On the Maximum and Minimum of Double Generalized Gamma Variates with Applications to the Performance of Free-space Optical Communication Systems

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Abstract—In this work, we derive the exact statistical characteristics of the maximum and the minimum of two modified¹ double generalized gamma variates in closed-form in terms of Meijer's G-function, Fox's H-function, the extended generalized bivariate Meijer's G-function and H-function in addition to simple closed-form asymptotic results in terms of elementary functions. Then, we rely on these new results to present the performance analysis of (i) a dual-branch free-space optical selection combining diversity and of (ii) a dual-hop free-space optical relay transmission system over double generalized gamma fading channels with the impact of pointing errors. In addition, we provide asymptotic results of the bit error rate of the two systems at high SNR regime. Computer-based Monte-Carlo simulations verify our new analytical results.

Index Terms—Free-space optical, double generalized Gamma, turbulence, pointing errors, dual-branch diversity, selection combining, dual-hop relay system, hybrid RF/FSO.

I. INTRODUCTION

Recently, radio frequency spectrum for wireless communications is facing a crunch. Research is driven to find solutions such as utilizing the spectrum more efficiently or using unregulated one such as the optical spectrum under which free-space optical (FSO) communications operates [5], [6]. FSO can complement or substitute radio frequency (RF) systems due to their higher bandwidth, cost effective, resistivity to

interference and efficient employment time [7], [8]. This makes FSO applicable for broadband wireless technologies such as optical fiber backup, metropolitan area network, and last mile access [9]. On the other hand, FSO still have some challenges that can affect link range and system reliability such as atmospheric turbulence conditions (scintillation) that is comparable to the fading phenomena in the RF systems. Weather conditions can also affect signal strength as the air being the medium of the transmission. This can be solved by employing network redundancy along the transmission path. Moreover, thermal expansion, weak earth quakes, and dynamic wind loads can result in misalignment between the transmitter and the receiver introducing the pointing errors [2], [10]–[12].

Diversity, in which multiple copies of the same data is transmitted to the receiver such that each copy experiences different path/channel, has shown a great deal in mitigating fading effects in RF systems in addition to improving both performance and capacity [13]–[15]. For diversity combining techniques, selection combining (SC) is one of the most efficient and simple combining schemes. In SC, the receiver aims to process the branch with maximum signal-to-noise ratio (SNR). Diversity schemes in FSO systems was first introduced by Ibrahim [16] followed by great number of research instigating diversity over weak and strong turbulence conditions. Some investigated the spatial diversity and combining techniques of correlated and independent log-normal turbulence channels [17]–[20]. Also, spatial diversity was studied over K-distributed channels for different combining schemes in [21]. Moreover, SC over Gamma-Gamma under the impact of pointing errors has been investigated [4].

Relaying also has been an interesting area of research as it enhances the capacity of the system and provides energy efficient coverage. In this technique, a relay node is positioned in the way between transmitter and receiver to support direct data transmission. Multiple research works have been reported to analyze the relay system on both symmetric and asymmetric links (i.e. symmetric links when source-relay (S~R) and relay-destination (R~D) links fall under the same fading model) [22]–[25]. Moreover, relaying over FSO system was introduced and then followed by several research work concerning symmetric multi-hop FSO systems over K-distributed, Gaussian and Gamma-Gamma turbulence channel [26]–[28]. On the other hand, several studies have been conducted to

¹Under the impact of pointing error (i.e. $I = I_p I_a$ where I_a is the irradiance modeled as double generalized gamma random variable [1] and I_p is the pointing error whose probability density function (PDF) can be found in [2, Eq. (10)]). The PDF of I can be found in [3, Eq. (8)]

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analyze asymmetric fixed and variable gain FSO relay systems as such integrating RF links in FSO systems. Specifically, assuming that RF and FSO links are, respectively, subject to Rayleigh and Gamma-Gamma or M-distributed turbulence channels [29]–[31]. Analyzing variable gain relay system analytically might not be tractable and mathematically not feasible. Throughout the years, the end-to-end SNR has been upper bounded by the minimum SNR among the sublinks [32] and harmonic mean of the each link SNR [33]. Few research work have been reported focusing on variable gain FSO relay systems over Gamma-Gamma with pointing errors [4], [34], [35].

Double generalized gamma channel model for free-space optical communication systems developed by [1] covers a very wide range of turbulence conditions and makes it generic to describe the FSO channel. Also, pointing errors was integrated to this model to better show the impact of pointing error impairments on the FSO system [3]. Hence, in this work, we study the dual-branch FSO selection combining and dual-hop variable gain FSO relay operating on such channels with the impact of pointing errors to show diversity enhancement on the system performance and capacity. First, we express the statistical properties of the maximum of the minimum of double Generalized Gamma random variables under the impact of pointing errors in terms of H-function and G-function. In particular, we find the cumulative distribution function (CDF), the probability density function (PDF), the moment generating function (MGF), and the moments in closed-form. We then use these results to evaluate performance measures such as the average bit error rate (BER) and the ergodic capacity (EC) of both FSO systems. Finally, we multiplex two independent FSO links with desired priority on the data rate of each links, thereby, utilizing the resources efficiently and effectively.

II. CHANNEL AND SYSTEM MODELS

In this work, we consider a single FSO branch with two types of detection techniques, heterodyne and intensity modulation/direct detection (IM/DD). Data transmission is affected by path loss, atmospheric turbulence conditions, pointing errors, and additive white Gaussian noise (AWGN) that can be modeled as

$$y_i = \eta I_i x + w_i, \quad (1)$$

where η is the effective photoelectric conversion ratio, w_i refers to the AWGN sample with power spectral density equals to N_0 , and I_i is the receiver irradiance that is defined as $I_i = I_{a,i} I_p$ where $I_{a,i}$ and I_p reflect the turbulence-induced fading and the pointing error effect, respectively [36]. The end-to-end SNR is defined as $\gamma_i = \frac{I_i^2}{\mathbb{E}[I_i] r_i} \mu_{r_i,i}$ where r_i refers to the detection method (i.e. $r_i = 1$ heterodyne detection and $r_i = 2$ IM/DD), $\mu_{r_i,i}$ is the average SNR defined in [37, Eq. (13)], and $\mathbb{E}[\cdot]$ is the expectation operator. If we assume that the turbulence channel is modeled as double generalized Gamma under the assumption of pointing errors, then the CDF

of γ_i can be given as [37, Eq. (14)]

$$F_{\gamma_i}(\gamma) = \frac{\xi_i^2 \sigma_i^{\beta_{1i}-\frac{1}{2}} \lambda_i^{\beta_{2i}-\frac{1}{2}} (2\pi)^{1-\frac{r_i(\lambda_i+\sigma_i)}{2}} r_i^{\beta_{1i}+\beta_{2i}-2}}{\alpha_{2i} \lambda_i \Gamma(\beta_{1i}) \Gamma(\beta_{2i})} \times G_{r_i+1, u_i+1}^{u_i, 1} \left[C_i \left(\frac{\gamma}{\mu_{r_i,i}} \right)^{v_i} \middle| \begin{matrix} 1, \kappa_{3i} \\ \kappa_{4i}, 0 \end{matrix} \right], \quad (2)$$

