Multi-Parameter Estimation for Orthorhombic Media

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SUMMARY

Building reliable anisotropy models is crucial in seismic modeling, imaging and full waveform inversion. However, estimating anisotropy parameters is often hampered by the trade off between inhomogeneity and anisotropy. For instance, one way to estimate the anisotropy parameters is to relate them analytically to traveltimes, which is challenging in inhomogeneous media. Using perturbation theory, we develop traveltime approximations for orthorhombic media as explicit functions of the anellipticity parameters η_1 , η_2 and a parameter $\Delta\gamma$ in inhomogeneous background media. Specifically, our expansion assumes inhomogeneous ellipsoidal anisotropic background model, which can be obtained from well information and stacking velocity analysis. This approach has two main advantages: in one hand, it provides a computationally efficient tool to solve the orthorhombic eikonal equation, on the other hand, it provides a mechanism to scan for the best fitting anisotropy parameters without the need for repetitive modeling of traveltimes, because the coefficients of the traveltime expansion are independent of the perturbed parameters. Furthermore, the coefficients of the traveltime expansion provide insights on the sensitivity of the traveltime with respect to the perturbed parameters. We show the accuracy of the traveltime approximations as well as an approach for multi-parameter scanning in orthorhombic media.

INTRODUCTION

With wide azimuth 3D surveys becoming very common, it is becoming more apparent that transverse isotropy (TI) is often insufficient to characterize many of the geological settings needed to describe the wavefields we observe, and as result, orthorhombic anisotropy has emerged as a better approximation for complicated structures specifically in describing sedimentary basins with parallel vertical cracks (Schoenberg and Helbig (1997), Bakulin et al. (2000), Tsvankin (2012)). As a result, we are now facing more challenging objectives, chief among them is the estimation of the anisotropy parameters corresponding to the orthorhombic model.

Luckily, one way to estimate anisotropy parameters is using traveltimes. Under the acoustic approximation, traveltime computation for orthorhombic symmetry depends on six parameters (Tsvankin (1997), Alkhalifah (2003)): the symmetry-plane NMO velocities, v_1 and v_2 , the vertical velocity v_v , the anellipticity parameters η_1 and η_2 and the δ parameter. Traveltimes are generally evaluated by solving a nonlinear partial differential equation referred to as the eikonal equation. Solving the orthorhombic eikonal equation is very difficult because it requires finding roots of a sextic polynomial at each computational step in a finite difference implementation. Recently, Waheed et al. (2014) alleviated the complexity of solving this eikonal equation by iteratively evaluating a sequence of simpler elliptically anisotropic eikonal equations.

Estimating anisotropy parameters is possible through a scan search, but this requires repetitive modeling of traveltimes. Previously, Alkhalifah (2011) proposed an eikonal based scanning approach to search for the anisotropy parameter η in vertical TI media, which provides the best traveltime fit to the data in a general inhomogeneous background medium. Later, Alkhalifah (2013) and Masmoudi and Alkhalifah (2014) developed this concept to approximate traveltimes in horizontal TI media with arbitrary symmetry-axis azimuth ϕ and estimated the anisotropy parameter η and the azimuthal angle ϕ . Furthermore, Masmoudi and Alkhalifah (2015) derived traveltime approximations for orthorhombic media as explicit functions of the anisotropy parameters. This is accomplished by perturbing the solution of the orthorhombic eikonal equation with respect to the anellipticity parameters η_1 , η_2 and a parameter $\Delta\gamma$ from a generally inhomogeneous, ellipsoidal background medium. Though Masmoudi and Alkhalifah (2015) equations are developed for a general inhomogeneous background medium, the analytical description of the traveltimes for the homogeneous orthorhombic case provides highly accurate description of reflection traveltimes. They can also be used to estimate the anisotropy parameters.

In this abstract, we further assess the accuracy of Masmoudi and Alkhalifah (2015) traveltime approximations in inhomogeneous orthorhombic models. We also propose an approach to invert for the perturbed parameters through a scan search and demonstrate the efficiency of the method on the realistic 3D SEG-EAGE salt model as well as a three layer orthorhombic model.

