 3353GSM (unit is g/m²).

To produce 3D woven composite plates, the dry fabric was infiltrated by MTM 57 epoxy matrix using resin film infusion (RFI) process. Infusion process was carried out at 70°C for 1 h; while curing process was done at 120°C for 1 h. Minimum pressure for the MTM57 was set at 2.8 bars. Once the 3D plates with the size of 300 mm x 200 mm were cured, in order to achieve the objective of studying loading direction sensitivity, plates were cut into two different orientations (see Fig. 1b). Thickness of cured composite plates was 3.6 mm. Samples required for on-axis testing were then cut along 0° direction (warp) and 90° direction (weft).

2.2 Testing and characterisation

2.2.1 Mechanical Testing

Rectangular specimens (250 x 25 mm²) were cut from manufactured panels (300 x 200 mm²). The testing test setup shown in Fig. 2a consists of two main data acquisition channels for load and displacement within Instron 5882. Strains were acquired using digital image correlation (DIC) system utilising SensiCam 12-bit CCD camera (PCO) with TC-2336 bi-telecentric lenses (The Telecentric Company) shown in Fig. 2a. The resolution of CCD camera was 1376x1040 pixels. The bi-telecentric lenses has a depth of field of 11 mm, diameter of 61 mm and magnification of 0.234 x. Fig. 2b shows a typical speckle pattern on a specimen's surface.

CamWare V3.11 software was used to capture the speckle pattern images acquired from the CCD camera.

Tensile testing was conducted in accordance to D 3039 ASTM standards for “Tensile Properties of Polymer Matrix Composite Materials” in a displacement controlled mode with a rate of 2 mm/min. At least three specimens are tested for each orientation (0° and 90°). All tests were performed in a controlled environment where the temperature was 21°C and the relative humidity (RH) was 45%. The speckle images were then processed using Vic2D. In processing these images, the subset size was set to the maximum value of 101x101 pixels adequate for woven specimens. In addition, the step size (distance between subsets) was set to 5 pixels. The observation window of approximately 25x 22 mm² produced an image with dimensions of 1040x896 pixels. Global mean values of strains (ϵ_{xx} ; ϵ_{yy} ; ϵ_{xy}) are obtained from DIC analysis using Vic2D. The fibre volume fraction analysis was carried out using acid digestion method according to ASTM standards (ASTM D 3171) for “Constituent Content of Composite Materials”.

2.2.2 Unit cell visualisation using X-CT:

X-ray CT scan was performed on the orthogonal woven sample using a Zeiss Xradia VersaXRM-510. To obtain a sufficient resolution, a square cross section of (30 mm x 30 mm) was cut from the sample. The 0.4x objective of the scanner was used. The total volume in the field of view was 10 x 10 x 10 mm³, resulting in a voxel size of 5 µm. The source voltage and current were set to 40 kV and 75 µA respectively. The exposure time for each radiograph was 10 s, with 1600 radiographs being collected over 360°. The total data acquisition time was 7 hours. After scanning, the 3D image (Fig. 3a) was reconstructed out of the 2D X-ray slices. To accurately capture the unit cell geometry, all dimensions, used to implement the model (Section 3.2), are measured from the X-CT images. Using a cutting plane parallel to the warp yarns, X-CT section view (Fig. 3b) clearly shows voids and binder distortion, due to RFI process and fabric compaction, which is different from the idealized simulated geometry. The effect of simulating an idealized unit cell model is discussed in (Section 4.3).

3 A damage mechanics approach for 3D orthogonal woven composites

Damage initiation criteria and damage evolution laws have to be specified for both composite constituents', yarns and matrix.

For matrix failure, pressure dependent modified von Mises criterion[7] is used (see Eq. 1)

$$F_m = \frac{S_m^c - S_m^t}{S_m^c S_m^t} (\sigma_1 + \sigma_2 + \sigma_3) + \frac{1}{2S_m^c S_m^t} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2] \quad (1)$$

where σ_i with $i = 1, 2, 3$ are the principal stresses and S_m^j with $j = t, c$ are the tensile and compressive strengths of the matrix. Matrix is assumed to fail once the damage initiation parameter F_m exceeds 1.