where $\xi_i = \frac{w_e}{2\sigma_s}$ is the ratio between the equivalent beam width at the receiver and the pointing error displacement standard deviation (jitter), α_x , β_x , and Ω_x are identified using the variance of the small and large scale fluctuations of the laser beam from [38, Eq. (18)-(20)] inserted in [1, Eq. (8a), (8b), (9)], λ_i and σ_i are positive integers such as $\frac{\lambda_i}{\sigma_i} = \frac{\alpha_{1i}}{\alpha_{2i} \beta_i}$, $u_i = r_i(1 + \lambda_i + \sigma_i)$, $v_i = \alpha_{2i} \lambda_i$, $C_i = \left(\frac{A_{2i} h_i v_i}{r_i (\lambda_i + \sigma_i)} \right)^{r_i}$, $h_i = \frac{A_{1i} B_{1i}}{(1 + \xi_i^2) A_{2i}^{1/v_i}}$, $A_{1i} = \frac{\xi_i^2 \sigma_i^{\beta_{1i}-\frac{1}{2}} \lambda_i^{\beta_{2i}-\frac{1}{2}} (2\pi)^{1-\frac{(\lambda_i+\sigma_i)}{2}}}{\Gamma(\beta_{1i}) \Gamma(\beta_{2i})}$, $A_{2i} = \frac{\beta_{1i}^{\sigma_i} \beta_{2i}^{\lambda_i}}{\lambda_i^{\lambda_i} \sigma_i^{\sigma_i} \Omega_{1i}^{\sigma_i} \Omega_{2i}^{\lambda_i}}$, $B_{1i} = \prod_{j=1}^{\sigma_i+\lambda_i} \Gamma\left(\frac{1}{v_i} + \kappa_{1i,j}\right)$, where $\kappa_{i,j}$ is the j th-term of κ_i , $\kappa_{1i} = \Delta(\sigma_i : \beta_{1i})$, $\Delta(\lambda_i : \beta_{2i})$, $\kappa_{3i} = \Delta(r_i : \frac{\xi_i^2 + v_i}{v_i})$ comprising of r_i terms, and $\kappa_{4i} = [\Delta(r_i : \kappa_{2i})]$ comprising of u_i terms such that $\kappa_{2i} = \xi_i^2 / v_i$, κ_{1i} , $\Delta(x : y) \triangleq \frac{y}{x}, \frac{y+1}{x}, \dots, \frac{y+x-1}{x}$ and $[\Delta(z : a_m)] \triangleq \Delta(z : a_1), \Delta(z : a_2), \dots, \Delta(z : a_m)$.

By setting $\sigma_i = \lambda_i = \Omega_i = 1$, $\beta_{1i} = \alpha_i$, $\beta_{2i} = \beta_i$, $h_i = \frac{\xi_i^2}{\xi_i^2 + 1}$, $\xi_i \gg 1$ then $h_i = 1$, $C_i = \left(\frac{\alpha_i \beta_i}{r_i^2} \right)^{r_i}$, $u_i = 3r_i$, $v_i = 1$, $\kappa_{3i} = \Delta(r_i : \xi_i^2 + 1)$, $\kappa_{4i} = \Delta(r_i : \xi_i^2)$, $\Delta(r_i : \alpha_i)$, $\Delta(r_i : \beta_i)$ we have the special case of Gamma-Gamma (GG) turbulence perturbed by pointing errors [4].

III. CLOSED-FORM STATISTICAL CHARACTERISTICS

A. Maximum of Two Variates

Here, we study the statistical characteristic of the maximum of two modified generalized gamma random variates γ_M which is given by

$$\gamma_M = \max(\gamma_a, \gamma_b), \quad (3)$$

assuming that γ_a and γ_b are independent not necessarily identically distributed (i.n.i.d) random variables (RVs).

1) *Cumulative Distribution Function:* The CDF can be easily derived as

$$F_{\gamma_M}(\gamma) = A_{3a} A_{3b} G_{r_a+1, u_a+1}^{u_a, 1} \left[C_a \left(\frac{\gamma}{\mu_{r_a,a}} \right)^{v_a} \middle| \begin{matrix} 1, \kappa_{3a} \\ \kappa_{4a}, 0 \end{matrix} \right] \times G_{r_b+1, u_b+1}^{u_b, 1} \left[C_b \left(\frac{\gamma}{\mu_{r_b,b}} \right)^{v_b} \middle| \begin{matrix} 1, \kappa_{3b} \\ \kappa_{4b}, 0 \end{matrix} \right], \quad (4)$$

where $A_{3i} = \frac{\xi_i^2 \sigma_i^{\beta_{1i}-\frac{1}{2}} \lambda_i^{\beta_{2i}-\frac{1}{2}} (2\pi)^{1-\frac{r_i(\lambda_i+\sigma_i)}{2}} r_i^{\beta_{1i}+\beta_{2i}-2}}{\alpha_{2i} \lambda_i \Gamma(\beta_{1i}) \Gamma(\beta_{2i})}$. For the case of Gamma-Gamma turbulence model, we have $A_{3i} = \frac{\xi_i^2 r_i^{\alpha_i + \beta_i - 2}}{\Gamma(\alpha_i) \Gamma(\beta_i) (2\pi)^{r_i - 1}}$.

Moreover, an asymptotic expression can be obtained via the expansion of the Meijer's G-function [39, Eq. (26)] as $C_i \left(\frac{\gamma}{\mu_{r_i,i}} \right)^{v_i} \rightarrow 0$ (i.e high SNR) as

$$\tilde{F}_{\gamma_M}(\gamma) \underset{\mu_{r_i,i} \gg 1}{\approx} A_{3a} A_{3b} \sum_{k=1}^{u_a} \sum_{x=1}^{u_b} \left(\frac{\gamma^{v_a} C_a}{\mu_{r_a,a}^{v_a}} \right)^{\kappa_{4a,k}} \left(\frac{\gamma^{v_b} C_b}{\mu_{r_b,b}^{v_b}} \right)^{\kappa_{4b,x}} \times \frac{\prod_{l=1; l \neq k}^{u_a} \Gamma(\kappa_{4a,l} - \kappa_{4a,k}) \prod_{l=1; l \neq x}^{u_b} \Gamma(\kappa_{4b,l} - \kappa_{4b,x})}{\kappa_{4a,k} \kappa_{4b,x} \prod_{l=1}^{r_a} \Gamma(\kappa_{3a,l} - \kappa_{4a,k}) \prod_{l=1}^{r_b} \Gamma(\kappa_{3b,l} - \kappa_{4b,k})}. \quad (5)$$

The asymptotic expression in (5) is simple and is in terms of elementary functions that can be evaluated using any computer software.