THEORY

The eikonal equation for orthorhombic media, under the acoustic assumption, can be written as (Alkhalifah (2003), Ibanes-Jacome et al. (2014)):

$$\begin{aligned} A \left(\frac{\partial \tau}{\partial x} \right)^2 + B \left(\frac{\partial \tau}{\partial y} \right)^2 + C \left(\frac{\partial \tau}{\partial z} \right)^2 + D \left(\frac{\partial \tau}{\partial x} \right)^2 \left(\frac{\partial \tau}{\partial y} \right)^2 \\ + E \left(\frac{\partial \tau}{\partial x} \right)^2 \left(\frac{\partial \tau}{\partial z} \right)^2 + F \left(\frac{\partial \tau}{\partial y} \right)^2 \left(\frac{\partial \tau}{\partial z} \right)^2 \\ + G \left(\frac{\partial \tau}{\partial x} \right)^2 \left(\frac{\partial \tau}{\partial y} \right)^2 \left(\frac{\partial \tau}{\partial z} \right)^2 = 1, \end{aligned} \quad (1)$$

where $\tau(x, y, z)$ is the traveltime measured from the source to a point with coordinates (x, y, z) . The following notations are used for simplification:

$$\begin{aligned} A &= v_1^2(1 + 2\eta_1), \quad B = v_2^2(1 + 2\eta_2), \quad C = v_v^2, \\ D &= v_1^2(1 + 2\eta_1) \left((1 + 2\eta_1)\gamma^2 v_1^2 - (1 + 2\eta_2)v_2^2 \right), \\ E &= -2\eta_1 v_1^2 v_v^2, \quad F = -2\eta_2 v_2^2 v_v^2, \\ G &= -v_v^2 v_1^2 \left((1 + 2\eta_1)^2 v_1^2 \gamma^2 - 2(1 + 2\eta_1)v_1 v_2 \gamma + v_2^2(1 - 4\eta_1 \eta_2) \right). \end{aligned}$$

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Here v_v is the P-wave vertical velocity, v_1 and v_2 are the NMO P-wave velocities for horizontal reflectors in the $[x, z]$ and $[y, z]$ planes, respectively, η_1 and η_2 are the anellipticity anisotropy parameters in the $[x, z]$ and $[y, z]$ planes, respectively, and γ is related to Thomsen (1986) δ parameter defined in the $[x, y]$ plane through the relation $\gamma = \sqrt{1 + 2\delta}$.

Numerically solving equation (1) is very hard because this requires finding roots of a sextic polynomial at each computational step in a finite difference implementation scheme. However, traveltime computation for the simple ellipsoidal anisotropic (EA) model is cheaper and more efficient (Waheed et al. (2014)). In fact, by setting $\eta_1 = 0$, $\eta_2 = 0$ and $\gamma = \frac{v_2}{v_1}$, equation (1) reduces to the eikonal equation for an EA medium:

$$v_1^2 \left(\frac{\partial \tau}{\partial x} \right)^2 + v_2^2 \left(\frac{\partial \tau}{\partial y} \right)^2 + v_v^2 \left(\frac{\partial \tau}{\partial z} \right)^2 = 1. \quad (2)$$

Instead of solving directly the six order nonlinear partial differential equation (1), we solve it approximately by solving a series of simpler linear equations using perturbation theory. By considering η_1 , η_2 and $\Delta\gamma = \gamma - \gamma_0$ (where $\gamma_0 = \frac{v_2}{v_1}$) to be constant and small, we expand the traveltime solution as a series expansion in η_1 , η_2 and $\Delta\gamma$. However, the velocities v_1 , v_2 and v_v are allowed to vary freely representing an inhomogeneous background medium. Specifically, by substituting the following solution:

$$\begin{aligned} \tau(x, y, z) \approx & \tau_0(x, y, z) + \tau_{\eta_1}(x, y, z) \eta_1 + \tau_{\eta_2}(x, y, z) \eta_2 \\ & + \tau_{\eta_1^2}(x, y, z) \eta_1^2 + \tau_{\eta_2^2}(x, y, z) \eta_2^2 + \tau_{\eta_1 \eta_2}(x, y, z) \eta_1 \eta_2 \\ & + \Delta\gamma \tau_{\Delta\gamma}(x, y, z). \end{aligned} \quad (3)$$