For yarns, three damage mechanisms are considered: fibre breakage, transverse damage and shear diffused damaged. The damage initiation criteria assumed are the critical energy release rates for each mechanism (see Section 3.2.3). Fibre breakage is only associated with reduction in longitudinal stiffness. However, transverse cracking and shear diffused damage result in reduction of the transverse and the in-plane shear moduli respectively. Continuum damage mechanics (CDM) is used to derive the damage evolution laws, so the stiffness properties are degraded in a continuous manner as damage propagates in each constituent. Poisson's ratios ($\nu_{12}, \nu_{23}, \nu_{13}$) are assumed to remain constant.

3.1 Model considerations

To derive a damage evolution law for the yarns, the proposed model simplifies the 3D orthogonal woven structure (see Fig. 4a) as a cross-ply laminate. The reason behind this simplification is that previous studies [13–15] have shown that the damage and cracking progression in 3D woven composites is very similar to cross-ply laminates when loaded uniaxially along the warp or the weft directions as detailed in (Section 4.3.1). Another motivation for this assumption is that the in-plane properties of 3D orthogonal woven composite are not significantly affected by the binder removal as demonstrated in Table 1. Only the out-of-plane properties are significantly affected as the binder yarns provide through thickness reinforcement in the first place. Experimental results are provided in the table along with analytical approximation calculated by rule of mixture taking into account the directional fibre volume

fraction in the warp, weft and binder directions. A typical fibre packing factor “volume fraction” inside yarns of ~75% [16] is assumed for the rest of the simulation.

Thus, the model considers a warp/weft/warp laminate as described in Fig. 4b where h_f and h_w are the weft and warp layers thicknesses respectively.

Both the weft and warp layers are assumed to be orthotropic expressed in the form of 4th order stiffness tensors $\underline{\underline{K}}_f^0$ and $\underline{\underline{K}}_w^0$ respectively. The laminate is subjected to a macroscopic in-plane strain $\underline{\underline{\epsilon}}^p$ (with 1 being the fibre direction in the weft layer and 2 is the transverse direction):

$$\underline{\underline{\epsilon}}^p = \epsilon_{11}^p \underline{e}_1 \otimes \underline{e}_1 + \epsilon_{22}^p \underline{e}_2 \otimes \underline{e}_2 + \epsilon_{12}^p (\underline{e}_1 \otimes \underline{e}_2 + \underline{e}_2 \otimes \underline{e}_1) \quad (2)$$

Remark: The thermal residual stresses due to manufacturing process are not described here, however they can be accounted for, if needed, by modifying the effective stress described in Section 3.2.

This laminate is then perturbed by a periodical network of transverse cracks quantified by a dimensionless cracking rate ρ ($\rho = h_f/L$, L being the average inter crack spacing) (Fig. 4c). Those cracks are assumed to span through the thickness of the weft layer and are infinite about the $\underline{e}_1$ direction.

3.2 Micro-mechanics damage model

In this part, the model for the multiplication of transverse and shear damage is derived, taking into account the geometrical and loading assumptions described earlier. Three steps are required for the model derivation. First, the structure of the mechanical fields (displacements, strains and stresses) in the cracked structure is highlighted. Second, the homogenisation step that defines an equivalent damage layer to the cracked one is explained in detail. Finally, the evolution laws, for transverse damage and shear damage under the loading described by Eq. 2, are derived.