2) *Probability Density Function*: The PDF is then obtained by differentiating (4) with respect to γ yielding

$$\begin{aligned} f_{\gamma_M}(\gamma) = & \frac{A_{3a}A_{3b}}{\gamma} \left(v_b G_{r_b, u_b}^{u_b, 0} \left[C_b \left(\frac{\gamma}{\mu_{r_b, b}} \right)^{v_b} \middle| \begin{matrix} \kappa_{3b} \\ \kappa_{4b} \end{matrix} \right] \right. \\ & \times G_{r_a+1, u_a+1}^{u_a, 1} \left[C_a \left(\frac{\gamma}{\mu_{r_a, a}} \right)^{v_a} \middle| \begin{matrix} 1, \kappa_{3a} \\ \kappa_{4a}, 0 \end{matrix} \right] \\ & + v_a G_{r_a, u_a}^{u_a, 0} \left[C_a \left(\frac{\gamma}{\mu_{r_a, a}} \right)^{v_a} \middle| \begin{matrix} \kappa_{3a} \\ \kappa_{4a} \end{matrix} \right] \\ & \left. \times G_{r_b+1, u_b+1}^{u_b, 1} \left[C_b \left(\frac{\gamma}{\mu_{r_b, b}} \right)^{v_b} \middle| \begin{matrix} 1, \kappa_{3b} \\ \kappa_{4b}, 0 \end{matrix} \right] \right). \end{aligned} \quad (6)$$

An asymptotic expression of the PDF can be obtained by the same way done earlier for the CDF. In other words, by using the expansion of the Meijer's G-function, we can reach to an asymptotic result as

$$\begin{aligned} \tilde{f}_{\gamma_M}(\gamma) & \approx_{\mu_{r_i, i} > 1} A_{3a}A_{3b} \sum_{k=1}^{u_a} \sum_{x=1}^{u_b} \left(\frac{C_a}{\mu_{r_a, a}} \right)^{\kappa_{4a, k}} \left(\frac{C_b}{\mu_{r_b, b}} \right)^{\kappa_{4b, x}} \\ & \times \frac{\prod_{l=1, l \neq k}^{u_a} \Gamma(\kappa_{4a, l} - \kappa_{4a, k}) \prod_{l=1, l \neq x}^{u_b} \Gamma(\kappa_{4b, l} - \kappa_{4b, x})}{\prod_{l=1}^{r_a} \Gamma(\kappa_{3a, l} - \kappa_{4a, k}) \prod_{l=1}^{r_b} \Gamma(\kappa_{3b, l} - \kappa_{4b, x})} \\ & \times \left(\frac{v_b}{\kappa_{4a, k}} + \frac{v_a}{\kappa_{4b, x}} \right) \gamma^{v_a \kappa_{4a, k} + v_b \kappa_{4b, x} - 1}. \end{aligned} \quad (7)$$

Derived PDFs were verified via Monte-Carlo simulations as

Figure 1: PDF validation for the maximum of two double GG variates

shown in Fig. 1.

3) *Moment Generating Function*: The MGF defined as $\mathcal{M}_{\gamma_M}(s) \triangleq \mathbb{E}[e^{-\gamma_M s}]$ can be expressed in terms of CDF using integration by parts as [40]

$$\mathcal{M}_{\gamma_M}(s) = s \int_0^\infty \exp(-\gamma s) F_{\gamma_M}(\gamma) d\gamma. \quad (8)$$

Using the property $\frac{1}{C} G_{p,q}^{m,n} \left[z^{1/C} \middle| \begin{matrix} \mathbf{a} \\ \mathbf{b} \end{matrix} \right] = H_{p,q}^{m,n} \left[z \middle| \begin{matrix} (a, C) \\ (b, C) \end{matrix} \right]$, we transform each term to the Fox's H-function that can be evaluated using Mathematica [41]. As such, the CDF in (4)

can be written as

$$\begin{aligned} F_{\gamma_M}(\gamma) = & \frac{A_{3a}A_{3b}}{v_a v_b} H_{r_a+1, u_a+1}^{u_a, 1} \left[\frac{C_a^{1/v_a} \gamma}{\mu_{r_a, a}} \middle| \begin{matrix} (\kappa_{5a}, 1/v_a) \\ (\kappa_{6a}, 1/v_a) \end{matrix} \right] \\ & \times H_{r_b+1, u_b+1}^{u_b, 1} \left[\frac{C_b^{1/v_b} \gamma}{\mu_{r_b, b}} \middle| \begin{matrix} (\kappa_{5b}, 1/v_b) \\ (\kappa_{6b}, 1/v_b) \end{matrix} \right], \end{aligned} \quad (9)$$

where $\kappa_{5i} = 1, \kappa_{3i}$ and $\kappa_{6i} = \kappa_{4i}, 0$. Afterward, we apply the identity [42, Eq. (2.2)], after inserting (9) in (8) to obtain

$$\begin{aligned} \mathcal{M}_{\gamma_M}(s) = & \frac{A_{3a}A_{3b}}{v_a v_b} \\ & \times \mathbf{H} \left[\begin{matrix} \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix} \\ \begin{pmatrix} u_a, 1 \\ r_a+1, u_a+1 \end{pmatrix} \\ \begin{pmatrix} u_b, 1 \\ r_b+1, u_b+1 \end{pmatrix} \end{matrix} \middle| \begin{matrix} \begin{pmatrix} 0; 1, 1 \\ (-; -, -) \end{pmatrix} \\ \begin{pmatrix} (\kappa_{5a}, [v_a^{-1}]) \\ (\kappa_{6a}, [v_a^{-1}]) \end{pmatrix} \\ \begin{pmatrix} (\kappa_{5b}, [v_b^{-1}]) \\ (\kappa_{6b}, [v_b^{-1}]) \end{pmatrix} \end{matrix} \right] \begin{matrix} \frac{C_a^{1/v_a}}{s \mu_{r_a, a}} \\ \frac{C_b^{1/v_b}}{s \mu_{r_b, b}} \end{matrix}, \end{aligned} \quad (10)$$

where $\mathbf{H}[\cdot]$ is the bivariate H-function defined in [42]. It can be evaluated efficiently using MATHEMATICA[®] implementation in [43] or MATLAB[®] implementation in [44]. In this work, we have implemented the bivariate H-function in MATHEMATICA[®] and easily evaluated the expressions.

Simplified formula can be obtained if solving (8) but using the asymptotic CDF (5) to reach to the following result

$$\begin{aligned} \tilde{\mathcal{M}}_{\gamma_M}(s) & \approx_{\mu_{r_i, i} > 1} A_{3a}A_{3b} \sum_{k=1}^{u_a} \sum_{x=1}^{u_b} \left(\frac{C_a}{\mu_{r_a, a}} \right)^{\kappa_{4a, k}} \left(\frac{C_b}{\mu_{r_b, b}} \right)^{\kappa_{4b, x}} \\ & \times \frac{\prod_{l=1, l \neq k}^{u_a} \Gamma(\kappa_{4a, l} - \kappa_{4a, k}) \prod_{l=1, l \neq x}^{u_b} \Gamma(\kappa_{4b, l} - \kappa_{4b, x})}{\prod_{l=1}^{r_a} \Gamma(\kappa_{3a, l} - \kappa_{4a, k}) \prod_{l=1}^{r_b} \Gamma(\kappa_{3b, l} - \kappa_{4b, x})} \\ & \times \frac{\Gamma(v_a \kappa_{4a, k} + v_b \kappa_{4b, x} + 1) s^{-(v_a \kappa_{4a, k} + v_b \kappa_{4b, x})}}{\kappa_{4a, k} \kappa_{4b, x}}. \end{aligned} \quad (11)$$

Setting the same special parameters listed in Sec. II for the Gamma-Gamma case, the MGF can be expressed as

$$\begin{aligned} \mathcal{M}_{\gamma_M}^*(s) = & A_{3a}A_{3b} \\ & \times G_{1, 0: 3r_a, 1: 3r_b, 1}^{1, 0: r_a+1, 3r_a+1: r_b+1, 3r_b+1} \left[\frac{1}{s} \middle| \begin{matrix} \kappa_{5a} \\ \kappa_{6a} \end{matrix} \middle| \begin{matrix} \kappa_{5b} \\ \kappa_{6b} \end{matrix} \middle| \frac{C_a}{s \mu_{r_a, a}}, \frac{C_b}{s \mu_{r_b, b}} \right], \end{aligned} \quad (12)$$

in agreement with [4, Eq. (8)] where $G_{p,q}^{m,n}[\cdot]$ is the extended generalized bivariate Meijer's G-function (EGBMGF) that can be evaluated using the code given in [43, Table II].