into the eikonal equation (1), we obtain linear first-order partial differential equations having the following general form:

$$v_1^2 \frac{\partial \tau_0}{\partial x} \frac{\partial \tau_i}{\partial x} + v_2^2 \frac{\partial \tau_0}{\partial y} \frac{\partial \tau_i}{\partial y} + v_v^2 \frac{\partial \tau_0}{\partial z} \frac{\partial \tau_i}{\partial z} = f_i(x, y, z) \quad (4)$$

with $i = \eta_1, \eta_2, \eta_1^2, \eta_2^2, \eta_1 \eta_2, \Delta\gamma$ and τ_0 satisfies the eikonal equation (2) for an EA background model where $f_0 = 1$. Due to the small influence that the parameter δ has on reflection seismic data, only the first order term $\tau_{\Delta\gamma}$ is considered in the expansion (3). As shown in Appendix A, the function $f_i(x, y, z)$ becomes more complicated for i corresponding to the second order term and it depends on terms for the first-order and background solutions. Thus these equations must be solved in succession. To improve the accuracy of the traveltime expansion (3), Masmoudi and Alkhalifah (2015) proposed to use Shanks transform (Bender and Orszag, 1978). Specifically, the following hybrid form of traveltime approximations is very efficient:

$$\tau(x, y, z) = \tau_1(x, y, z) \cos^2 \alpha + \tau_2(x, y, z) \sin^2 \alpha \quad (5)$$

where $\tau_1(x, y, z)$, respectively $\tau_2(x, y, z)$, is a Shanks transform expansion with respect to η_1 , respectively η_2 and α is the source to receiver azimuth with respect to the acquisition frame (see Masmoudi and Alkhalifah (2015) for more details).

SCANNING FOR EFFECTIVE PARAMETERS

To scan for effective η_1 , η_2 and $\Delta\gamma$, the coefficients τ_i need to be evaluated only once, as they do not depend on the perturbed

parameters, and can be used with equation (5) to search for the best traveltime fit extracted from the data. For instance, the minimum of the following root-mean-square error (RMSE)

$$\text{RMSE}(\eta_1, \eta_2, \Delta\gamma) = \sqrt{\frac{\sum_{i,j}^{n_1, n_2} (\widehat{\tau}_{ij}(\eta_1, \eta_2, \Delta\gamma) - \tau_{ij})^2}{n_1 \times n_2}} \quad (6)$$

can be used to predict the best fitting parameters η_1 , η_2 and $\Delta\gamma$. In equation (6), τ_{ij} and $\widehat{\tau}_{ij}$ are respectively the exact (observed) and approximated (predicted) traveltimes at the i th inline and j th crossline location of a recording receiver, n_1 and n_2 are the two recording directions.

NUMERICAL EXAMPLES

In this section, we test the accuracy of the traveltime approximation (5) and demonstrate its utility in estimating effective parameters in complex media. Specifically, we show applications of our approach on two models. The first example is the 3D SEG-EAGE salt body model extended to an orthorhombic media. This model is interesting since seismic waves are diffracted at the boundaries of the sharp salt body and the diffracted energy recorded on the surface can be used to estimate the anisotropy parameters. The model is also convenient to test the accuracy of the traveltime approximation (5) as well as to pinpoint the importance of the sensitivity of these parameters by analyzing the traveltime coefficients of the expansion. The second example is a 3D layer orthorhombic model with which we estimate the effective parameters at each layer and assess the accuracy of the inverted interval parameters.

SEG-EAGE Salt model

The orthorhombic model considered in this example is described by the inhomogeneous vertical velocity v_v as shown in figure (1) as well as five homogeneous anisotropy parameters given as follows: $\eta_1 = 0.15$, $\eta_2 = 0.1$, $\delta_1 = 0.1$, $\delta_2 = 0.15$, $\delta = 0.1$. Here, δ_1 and δ_2 are two Thomsen (1986) parameters defined respectively in the $[x, z]$ and $[y, z]$ planes. We consider