3.2.1 Structure of the mechanical fields

Let $(\underline{u}^f, \underline{\varepsilon}^f, \underline{\sigma}^f)$ be the displacement field, the infinitesimal strain field and the Cauchy stress field in the cracked laminate when subjected to $\underline{\varepsilon}^p$ (Fig. 4c). Also, $(\bar{u}, \bar{\varepsilon}, \bar{\sigma})$ are assumed to be the displacement field, the infinitesimal strain field and the Cauchy stress fields in the uncracked laminate (the same structure without cracks) when subjected to $\underline{\varepsilon}^p$ (Fig. 4c). Just as a naming convention, $(\bar{u}, \bar{\varepsilon}, \bar{\sigma})$ are referred to, in this paper, as the “effective” mechanical fields. Cracked surfaces are traction free surfaces ($\underline{\sigma}^f \cdot \underline{n} = \underline{0}$) in the actual damaged weft layer while in the case of non-cracked weft layer, internal traction can be expressed as $(\bar{\sigma} \cdot \underline{n} = \bar{\tau})$. So, the perturbation problem introduced should have the counter traction $(\tilde{\sigma} \cdot \underline{n} = -\bar{\tau})$ experienced by the non-cracked layer to end up simulating the traction free surfaces in the cracked weft layer.

Thus, the solution of the actual cracked laminate $(\underline{u}^f, \underline{\varepsilon}^f, \underline{\sigma}^f)$ can be written as the superposition (Fig. 4c) of the non-cracked laminate solution $(\bar{u}, \bar{\varepsilon}, \bar{\sigma})$ and the periodical perturbation solution $(\tilde{u}, \tilde{\varepsilon}, \tilde{\sigma})$ induced by the transverse cracks such that:

$$\underline{u}^f = \bar{u} + \tilde{u} \quad ; \quad \underline{\varepsilon}^f = \bar{\varepsilon} + \tilde{\varepsilon} \quad ; \quad \underline{\sigma}^f = \bar{\sigma} + \tilde{\sigma} \quad (3)$$

So, the problem now is reduced to solving the residual “perturbation” problem (Fig. 4c) to determine how the stiffness tensor of the weft layer evolves with increasing crack density in the yarns as it will be highlighted in the following section.

3.2.2 Strain energy density and homogenisation of the damaged layer

The homogenisation approach utilised here was initially proposed by [17], and then developed in [18–20] for the analysis of transverse cracking in laminated composites. This technique can be extended to the proposed framework. The key points of this approach is highlighted here, however the reader can refer to [17] for extensive details and related proofs.

Following this technique, the cracked weft layer (see Fig. 5a) can be replaced by an equivalent layer, homogeneous but with mechanical damage (see Fig. 5b). The criterion of this homogenisation process is that the total strain energy of the new laminated structure (see Fig. 5b)

is equal to the strain energy of the initial cracked structure (see Fig. 5a), when subjected to $\underline{\underline{\varepsilon}}^p$.

From [17,18], the strain energy density e_f of the equivalent weft layer is written:

$$e_f = e_f^0 - \left[\frac{f_+(\bar{\sigma}_{11})d_f}{2E_{11f}^0} \bar{\sigma}_{11}^2 + \frac{f_+(\bar{\sigma}_{22})d_t}{2E_{22f}^0} \bar{\sigma}_{22}^2 + \frac{d_s}{2G_{12f}^0} \bar{\sigma}_{12}^2 \right] \quad (4)$$

where:

- e_f^0 is the *effective strain energy density* (the strain energy density of the undamaged weft layer experiencing the effective fields $(\underline{\underline{\varepsilon}}, \underline{\underline{\sigma}})$):

$$e_f^0 = \frac{1}{2} \underline{\underline{\sigma}} : \underline{\underline{K}}_f^0 : \underline{\underline{\sigma}} \quad (5)$$

- $\bar{\sigma}_{11}$ is the *effective longitudinal stress* in the weft layer ($\bar{\sigma}_{11} = \underline{e}_1 \cdot \underline{\underline{\sigma}} \cdot \underline{e}_1$)
- $\bar{\sigma}_{22}$ is the *effective transverse stress* in the weft layer ($\bar{\sigma}_{22} = \underline{e}_2 \cdot \underline{\underline{\sigma}} \cdot \underline{e}_2$)
- $\bar{\sigma}_{12}$ is the *effective shear stress* in the weft layer ($\bar{\sigma}_{12} = \underline{e}_1 \cdot \underline{\underline{\sigma}} \cdot \underline{e}_2$)
- $f_+(\cdot)$ is defined as:

$$\forall x \in R, \quad f_+(x) = \frac{x + |x|}{2} \quad (6)$$

where $|\cdot|$ is the absolute value function and R the real numbers space. This function accounts for crack closure effects (the mechanical damage associated with cracks vanishes when those cracks get closed).