4) *Moments*: The moments is defined as

$$\mathbb{E}[\gamma^n] \triangleq \int_0^\infty \gamma^n f_{\gamma}(\gamma) d\gamma. \quad (13)$$

Placing (6) in (13) and utilizing equation [45, Eq. (1.7)], we obtain the following

$$\begin{aligned} \mathbb{E}[\gamma_M^n] &= A_{3a} A_{3b} \left[\left(\frac{\mu_{r_a,a}}{C_a^{1/v_a}} \right)^n v_b \right. \\ &\times H_{u_a+r_b+1, u_b+r_a+1}^{\left(\frac{C_b^{1/v_b} \mu_{r_a,a}}{C_a^{1/v_a} \mu_{r_b,b}} \right)^{v_a v_b} \left| \begin{matrix} (\kappa_{7a}, v_b), (\kappa_{3b}, v_a) \\ (\kappa_{4b}, v_a), (\kappa_{8a}, v_b) \end{matrix} \right.} \\ &+ \left(\frac{\mu_{r_b,b}}{C_b^{1/v_b}} \right)^n v_a \\ &\times H_{u_b+r_a+1, u_a+r_b+1}^{\left(\frac{C_a^{1/v_a} \mu_{r_b,b}}{C_b^{1/v_b} \mu_{r_a,a}} \right)^{v_a v_b} \left| \begin{matrix} (\kappa_{7b}, v_a), (\kappa_{3a}, v_b) \\ (\kappa_{4a}, v_b), (\kappa_{8b}, v_a) \end{matrix} \right.} \Big]. \end{aligned} \quad (14)$$

where $\kappa_{7i} = 1 - \kappa_{6i} - \frac{n}{v_i}$ and $\kappa_{8i} = 1 - \kappa_{5i} - \frac{n}{v_i}$. Setting the same parameters listed in Sec. II, (14) simplifies to

$$\begin{aligned} \mathbb{E}[\gamma_M^n]^* &= A_{3a} A_{3b} \\ &\times \left[\left(\frac{\mu_{r_a,a}}{C_a} \right)^n G_{3r_a+r_b+1, 3r_b+r_a+1}^{\left(\frac{C_b \mu_{r_a,a}}{C_a \mu_{r_b,b}} \right) \left| \begin{matrix} \kappa_{7a}, \kappa_{3b} \\ \kappa_{4b}, \kappa_{8a} \end{matrix} \right.} \right. \\ &+ \left(\frac{\mu_{r_b,b}}{C_b} \right)^n G_{3r_b+r_a+1, 3r_a+r_b+1}^{\left(\frac{C_a \mu_{r_b,b}}{C_b \mu_{r_a,a}} \right) \left| \begin{matrix} \kappa_{7b}, \kappa_{3a} \\ \kappa_{4a}, \kappa_{8b} \end{matrix} \right.} \Big], \end{aligned} \quad (15)$$

in agreement with [4, Eq. (10)].

B. Minimum of Two Variates

In this section, we list the statistical properties of the minimum of two modified generalized gamma random variables γ_m given by

$$\gamma_m = \min(\gamma_1, \gamma_2). \quad (16)$$

1) *Cumulative Distribution Function*: The CDF is given by

$$\begin{aligned} F_{\gamma_m}(\gamma) &= 1 - \Pr(\min(\gamma_1, \gamma_2) > \gamma) \\ &= F_{\gamma_1}(\gamma) + F_{\gamma_2}(\gamma) - F_{\gamma_1}(\gamma)F_{\gamma_2}(\gamma), \end{aligned} \quad (17)$$

under the assumption that γ_1 and γ_2 are i.n.i.d. Substituting (2) in (17), we obtain

$$\begin{aligned} F_{\gamma_m}(\gamma) &= A_{3a} G_{r_a+1, u_a+1}^{u_a, 1} \left[C_a \left(\frac{\gamma}{\mu_{r_a,a}} \right)^{v_a} \left| \begin{matrix} 1, \kappa_{3a} \\ \kappa_{4a}, 0 \end{matrix} \right. \right] \\ &+ A_{3b} G_{r_b+1, u_b+1}^{u_b, 1} \left[C_b \left(\frac{\gamma}{\mu_{r_b,b}} \right)^{v_b} \left| \begin{matrix} 1, \kappa_{3b} \\ \kappa_{4b}, 0 \end{matrix} \right. \right] \\ &- A_{3a} A_{3b} G_{r_a+1, u_a+1}^{u_a, 1} \left[C_a \left(\frac{\gamma}{\mu_{r_a,a}} \right)^{v_a} \left| \begin{matrix} 1, \kappa_{3a} \\ \kappa_{4a}, 0 \end{matrix} \right. \right] \\ &\times G_{r_b+1, u_b+1}^{u_b, 1} \left[C_b \left(\frac{\gamma}{\mu_{r_b,b}} \right)^{v_b} \left| \begin{matrix} 1, \kappa_{3b} \\ \kappa_{4b}, 0 \end{matrix} \right. \right]. \end{aligned} \quad (18)$$

In addition, an asymptotic expression can be obtained via the expansion of the Meijer's G function as

$$\begin{aligned} \tilde{F}_{\gamma_m}(\gamma) &\underset{\mu_{r_i,i} \gg 1}{\approx} A_{3a} \sum_{k=1}^{u_a} \left(\frac{\gamma^{v_a} C_a}{\mu_{r_a,a}^{v_a}} \right)^{\kappa_{4a,k}} \\ &\times \frac{\prod_{l=1; l \neq k}^{u_a} \Gamma(\kappa_{4a,l} - \kappa_{4a,k})}{\kappa_{4a,k} \prod_{l=1}^{r_a} \Gamma(\kappa_{3a,l} - \kappa_{4a,k})} + A_{3b} \sum_{x=1}^{u_b} \left(\frac{\gamma^{v_b} C_b}{\mu_{r_b,b}^{v_b}} \right)^{\kappa_{4b,x}} \\ &\times \frac{\prod_{l=1; l \neq x}^{u_b} \Gamma(\kappa_{4b,l} - \kappa_{4b,x})}{\kappa_{4b,x} \prod_{l=1}^{r_b} \Gamma(\kappa_{3b,l} - \kappa_{4b,x})} - \tilde{F}_{\gamma_M}(\gamma). \end{aligned} \quad (19)$$