Figure 1: Velocity of the SEG-EAGE salt model

one scattering point located at $(x = 3.6 \text{ km}, y = 3.6 \text{ km}, z = 1.4 \text{ km})$. We solve numerically the coefficients of the traveltime expansion given in Appendix A. Figure 2 shows the maps of the coefficients τ_{η_1} , τ_{η_2} , $\tau_{\eta_1^2}$, $\tau_{\eta_2^2}$, $\tau_{\eta_1 \eta_2}$ and $\tau_{\Delta\gamma}$. The behavior of these coefficients is interesting as it reveals the sensitivity of the traveltimes with respect to each parameter. For instance, figures 2(a) and 2(c) corresponding to the coefficients τ_{η_1} and

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Figure 2: Coefficients of the traveltime expansion (3) for the SEG-EAGE model with a source located at $(x = 3.6 \text{ km}, y = 3.6 \text{ km}, z = 1.4 \text{ km})$, respectively τ_{η_1} (Fig. 2(a)), τ_{η_2} (Fig. 2(b)), $\tau_{\eta_1^2}$ (Fig. 2(c)), $\tau_{\eta_2^2}$ (Fig. 2(d)), $\tau_{\eta_1\eta_2}$ (Fig. 2(e)), $\tau_{\Delta\gamma}$ (Fig. 2(f)).

$\tau_{\eta_1^2}$, respectively, show that corrections to the background traveltimes τ_0 increase with larger x-offset while they remain almost constant in the y-offset direction. This is expected since η_1 is defined in the $[x, z]$ plane. Similar conclusions could be made on the behavior of τ_{η_2} and $\tau_{\eta_2^2}$. More detailed analysis of the behavior of the coefficients will be addressed during the presentation.

Furthermore, we compute the exact (numerically) traveltimes recorded on the surface using the iterative approach proposed by (Waheed et al., 2014) and we invert for the perturbed parameters η_1 , η_2 and $\Delta\gamma$ using the traveltime approximations (5) and the scanning approach (6). Figure (3) shows the RMSE as a function of η_1 , η_2 and $\Delta\gamma$. In this case, a step of 0.01 unit is used for the three perturbed parameters for the scanning. Clearly, figure (3) indicates that the best fitting anisotropy parameters η_1 and η_2 are respectively 0.15 and 0.09, which is consistent with the exact values. Also, figure (3) reveals that the minimum RMSE is not clearly located in the $\Delta\gamma$ panel, which confirms the small influence of δ on the traveltimes. Yet, the minimum RMSE indicates a value of $\Delta\gamma = 0.04$ corresponding to $\delta = 0.084$, which is very close to the exact value.

Figure 3: Root-Mean-Square error between the exact and approximated traveltimes as a function of η_1 , η_2 and $\Delta\gamma$.

Using the recovered anisotropy parameters, we compare the accuracy of the new traveltime approximations with more accurate traveltimes (Waheed et al., 2014). Figure (4) shows a remarkable fit between the traveltimes, keeping in view the fact that the strength of anisotropy is not negligible.

Figure 4: Traveltime contours for the SEG-EAGE salt model using the approximated traveltimes with the recovered anisotropy parameters (dashed lines) and the accurate traveltimes computation (Waheed et al., 2014) (solid lines).

The above example demonstrate the potential of equation (5) in yielding highly accurate traveltimes and its role in estimating the anisotropy parameters in complex media.

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Three-layer Orthorhombic model

The three layer model considered here has the following interval parameter values:

	v_v	v_1	v_2	η_1	η_2	δ
Layer 1	1800	1900	2000	0.1	0.05	0.1
Layer 2	1900	2000	2100	0.15	0.1	0.15
Layer 3	2000	2100	2200	0.2	0.15	0.2

The size of the model is 4km $\times$ 4km laterally and 3km in depth and the thickness of each layer is 1km. A source is considered at the top of the model ($x = 0$ km, $y = 0$ km, $z = 0$ km) and the receivers are placed on the surface. The idea here is to use the reflection traveltimes at the 3 layer boundaries to infer the effective parameters. Once we estimate the effective parameters at each layer, we convert them to their interval values. The results are summarized in the following table:

	η_{1eff}	η_{1int}	η_{2eff}	η_{2int}	$\Delta\gamma_{eff}$
Layer 1	0.1	0.1	0.05	0.05	0.042
Layer 2	0.131	0.143	0.087	0.102	0.066
Layer 3	0.166	0.191	0.12	0.144	0.089

For the sake of comparison, we only compute the interval parameters η_{1int} and η_{2int} from the recovered effective η_{1eff} and η_{2eff} using the VTI Dix equations (Grechka et al., 1999) as the vertical planes in orthorhombic media can be approximated by the corresponding VTI equations. The resulting interval parameters turn out to be very close to the true interval values.