• d_f, d_t, d_s are the internal variable that quantifies longitudinal, transverse and shear damage respectively. $E_{11f}^0, E_{22f}^0, G_{12f}^0$ are the longitudinal, transverse and in-plane shear modulus of the non-cracked weft layer.

Fibre breakage failure is simulated as brittle fracture so direct reduction of longitudinal stiffness is assumed ($d_f = 0.999$) while damage evolution for the transverse and shear damage

mechanisms is detailed here. The derivation details first the damage evolution law for transverse loading (Only $\bar{\sigma}_{22} \neq 0$) followed by the shear loading (Only $\bar{\sigma}_{12} \neq 0$).

For transverse damage, d_t depends on ρ through a bijective function $\mathbf{c}_t$, such that:

$$\forall \rho \in R^+, d_t = \mathbf{c}_t(\rho) \quad (7)$$

Function $\mathbf{c}_t$ is calculated by solving the residual problem defining the periodical perturbation

$(\underline{u}, \underline{\varepsilon}, \underline{\sigma})$ over the periodical cracked cell. $\mathbf{c}_t^{-1}$ is defined as the reciprocal function of $\mathbf{c}_t$, and $\mathbf{c}_t'$ as the derivative of $\mathbf{c}_t$ such that:

$$\forall \rho \in R^+, \mathbf{c}_t^{-1}(\mathbf{c}_t(\rho)) = \rho ; \mathbf{c}_t'(\rho) = \frac{d\mathbf{c}_t}{d\rho}(\rho) \quad (8)$$

From Eq. 4 and 5, e_f can be rewritten in terms of the effective stress to derive the damage evolution law as needed in Section 3.2.3:

$$e_f = \frac{1}{2} \underline{\underline{\sigma}} : \left(\underline{\underline{K}}_f^0{}^{-1} - \frac{f_+(\bar{\sigma}_{22})d_t}{E_{22f}^0} \underline{e}_2 \otimes \underline{e}_2 \right) : \underline{\underline{\sigma}} \quad (9)$$

However, it is also convenient to rewrite the strain energy density as a function of the stress in the equivalent damaged layer, as this defines the effective stiffness of this layer. Let's denote $\underline{\underline{\sigma}}$ the stress in the equivalent damaged layer when the structure is subjected to the in-the-plane extension $\underline{\underline{\varepsilon}}^p$ (Fig. 5b). Then, introducing $\underline{\underline{\bar{K}}}_f^0$ as the plane-stress stiffness operator for the undamaged layer, and $\underline{\underline{\bar{K}}}_f$ as the plane-stress stiffness operator for the damaged layer, stresses can be expressed as:

$$\underline{\underline{\sigma}} = \underline{\underline{\bar{K}}}_f^0 : \underline{\underline{\varepsilon}}^p ; \underline{\underline{\sigma}} = \underline{\underline{\bar{K}}}_f : \underline{\underline{\varepsilon}}^p \quad (10)$$

Introducing Eq. 10 into Eq. 9, we get:

$$e_f = \frac{1}{2} \underline{\underline{\sigma}} : \underline{\underline{\bar{K}_f}}^{-1} : \underline{\underline{\sigma}} \quad (11)$$

where

$$\underline{\underline{\bar{K}_f}} = \underline{\underline{\bar{K}_f^0}} : \left(\underline{\underline{K_f^0}}^{-1} - \frac{f_+(\bar{\sigma}_{22})d_t}{E_{22f}^0} \underline{\underline{e_2}} \otimes \underline{\underline{e_2}} \right) : \underline{\underline{\bar{K}_f^0}} \quad (12)$$

Eq. 12 directly defines the stiffness operator of the equivalent damaged layer as function of the transverse cracks induced damage d_t .