2) *Probability Density Function*: Differentiating (18) with respect to γ , a closed-form expression of the PDF of γ_m is

obtained as

$$\begin{aligned} f_{\gamma_m}(\gamma) &= \frac{A_{3a}}{\gamma} v_a G_{r_a+1, u_a}^{u_a, 0} \left[C_a \left(\frac{\gamma}{\mu_{r_a,a}} \right)^{v_a} \left| \begin{matrix} \kappa_{3a} \\ \kappa_{4a} \end{matrix} \right. \right] \\ &+ \frac{A_{3b}}{\gamma} v_b G_{r_b+1, u_b}^{u_b, 0} \left[C_b \left(\frac{\gamma}{\mu_{r_b,b}} \right)^{v_b} \left| \begin{matrix} \kappa_{3b} \\ \kappa_{4b} \end{matrix} \right. \right] - f_{\gamma_M}(\gamma), \end{aligned} \quad (20)$$

where $f_{\gamma_M}(\gamma)$ is defined in (6). Similarly, an asymptotic expression of the PDF can be obtained when expanding the Meijer's G-function leading to

$$\begin{aligned} \tilde{f}_{\gamma_m}(\gamma) &\underset{\mu_{r_i,i} \gg 1}{\approx} A_{3a} v_a \sum_{k=1}^{u_a} \left(\frac{C_a}{\mu_{r_a,a}^{v_a}} \right)^{\kappa_{4a,k}} \gamma^{v_a \kappa_{4a,k} - 1} \\ &\times \frac{\prod_{l=1; l \neq k}^{u_a} \Gamma(\kappa_{4a,l} - \kappa_{4a,k})}{\prod_{l=1}^{r_a} \Gamma(\kappa_{3a,l} - \kappa_{4a,k})} \\ &+ A_{3b} v_b \sum_{x=1}^{u_b} \left(\frac{C_b}{\mu_{r_b,b}^{v_b}} \right)^{\kappa_{4b,x}} \gamma^{v_b \kappa_{4b,x} - 1} \\ &\times \frac{\prod_{l=1; l \neq x}^{u_b} \Gamma(\kappa_{4b,l} - \kappa_{4b,x})}{\prod_{l=1}^{r_b} \Gamma(\kappa_{3b,l} - \kappa_{4b,x})} - \tilde{f}_{\gamma_M}(\gamma). \end{aligned} \quad (21)$$

Monte-Carlo simulations were utilized again to verify the

Figure 2: PDF validation for the minimum of two double GG variates

PDFs as in Fig. 2.

3) *Moment Generating Function*: Placing (18) in the definition of the MGF, and utilizing Eq. (10) and [46, Eq. (07.34.21.0088.01)], the MGF can be expressed as

$$\begin{aligned} \mathcal{M}_{\gamma_m}(s) &= \frac{A_{3a} \sqrt{v_a}}{(2\pi)^{\frac{v_a-1}{2}}} G_{r_a+v_a, u_a}^{u_a, v_a} \left[C_a \left(\frac{v_a}{s \mu_{r_a,a}} \right)^{v_a} \left| \begin{matrix} \Delta(v_a, 1), \kappa_{3a} \\ \kappa_{4a} \end{matrix} \right. \right] \\ &+ \frac{A_{3b} \sqrt{v_b}}{(2\pi)^{\frac{v_b-1}{2}}} G_{r_b+v_b, u_b}^{u_b, v_b} \left[C_b \left(\frac{v_b}{s \mu_{r_b,b}} \right)^{v_b} \left| \begin{matrix} \Delta(v_b, 1), \kappa_{3b} \\ \kappa_{4b} \end{matrix} \right. \right] - \mathcal{M}_{\gamma_M}(s). \end{aligned} \quad (22)$$

More simplified expression for the MGF can be obtained if the expansion of the Meijer's G-function were utilized in a

similar way to find the MGF of the maximum

$$\begin{aligned} \tilde{\mathcal{M}}_{\gamma_m}(s) &\underset{\mu_{r_i,i} > 1}{\approx} A_{3a} v_a \sum_{k=1}^{u_a} \left(\frac{C_a}{(s\mu_{r_a,a})^{v_a}} \right)^{\kappa_{4a,k}} \Gamma(v_a \kappa_{4a,k}) \\ &\times \frac{\prod_{l=1; l \neq k}^{u_a} \Gamma(\kappa_{4a,l} - \kappa_{4a,k})}{\prod_{l=1}^{r_a} \Gamma(\kappa_{3a,l} - \kappa_{4a,k})} + A_{3b} v_b \sum_{x=1}^{u_b} \left(\frac{C_b}{(s\mu_{r_b,b})^{v_b}} \right)^{\kappa_{4b,x}} \\ &\times \frac{\Gamma(v_b \kappa_{4b,x}) \prod_{l=1; l \neq x}^{u_b} \Gamma(\kappa_{4b,l} - \kappa_{4b,x})}{\prod_{l=1}^{r_b} \Gamma(\kappa_{3b,l} - \kappa_{4b,x})} - \tilde{\mathcal{M}}_{\gamma_M}(s). \end{aligned} \quad (23)$$

The MGF for Gamma-Gamma model can be obtained by setting the same parameters listed in Sec. II

$$\begin{aligned} \mathcal{M}_{\gamma_m}^*(s) &= A_{3a} G_{r_a+1, 3r_a}^{\kappa_{3a,1}} \left[\frac{C_a}{s\mu_{r_a,a}} \middle| \begin{matrix} 1, \kappa_{3a} \\ \kappa_{4a} \end{matrix} \right] \\ &+ A_{3b} G_{r_b+1, 3r_b}^{\kappa_{3b,1}} \left[\frac{C_b}{s\mu_{r_b,b}} \middle| \begin{matrix} 1, \kappa_{3b} \\ \kappa_{4b} \end{matrix} \right] - A_{3a} A_{3b} \\ &\times G_{1,0; r_a+1, 3r_a+1; r_b+1, 3r_b+1}^{1,0; 3r_a, 1; 3r_b, 1} \left[- \middle| \begin{matrix} \kappa_{5a} & \kappa_{5b} \\ \kappa_{6a} & \kappa_{6b} \end{matrix} \middle| \frac{C_a}{s\mu_{r_a,a}}, \frac{C_b}{s\mu_{r_b,b}} \right]. \end{aligned} \quad (24)$$

in agreement with [4, Eq. (15)].

4) *Moments*: Substituting (20) in the definition of the moments (13) and utilizing [46, Eq. (2.25.2.1)] and Eq. (14), we obtain the moments as

$$\begin{aligned} \mathbb{E}[\gamma_m^n] &= A_{3a} \left(\frac{\mu_{r_a,a}}{C_a^{1/v_a}} \right)^n \Gamma \left(\frac{\kappa_{4a} + \frac{n}{v_a}}{\kappa_{3a} + \frac{n}{v_a}} \right) + A_{3b} \left(\frac{\mu_{r_b,b}}{C_b^{1/v_b}} \right)^n \\ &\times \Gamma \left(\frac{\kappa_{4b} + \frac{n}{v_b}}{\kappa_{3b} + \frac{n}{v_b}} \right) - \mathbb{E}[\gamma_M^n] \end{aligned} \quad (25)$$

where $\Gamma \left(\frac{\mathbf{x}}{\mathbf{y}} \right) = \frac{\prod_{i=1}^m \Gamma(x_i)}{\prod_{i=1}^n \Gamma(y_i)}$ such that m and n are the lengths of $\mathbf{x}$ and $\mathbf{y}$, respectively.