DISCUSSION

The proposed traveltime perturbation is a shortcut that provides traveltimes for various models using an analytic formula. In many applications such as parameter estimation or travel-time tomography, the cost saving associated with traveltime computation is significant. Moreover, developing the background velocity model for such perturbation can be obtained using stacking velocity analysis and well velocities. Since surface seismic data is not very sensitive to Thomsen δ parameter, the EA background model can be limited to an isotropic model. Furthermore, the tilt angle can be estimated from the model structure and incorporated to the background model without extra cost. Finally, the azimuthal angle can also be estimated with a similar approach as proposed by Masmoudi and Alkhalifah (2014).

CONCLUSION

By using an expansion of the traveltime solution of the orthorhombic eikonal equation in terms of the anelasticity parameters η_1 , η_2 and the parameter $\Delta\gamma$, we developed an efficient tool to estimate these parameters in a generally inhomogeneous background medium.

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APPENDIX A

HIGHER ORDER EXPANSIONS

By inserting the traveltime expansion (3) into the eikonal equation (1) and equating the coefficients of the powers of the independent parameters η_1 , η_2 and $\Delta\gamma$ in succession starting with the first power of the parameters, we obtain linear partial differential equations having the general form as given in equation (4). The source function of the first power term in η_1 and zeroth power in η_2 and $\Delta\gamma$ is given by:

$$f_{\eta_1} = v_1^2 \left(v_v^2 \left(\frac{\partial \tau_0}{\partial z} \right)^2 \left(\frac{\partial \tau_0}{\partial x} \right)^2 - v_2^2 \left(\frac{\partial \tau_0}{\partial y} \right)^2 \left(\frac{\partial \tau_0}{\partial x} \right)^2 - \left(\frac{\partial \tau_0}{\partial x} \right)^2 \right). \quad (A-1)$$

The source function of the first power term in η_2 and zeroth power in η_1 and $\Delta\gamma$ is given by:

$$f_{\eta_2} = v_2^2 \left(v_v^2 \left(\frac{\partial \tau_0}{\partial z} \right)^2 \left(\frac{\partial \tau_0}{\partial y} \right)^2 + v_1^2 \left(\frac{\partial \tau_0}{\partial y} \right)^2 \left(\frac{\partial \tau_0}{\partial x} \right)^2 - \left(\frac{\partial \tau_0}{\partial y} \right)^2 \right). \quad (A-2)$$

The source function of the first power term in $\Delta\gamma$ and zeroth power in η_1 and η_2 is given by:

$$f_{\Delta\gamma} = -v_1^3 v_2 \left(\frac{\partial \tau_0}{\partial y} \right)^2 \left(\frac{\partial \tau_0}{\partial x} \right)^2. \quad (A-3)$$

The source function of the second power term in η_1 and zeroth power in η_2 and $\Delta\gamma$ is given by:

$$\begin{aligned} f_{\eta_1^2} = & -\frac{1}{2} \left(v_v^2 \left(\frac{\partial \tau_{\eta_1}}{\partial z} \right)^2 + v_1^2 \left(\frac{\partial \tau_{\eta_1}}{\partial x} \right)^2 + v_2^2 \left(\frac{\partial \tau_{\eta_1}}{\partial y} \right)^2 \right) \\ & + 2v_1^2 \left(\frac{\partial \tau_0}{\partial x} \right)^2 \left(-v_2^2 \frac{\partial \tau_0}{\partial y} \frac{\partial \tau_{\eta_1}}{\partial y} + v_v^2 \frac{\partial \tau_0}{\partial z} \frac{\partial \tau_{\eta_1}}{\partial z} \right) \\ & + 2v_1^2 \frac{\partial \tau_0}{\partial x} \frac{\partial \tau_{\eta_1}}{\partial x} \left(v_v^2 \left(\frac{\partial \tau_0}{\partial z} \right)^2 - v_2^2 \left(\frac{\partial \tau_0}{\partial y} \right)^2 - 1 \right) \\ & + 2v_1^2 v_2^2 \left(\frac{\partial \tau_0}{\partial x} \right)^2 \left(\frac{\partial \tau_0}{\partial y} \right)^2 \left(v_v^2 \left(\frac{\partial \tau_0}{\partial z} \right)^2 - 1 \right). \end{aligned} \quad (A-4)$$