3.2.3 Micro-mechanics based damage evolution

The strain energy density defined by Eq. 9 is now used as a potential to derive the cracking evolution law. First, the damage force Y_{d_t} that drives the damage progression is defined. Then, the damage evolution law, that relates d_t to the damage evolution force Y_{d_t} , is calculated.

- *The damage force:* the damage force is the derivative of e_f with respect to d_t at constant effective stress.

$$Y_{d_t} = - \frac{\partial e_f}{\partial d_t} \Big|_{\underline{\underline{\sigma}}} = f_+(\bar{\sigma}_{22}) \frac{\bar{\sigma}_{22}^2}{2E_{22f}^0} \quad (13)$$

- *The damage evolution:* the cracking evolution is obtained by calculating the energy release rate G of the structure when cracks multiply. This is a classical approach in micro-mechanics of laminated composites[20]. In this case, a continuous definition of G is obtained as function of the continuous variable ρ such that:

$$G(\rho) = - \frac{\partial U}{\partial A}(\rho) \quad (14)$$

U is the potential energy of the structure and A is the total area of cracks. At a constant effective stresses, G can be expressed as function of the total strain energy of the structure, E :

$$G = - \frac{\partial U}{\partial A} = - \frac{\partial E}{\partial A} \Big|_{\underline{\underline{\sigma}}} \quad (15)$$

Remarks: (1) $E = E_f + E_w$, where E_f is the total strain energy in the weft layer and E_w is the total strain energy in the warp layer (2) E_w is constant when the effective stress $\bar{\sigma}$ is kept constant. So,

$$G = -\frac{\partial(E_f + E_w)}{\partial A} \Big|_{\bar{\sigma}} = -\frac{\partial E_f}{\partial A} \Big|_{\bar{\sigma}} \\ = |\Omega_f| \frac{\partial e_f}{\partial d_t} \mathbf{c}_t'(\rho) \frac{d\rho}{dA} = |\Omega_f| \frac{f_+(\bar{\sigma}_{22}) \bar{\sigma}_{22}^2}{2E_{22f}^0} \mathbf{c}_t'(\rho) \frac{d\rho}{dA} \quad (16)$$

where $|\Omega_f|$ is the volume of the weft layer. Considering the relation between the volume of the weft layer, its thickness, the cracking rate and the total cracked area ($A = \rho |\Omega_f| / h_f$), Eq. 16 can be further simplified to:

$$G = h_f \frac{f_+(\bar{\sigma}_{22}) \bar{\sigma}_{22}^2}{2E_{22f}^0} \mathbf{c}_t'(\rho) \quad (17)$$

Cracks multiply when G equals the fracture toughness G_f that represents the resistance of the weft layer to transverse cracking. Then, considering Eq. 13, and from Eq. 17 during cracks multiplication:

$$G_f = h_f Y_{d_t} \mathbf{c}_t'(\rho) \quad (18)$$

The equilibrium during cracking described by Eq. 18 can be turned into a meso scale evolution law for d_t . This would define the following pseudo-potential of dissipation f for the evolution of the meso scale damage variable d_t :

$$f(Y_{d_t}, d_t) = f_1(Y_{d_t}) - f_2(d_t) \quad (19)$$

$$f_1(Y_{d_t}) = Y_{d_t} \quad (20)$$

$$f_2(d_t) = \frac{G_f}{h_f \mathbf{c}_t'(\mathbf{c}_t^{-1}(d_t))} \quad (21)$$

To conclude this part, the transverse cracking evolution can be fully replaced by the calculation of the evolution of the equivalent variable d_t . The evolution law of d_t is a classical damage evolution law based on the pseudo-potential defined by Eq. 19.

Similarly to derive the damage evolution law of shear damage within a yarn, in Eq. 4, normal stress $\bar{\sigma}_{22}$ is set to zero and only shear stress $\bar{\sigma}_{12}$ is considered instead. The shear damage evolution law can be defined in a similar manner to Eq. 19. Here, those damage evolution laws (Fig. 6) are obtained from micro mechanical considerations and rely on few well-defined material parameters. This largely differs from classical damage mechanics approaches in which the evolution law is obtained by retrofitting global experimental observations.