In the Gamma-Gamma case, (25) simplifies to

$$\begin{aligned} \mathbb{E}[\gamma_m^n]^* &= A_{3a} \left(\frac{\mu_{r_a,a}}{C_a^{1/v_a}} \right)^n \Gamma \left(\frac{\kappa_{4a} + n}{\kappa_{3a} + n} \right) \\ &+ A_{3b} \left(\frac{\mu_{r_b,b}}{C_b^{1/v_b}} \right)^n \Gamma \left(\frac{\kappa_{4b} + n}{\kappa_{3b} + n} \right) \\ &- A_{3a} A_{3b} \left[\left(\frac{\mu_{r_a,a}}{C_a} \right)^n G_{3r_b+1, 3r_a}^{3r_b+1, 3r_a} \left[\frac{C_b \mu_{r_a,a}}{C_a \mu_{r_b,b}} \middle| \begin{matrix} \kappa_{7a}, \kappa_{3b} \\ \kappa_{4b}, \kappa_{8a} \end{matrix} \right] \right. \\ &\left. + \left(\frac{\mu_{r_b,b}}{C_b} \right)^n G_{3r_a+1, 3r_b}^{3r_a+1, 3r_b} \left[\frac{C_a \mu_{r_b,b}}{C_b \mu_{r_a,a}} \middle| \begin{matrix} \kappa_{7b}, \kappa_{3a} \\ \kappa_{4a}, \kappa_{8b} \end{matrix} \right] \right]. \end{aligned} \quad (26)$$

as found in [4, Eq. (16)].

IV. APPLICATIONS

A. Performance of Dual-Branch SC Systems

Based on our results in Section III-A, we now study the performance of the selection combining scheme over dual-branch FSO links.

1) *Outage Probability*: The outage probability P_{out} is defined as follows:

$$P_{out} = \Pr[\gamma < \gamma_{th}] = F_{\gamma}(\gamma_{th}). \quad (27)$$

Accordingly, the outage probability for the maximum SNR of two links is obtained by substituting (4) into (27).

2) *Average Bit Error Rate*: Using the definition of the average BER [43, Eq. (12)]

$$\bar{P} = \frac{q^p}{2\Gamma(p)} \int_0^\infty \exp(-q\gamma) \gamma^{p-1} F_{\gamma}(\gamma) d\gamma \quad (28)$$

and placing (4) into (28) then utilizing the identity [42, Eq. (2.2)], we obtain $\bar{P}_{SC}$ as

$$\bar{P}_{SC} = \frac{A_{3a} A_{3b}}{2\Gamma(p) v_a v_b} \times \mathbf{H} \left[\begin{matrix} \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix} \\ \begin{pmatrix} u_a, 1 \\ r_a+1, u_a+1 \end{pmatrix} \\ \begin{pmatrix} u_b, 1 \\ r_b+1, u_b+1 \end{pmatrix} \end{matrix} \middle| \begin{matrix} (1-p; 1, 1) \\ (-, -, -) \\ (\kappa_{5a}, [v_a^{-1}]) \\ (\kappa_{6a}, [v_a^{-1}]) \\ (\kappa_{5b}, [v_b^{-1}]) \\ (\kappa_{6b}, [v_b^{-1}]) \end{matrix} \middle| \begin{matrix} \frac{C_a^{1/v_a}}{q\mu_{r_a,a}} \\ \frac{C_b^{1/v_b}}{q\mu_{r_b,b}} \end{matrix} \right], \quad (29)$$

where p and q indicate different modulation schemes parameters [40, Table I].

An asymptotic expression of $\bar{P}$ at high SNR can be derived as

$$\bar{P} \underset{\mu_{r_i,i} > 1}{\approx} \frac{q^p}{2\Gamma(p)} \int_0^\infty \exp(-q\gamma) \gamma^{p-1} \tilde{F}(\gamma) d\gamma. \quad (30)$$

Solving the integration leads to the following:

$$\begin{aligned} \bar{P}_{SC} &\underset{\mu_{r_i,i} > 1}{\approx} \frac{A_{3a} A_{3b}}{2\Gamma(p)} \sum_{k=1}^{u_a} \sum_{x=1}^{u_b} \left(\frac{C_a}{\mu_{r_a,a}^{v_a}} \right)^{\kappa_{4a,k}} \left(\frac{C_b}{\mu_{r_b,b}^{v_b}} \right)^{\kappa_{4b,x}} \\ &\times \frac{\prod_{l=1; l \neq k}^{u_a} \Gamma(\kappa_{4a,l} - \kappa_{4a,k}) \prod_{l=1; l \neq x}^{u_b} \Gamma(\kappa_{4b,l} - \kappa_{4b,x})}{\prod_{l=1}^{r_a} \Gamma(\kappa_{3a,l} - \kappa_{4a,k}) \prod_{l=1}^{r_b} \Gamma(\kappa_{3b,l} - \kappa_{4b,x})} \\ &\times \frac{\Gamma(v_a \kappa_{4a,k} + v_b \kappa_{4b,x} + p)}{\kappa_{4a,k} \kappa_{4b,x}} q^{-(v_a \kappa_{4a,k} + v_b \kappa_{4b,x})}. \end{aligned} \quad (31)$$

Diversity and coding gain can be extracted from (31) if the expression consists of only one term. In particular when the BER is the form $\bar{P} \approx (G_c \mu)^{-G_d}$ [47] where G_c and G_d are the diversity and coding gain respectively, and μ is the average SNR. Upon our observation in other study [37], this is possible if the absolute difference between $\left\{ \frac{\xi_i^2}{v_i r_i}, \frac{\beta_{1i}}{r_i \sigma_i}, \frac{\beta_{2i}}{r_i \lambda_i} \right\} \in \kappa_{4i}$ is significance. If so, then the summation in (31) reduces to a single dominant term results from the $\min \left(\frac{\xi_i^2}{v_i r_i}, \frac{\beta_{1i}}{r_i \sigma_i}, \frac{\beta_{2i}}{r_i \lambda_i} \right)$. In this case, the diversity and coding gain can be written as

$$G_d = v_a \kappa_{4a,k} + v_b \kappa_{4b,x},$$

$$\begin{aligned} G_c &= q \left(\frac{A_{3a} A_{3b} C_a^{\kappa_{4a,k}} C_b^{\kappa_{4b,x}} \Gamma(v_a \kappa_{4a,k} + v_b \kappa_{4b,x} + p)}{2\Gamma(p) \kappa_{4a,k} \kappa_{4b,x}} \right. \\ &\times \left. \frac{\prod_{l=1; l \neq k}^{u_a} \Gamma(\kappa_{4a,l} - \kappa_{4a,k}) \prod_{l=1; l \neq x}^{u_b} \Gamma(\kappa_{4b,l} - \kappa_{4b,x})}{\prod_{l=1}^{r_a} \Gamma(\kappa_{3a,l} - \kappa_{4a,k}) \prod_{l=1}^{r_b} \Gamma(\kappa_{3b,l} - \kappa_{4b,k})} \right)^{-1/G_d}, \end{aligned} \quad (32)$$

where $k \in \{1, r_a + 1, r_a + r_a \sigma_a + 1\}$ and $x \in \{1, r_b + 1, r_b + r_b \sigma_b + 1\}$.