The source function of the second power term in η_2 and zeroth power in η_1 and $\Delta\gamma$ is given by:

$$\begin{aligned} f_{\eta_2^2} = & -\frac{1}{2} \left(v_v^2 \left(\frac{\partial \tau_{\eta_2}}{\partial z} \right)^2 + v_1^2 \left(\frac{\partial \tau_{\eta_2}}{\partial x} \right)^2 + v_2^2 \left(\frac{\partial \tau_{\eta_2}}{\partial y} \right)^2 \right) \\ & + 2v_2^2 \frac{\partial \tau_0}{\partial y} \frac{\partial \tau_{\eta_2}}{\partial y} \left(v_v^2 \left(\frac{\partial \tau_0}{\partial z} \right)^2 + v_1^2 \left(\frac{\partial \tau_0}{\partial x} \right)^2 - 1 \right) \\ & + 2v_2^2 \left(\frac{\partial \tau_0}{\partial y} \right)^2 \left(v_1^2 \frac{\partial \tau_0}{\partial x} \frac{\partial \tau_{\eta_2}}{\partial x} + v_v^2 \frac{\partial \tau_0}{\partial z} \frac{\partial \tau_{\eta_2}}{\partial z} \right). \end{aligned} \quad (A-5)$$

Finally, the source function of the first power term in both η_1 and η_2 and zeroth power in $\Delta\gamma$ is given by:

$$\begin{aligned} f_{\eta_1 \eta_2} = & -v_1^2 \frac{\partial \tau_{\eta_2}}{\partial x} \left(2 \frac{\partial \tau_0}{\partial x} + \frac{\partial \tau_{\eta_1}}{\partial x} \right) - v_2^2 \frac{\partial \tau_{\eta_1}}{\partial y} \left(2 \frac{\partial \tau_0}{\partial y} + \frac{\partial \tau_{\eta_2}}{\partial y} \right) \\ & - v_v^2 \frac{\partial \tau_{\eta_1}}{\partial z} \frac{\partial \tau_{\eta_2}}{\partial z} + 2v_v^2 \left(\frac{\partial \tau_0}{\partial z} \right)^2 \left(v_1^2 \frac{\partial \tau_0}{\partial x} \frac{\partial \tau_{\eta_2}}{\partial x} + v_2^2 \frac{\partial \tau_0}{\partial y} \frac{\partial \tau_{\eta_1}}{\partial y} \right) \\ & + 2v_1^2 \left(\frac{\partial \tau_0}{\partial x} \right)^2 \left(v_2^2 \frac{\partial \tau_0}{\partial y} \frac{\partial \tau_{\eta_1}}{\partial y} - v_2^2 \frac{\partial \tau_0}{\partial y} \frac{\partial \tau_{\eta_2}}{\partial y} + v_v^2 \frac{\partial \tau_0}{\partial z} \frac{\partial \tau_{\eta_1}}{\partial z} \right) \\ & + 2v_2^2 \left(\frac{\partial \tau_0}{\partial y} \right)^2 \left(v_1^2 \frac{\partial \tau_0}{\partial x} \frac{\partial \tau_{\eta_1}}{\partial x} - v_1^2 \frac{\partial \tau_0}{\partial x} \frac{\partial \tau_{\eta_2}}{\partial x} + v_v^2 \frac{\partial \tau_0}{\partial z} \frac{\partial \tau_{\eta_1}}{\partial z} \right) \\ & + 2v_1^2 v_2^2 \left(\frac{\partial \tau_0}{\partial x} \right)^2 \left(\frac{\partial \tau_0}{\partial y} \right)^2 \left(1 - v_v^2 \left(\frac{\partial \tau_0}{\partial z} \right)^2 \right). \end{aligned} \quad (A-6)$$

EDITED REFERENCES

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