After obtaining the damage evolution laws as function of the micro-cracking rate, they can be correlated with the applied strain (Fig. 7) via the energy release rate [21] for transverse cracking and shear diffused damage respectively. Finally, those damage evolution laws are incorporated into UMAT subroutine in ABAQUS Standard to simulate the damage evolution law for individual yarns.

4 Results and Discussion

4.1 Stress strain curves

The total fibre volume fraction measured experimentally was $51.35 \pm 0.45 \%$ with $2.63 \pm 0.02 \%$ void content. Specimens were tested along the warp 0° and weft 90° directions up to failure. The global stress strain behaviour of the orthogonal woven composite (Fig. 8a) shows a repeatable response of specimens tested along either the warp or the weft direction. However, the material seems to be stiffer along the weft direction as well as having higher strength and strain to failure. The stiffness (Fig. 8a) measured along the weft and warp directions is 68.2 ± 3.71 GPa and 56.2 ± 0.78 GPa respectively. Similarly, the strength along the weft direction is 849 ± 26.14 MPa compared to 704 ± 14.78 MPa along the warp. This is due to the different fibre volume fraction in warp (21.57%) and weft (26.00%) directions.

4.2 DIC Results

Digital Image Correlation (DIC) was mainly used as a non-contact strain measurement technique during testing. The strain contours over specimen's surfaces is plotted in Fig.

8b. However, in the testing of the 3D woven composites it was even more utilised to identify the unit cell for the tested architectures. The orthogonal woven composite has a unit cell (Fig. 8b) with approximate dimensions of 5 x 5 mm. The periodicity of the weaving pattern makes it useful for the modelling and simulation to be verified against the DIC strain contours in case the asymptotic expansion homogenisation “periodical homogenisation” is implemented [4,5,22,23]. One advantage of utilising the DIC is the determination of stress localisation regions which are mainly the interlacement points between the binding yarns and the warp, weft yarns. Those colour maps as well as the DIC images (Fig. 8b) were captured at failure. Although the global strain to failure reported in (Fig. 8a) varies between 1.2 to 1.3 % strain, the stress localisation regions experience much higher strains (~2%). Comparing the strain to failure at the interlacement points against the carbon fibre (IM7) strain to failure can give better understanding of the fracture limit. According to the manufacturer datasheet, the failure strain is 1.9 % which is well below the strain indicated by the DIC strain contours. This suggested that although the overall strain in the test specimen is below the strain to failure of the fibres, this local stress concentration led to the final fracture when tested uni-axially.

4.3 Micro-mechanical modelling

4.3.1 Damage progression

The predicted damage evolution for the 3DOWC unit cell loaded along the warp direction (x-axis) is shown in Fig. 9. Previous studies [13–15] have proven, experimentally, that the damage and cracking progression in 3D woven composites is very similar to cross-ply laminates when loaded uniaxially along the warp or the weft directions. The damage starts first at the interlacement points (Fig. 9a) between the Z-binder and the in-plane yarns “warp and weft”. Following this stress localization effect, in-plane yarns, perpendicular to the loading direction, start experiencing transverse cracking (Fig. 9b-d), which is characterized by cracks spanning the whole yarn length and multiplying in number within the yarn up to a saturation point. The damage mechanism changes to delamination between the in-plane yarns that happens at a very last stage of the loading right before the longitudinal fibre breakage (Fig. 9e-f) of the yarns. Damage progression has been captured with the proposed model and correlates well with the experimental observations reported in literature.

4.3.2 Global stress strain response

Experimental stress strain curves for specimens tested in warp (Fig. 10a) and weft (Fig. 10b) directions are plotted in solid lines while the model prediction is in dashed lines. In both testing directions, the predicted response overestimates the actual response of the material. This is due to the fact that the model does not take into account the geometrical distortions of the 3DOWC due to compaction and yarns waviness.