For the Gamma-Gamma case, we have

$$\bar{P}_{SC}^* = \frac{A_{3a}A_{3b}}{2\Gamma(p)} \times G_{1,0:3r_a,1:3r_b,1}^{1,0:r_a+1,3r_a+1:r_b+1,3r_b+1} \left[p \left| \begin{matrix} \kappa_{5a} & \kappa_{5b} \\ \kappa_{6a} & \kappa_{6b} \end{matrix} \right| \frac{C_a}{q\mu_{r_a,a}}, \frac{C_b}{q\mu_{r_b,b}} \right], \quad (33)$$

in agreement with [4, Eq. (19)]

3) *Ergodic Capacity*: The ergodic capacity in FSO systems is defined as

$$\bar{C} \triangleq \mathbb{E}[\log_2(1 + c\gamma)] = \int_0^\infty \log_2(1 + c\gamma) f_\gamma(\gamma) d\gamma, \quad (34)$$

where $c = 1$ for heterodyne detection $r = 1$ and $c = e/2\pi$ for IM/DD $r = 2$. It is very important to note that the expression in (34) is exact for $r = 1$ while it is a lower bound for the case of $r = 2$ [48, Eq. (26)] [49, Eq. (7.43)]. $\log_2(1 + c\gamma)$ is expressed as $\frac{\ln(1+c\gamma)}{\ln(2)}$. $\bar{C}_{SC}$ is obtained by using (34) along with

the identities $\ln(1 + x) = G_{2,2}^{1,2} \left[x \left| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right. \right] = H_{2,2}^{1,2} \left[x \left| \begin{matrix} (1,1),(1,1) \\ (1,1),(0,1) \end{matrix} \right. \right]$ and [42, Eq. (2.3)] yielding

$$\bar{C}_{SC} = \frac{A_{3a}A_{3b}}{v_a v_b \ln(2)} \times \left(v_b \mathbf{H} \left[\begin{matrix} \begin{pmatrix} 0, u_b \\ u_b, r_b \end{pmatrix} \\ \begin{pmatrix} 1,2 \\ 2,2 \end{pmatrix} \\ \begin{pmatrix} u_a, 1 \\ r_a+1, u_a+1 \end{pmatrix} \end{matrix} \left| \begin{matrix} (1-\kappa_{4b}; [v_b^{-1}], [v_b^{-1}]) \\ (1-\kappa_{3b}; [v_b^{-1}], [v_b^{-1}]) \\ (1,1; 1,1) \\ (1,0; 1,1) \\ (\kappa_{5a}; [v_a^{-1}]) \\ (\kappa_{6a}; [v_a^{-1}]) \end{matrix} \right| \begin{matrix} \frac{c\mu_{r_b,b}}{C_b^{1/v_b}} \\ \frac{C_a^{1/v_a} \mu_{r_b,b}}{C_b^{1/v_b} \mu_{r_a,a}} \end{matrix} \right] \right. \\ \left. + v_a \mathbf{H} \left[\begin{matrix} \begin{pmatrix} 0, u_a \\ u_a, r_a \end{pmatrix} \\ \begin{pmatrix} 1,2 \\ 2,2 \end{pmatrix} \\ \begin{pmatrix} u_b, 1 \\ r_b+1, u_b+1 \end{pmatrix} \end{matrix} \left| \begin{matrix} (1-\kappa_{4a}; [v_a^{-1}], [v_a^{-1}]) \\ (1-\kappa_{3a}; [v_a^{-1}], [v_a^{-1}]) \\ (1,1; 1,1) \\ (1,0; 1,1) \\ (\kappa_{5b}; [v_b^{-1}]) \\ (\kappa_{6b}; [v_b^{-1}]) \end{matrix} \right| \begin{matrix} \frac{c\mu_{r_a,a}}{C_a^{1/v_a}} \\ \frac{C_b^{1/v_b} \mu_{r_a,a}}{C_a^{1/v_a} \mu_{r_b,b}} \end{matrix} \right] \right). \quad (35)$$

For the Gamma-Gamma case, we have

$$\bar{C}_{SC}^* = \frac{A_{3a}A_{3b}}{\ln(2)} \times \left(G_{3r_b,0:3r_a,1:1,2}^{3r_b,r_b:r_a+1,3r_a+1:2,2} \left[\begin{matrix} \kappa_{4b} & \kappa_{5a} \\ \kappa_{3b} & \kappa_{6a} \end{matrix} \left| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right| \frac{C_a \mu_{r_b,b}}{C_b \mu_{r_a,a}}, \frac{c\mu_{r_b,b}}{C_b} \right] \right. \\ \left. + G_{3r_a,0:3r_b,1:1,2}^{3r_a,r_a:r_b+1,3r_b+1:2,2} \left[\begin{matrix} \kappa_{4a} & \kappa_{5b} \\ \kappa_{3a} & \kappa_{6b} \end{matrix} \left| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right| \frac{B_b \mu_{r_a,a}}{B_a \mu_{r_b,b}}, \frac{c\mu_{r_a,a}}{B_a} \right] \right). \quad (36)$$

in agreement with [4, Eq. (21)].

B. Performance of Dual-Hop Relay Systems

In this section, we present the performance analysis of a dual-hop (DH) FSO variable gain relay transmission systems whose end-to-end SNR is known to be given by $\gamma_{DH} = (\gamma_1 \gamma_2)/(1 + \gamma_1 + \gamma_2)$ and which is typically approximated by $\gamma_{DH} \simeq \min(\gamma_1, \gamma_2)$ [32]. Thus, results obtained in Section III-B facilitate the following analysis.

1) *Outage Probability*: The outage probability for the minimum SNR of two links is obtained by substituting (18) into (27).

2) *Average Bit Error Rate*: $\bar{P}_{DH}$ is obtained by placing (18) into (28), and utilizing (29) and [43, 2.24.3.2] yielding

$$\bar{P}_{DH} = \frac{A_{3a}v_a^{p-1/2}}{2\Gamma(p)(2\pi)^{\frac{v_a-1}{2}}} \times G_{r_a+v_a+1,u_a+1}^{u_a,v_a+1} \left[C_a \left(\frac{v_a}{q\mu_{r_a,a}} \right)^{v_a} \left| \begin{matrix} \Delta(v_a, 1-p), \kappa_{5a} \\ \kappa_{6a} \end{matrix} \right| \right] \\ + \frac{A_{3b}v_b^{p-1/2}}{2\Gamma(p)(2\pi)^{\frac{v_b-1}{2}}} G_{r_b+v_b+1,u_b+1}^{u_b,v_b+1} \left[C_b \left(\frac{v_b}{q\mu_{r_b,b}} \right)^{v_b} \left| \begin{matrix} \Delta(v_b, 1-p), \kappa_{5b} \\ \kappa_{6b} \end{matrix} \right| \right] \\ - \bar{P}_{SC}. \quad (37)$$