The global stress strain response of the 3DOWC loaded uni-axially along the warp and the weft direction does not demonstrate significant reduction of the axial modulus. However, the effect of transverse cracking in the weft (Fig. 11a) and warp (Fig. 11b) layers on the elastic material properties of the 3DOWC unit cell suggests that although the matrix cracking doesn't affect the axial modulus, less than 5% (a fibre dominated property), a 20% reduction in shear modulus is observed for the same amount of cracks at failure.

In order to evaluate the efficiency of the model, Table 2 summarises the key comparison parameters as well as the margin of error in the model. Although the model is a simplified model, yet it can capture, with less than 10 % margin of error, the global behaviour of the 3DOWC tested in uniaxial tension. The main advantage of the model is that it only relies on well-defined material parameters with no need for any fitting or calibration constants.

The proposed model does not take into account the effect of voids content as well as the actual compacted structure after manufacturing. In addition, in deriving the damage evolution law for transverse damage and shear damage, it assumes implicitly equal crack spacing by calculating the average micro-cracking rate. Those model limitations can be improved further by implementing an image based FEM model using the reconstructed 3D geometry from the X-CT. This definitely will increase the computational cost but should lead to more accurate and conservative results. Using voxel meshing technique in this model is not the most suitable approach to capture the damage initiation and localisation as sharp edges are susceptible to stress concentration. Some techniques for smoothing those stress concentrations out and effectively characterise damage on the meso-scale level have been investigated in [24,25]. To account for viscoelastic nature of the matrix, more advanced plasticity models should be incorporated as well. The scope of this work was to establish the basis for the damage evolution laws as function

of the material parameters of the yarns and the recommended model improvements will be the focus of a future work for off-axis tension simulation.

5 Concluding remarks

The aim of this study was to provide a predictive micro-mechanical model capable of simulating the tensile response of 3DOWC loaded along the warp or the weft direction. The proposed model differs from previous models introduced in the literature in the sense that damage evolution laws are defined only using material parameters rather than using penalty functions or fitting degradation schemes. The effect of transverse cracking and shear diffused damage on the in-plane transverse modulus and shear modulus was investigated for yarns while one failure criterion was introduced to simulate the matrix damage “cracking”. Although the proposed model simplifies the 3DOWC unit cell as a cross-ply laminate, deviation from experimental results was in the range of 10 % difference and managed to capture the damage evolution in the binder, weft and warp yarns as proposed by experimental work in the literature. DIC was used to evaluate the surface strain contour during testing. The strain contour distribution demonstrated clearly the stress localisation at interlacement points and periodicity of the weaving architecture. In addition, X-CT scan was conducted on undamaged specimen to visualise the “as-manufactured” panel and capture all the geometrical distortions due to waviness and fabric compaction. The predicted values from the model overestimated the experimental response of the tested specimens as it did not include the effect of geometrical distortions and voids content. This opens a room for improvement of the model using image-based FEM and incorporating plasticity models to account for the viscoelasticity of the matrix. This study provides more efficient and predictive capabilities to existing unit cell models, which in turn reduces the need for extensive experimental characterisation, with a high level of confidence in the modelling results.

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Table 1 Elastic properties of 3D orthogonal woven composite: Experimental vs Analytical

	Experiment	With Binder	Without Binder
E1 (GPa)	56.2 ± 0.78	58.54	58.29
E2 (GPa)	68.2 ± 3.71	69.17	68.92
E3 (GPa)	---	15.82	6.51
G12 (GPa)	2.98 ± 0.04	3.30	3.17
ν_{12}	---	0.31	0.32

Table 2 Comparison between experimental and modelling results

Parameter	Warp			Weft		
	Experiment	Model	Error (%)	Experiment	Model	Error (%)
Young's Modulus (GPa)	56.2 ± 0.78	61.2	8.9	68.2 ± 3.71	73.4	7.3
Tensile Strength (MPa)	704 ± 14.78	745	5.9	849 ± 26.14	879	3.5
Failure Strain (%)	1.19 ± 0.01	1.26	5.8	1.26 ± 0.03	1.21	3.9