Similar to the SC, we can obtain an asymptotic expression of the BER via Meijer's G-function expansion by solving (30) as follows:

$$\bar{P}_{DH} \underset{\mu_{r_i,i} \gg 1}{\approx} \frac{A_{3a}}{2\Gamma(p)} \sum_{k=1}^{u_a} \left(\frac{C_a}{(q\mu_{r_a,a})^{v_a}} \right)^{\kappa_{4a,k}} \times \frac{\Gamma(v_a \kappa_{4a,k} + p) \prod_{l=1, l \neq k}^{u_a} \Gamma(\kappa_{4a,l} - \kappa_{4a,k})}{\kappa_{4a,k} \prod_{l=1}^{r_a} \Gamma(\kappa_{3a,l} - \kappa_{4a,k})} \\ + \frac{A_{3b}}{2\Gamma(p)} \sum_{x=1}^{u_b} \left(\frac{C_b}{(q\mu_{r_b,b})^{v_b}} \right)^{\kappa_{4b,x}} \times \frac{\Gamma(v_b \kappa_{4b,x} + p) \prod_{l=1, l \neq x}^{u_b} \Gamma(\kappa_{4b,l} - \kappa_{4b,x})}{\kappa_{4b,x} \prod_{l=1}^{r_b} \Gamma(\kappa_{3b,l} - \kappa_{4b,x})} - \bar{P}_{SC}. \quad (38)$$

In this case, diversity or coding can not be extracted due to the fact that the expression consist of 3 terms and can not be reduced to the form of $\bar{P} \approx (G_c \mu)^{-G_d}$. For the case of Gamma-Gamma, we have

$$\bar{P}_{DH}^* = \frac{A_{3a}}{2\Gamma(p)} G_{r_a+2,3a+1}^{3r_a,2} \left[\frac{C_a}{q\mu_{r_a,a}} \left| \begin{matrix} 1-p, \kappa_{5a} \\ \kappa_{6a} \end{matrix} \right| \right] \\ + \frac{A_{3b}}{2\Gamma(p)} G_{r_b+2,3b+1}^{3r_b,2} \left[\frac{C_b}{q\mu_{r_b,b}} \left| \begin{matrix} 1-p, \kappa_{5b} \\ \kappa_{6b} \end{matrix} \right| \right] - \bar{P}_{SC}^*, \quad (39)$$

which agrees with [4, Eq. (22)]

3) *Ergodic Capacity*: Using (34) along with the identities $\ln(1 + x) = G_{2,2}^{1,2} \left[x \left| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right. \right]$, [50, Eq. (12)], and [46, Eq. (07.34.21.0011.01)], we obtain $\bar{C}_{DH}$ in closed- form as

$$\bar{C}_{DH} = \frac{A_{3b}}{\ln(2)(2\pi)^{v_b-1}} G_{r_b+2v_b,v_b}^{u_b+2v_b,v_b} \left[\frac{C_b}{(c\mu_{r_b,b})^{v_b}} \left| \begin{matrix} \kappa_{9b}, \kappa_{3b} \\ \kappa_{4b}, \kappa_{10b} \end{matrix} \right| \right] \\ + \frac{A_{3a}}{\ln(2)(2\pi)^{v_a-1}} G_{r_a+2v_a,v_a}^{u_a+2v_a,v_a} \left[\frac{C_a}{(c\mu_{r_a,a})^{v_a}} \left| \begin{matrix} \kappa_{9a}, \kappa_{3a} \\ \kappa_{4a}, \kappa_{10a} \end{matrix} \right| \right] \\ - \bar{C}_{SC}, \quad (40)$$

where $\kappa_{9i} = \Delta(v_i, 0)$, $\Delta(v_i, 1)$ and $\kappa_{10i} = \Delta(v_i, 0)$, $\Delta(v_i, 0)$.

For the special case of Gamma-Gamma, we have

$$\bar{C}_{DH}^* = \frac{A_{3b}}{\ln(2)} G_{r_b+2,3r_b+2}^{3r_b,2,1} \left[\frac{C_b}{c\mu_{r_b,b}} \left| \begin{matrix} 0,1, \kappa_{3b} \\ \kappa_{4b}, 0,0 \end{matrix} \right| \right] \\ + \frac{A_{3a}}{\ln(2)} G_{r_a+2,3r_a+2}^{3r_a,2,1} \left[\frac{C_a}{c\mu_{r_a,a}} \left| \begin{matrix} 0,1, \kappa_{3a} \\ \kappa_{4a}, 0,0 \end{matrix} \right| \right] - \bar{C}_{SC}^* \quad (41)$$

in agreement with [4, Eq. (23)].

C. Two Data Streams Multiplexing

Assuming two FSO links that transmit independent data to the same destination. The first link requires high data rate

D_1 compared to the second FSO link with a lower data rate D_2 . An initial thought is to use the results obtained from the maximum and the minimum in Sections IV-A and IV-B, respectively. The link providing the maximum capacity $\bar{C}_M$ can be identified and D_1 data rate can be assigned to it. Similarly, the link providing the minimum capacity $\bar{C}_m$ can be identified and D_2 data rate can be assigned to it. In this work, we try to accommodate each link's need and at the same time try to utilize the network resources efficiently and intelligently. We introduce the parameter $\epsilon \in [0, 1]$ which consists of the percentage of time the first data stream is assigned to the channel with the best SNR. Thus, D_1 and D_2 can be given as

$$D_1 = \epsilon \bar{C}_M + (1 - \epsilon) \bar{C}_m, \quad (42)$$

$$D_2 = (1 - \epsilon) \bar{C}_M + \epsilon \bar{C}_m. \quad (43)$$

V. RESULTS AND DISCUSSION

In this section, we verify and validate our analytical results of hybrid FSO communication systems, assuming IM/DD $r = 2$, using computer-based simulations. We also consider RF link experiencing the same channel fading model.

A. Dual-Branch SC Systems

First, we investigate the BER performance of dual-branch FSO/FSO ($r_a = r_b \in \{1, 2\}$) and RF/FSO ($r_a = 1$ and $r_b \in \{1, 2\}$) SC systems under differentially coherent binary phase shift keying (DPSK) modulation in which $p = 1$ and $q = 1$. In this work, we consider two scenarios of atmospheric turbulence conditions, strong (consider the following set of parameters: $\alpha_{1,i} = 1.8621$, $\alpha_{2,i} = 1$, $\beta_{1,i} = 0.5$, $\beta_{2,i} = 1.8$, $\Omega_{1,i} = 1.5074$, and $\Omega_{2,i} = 0.9280$ such that $\lambda_i = 17$ and $\sigma_i = 9$) and moderate (consider the following set of parameters: $\alpha_{1,i} = 2.1690$, $\alpha_{2,i} = 1$, $\beta_{1,i} = 0.55$, $\beta_{2,i} = 2.35$, $\Omega_{1,i} = 1.5793$, and $\Omega_{2,i} = 0.9671$ where $\lambda_i = 28$ and $\sigma_i = 13$).

Figure 3: The impact of pointing error ($\xi = 1.2$) on the average BER of DPSK over (a) single FSO link, (b) dual-branch FSO/FSO and (c) RF/FSO SC in strong turbulence conditions.

First, Fig. 3 shows the impact of pointing errors (i.e strong effect of pointing errors as $\xi \rightarrow 0$) on the performance of

single FSO link and dual-branch SC of FSO/FSO and RF/FSO. We can notice instantly that SC helps to improve the performance comparing to the single link with expense of utilizing more hardware at the receive side. Also, dual-branch RF/FSO SC performance better than symmetric branches FSO/FSO. This encourages the integration of RF into FSO systems or vice versa. Regarding the asymptotic results, they converge fast to exact ones even for low SNR (i.e starting at 30 dB). Interestingly, the asymptotic results representing of all terms in the summation matches perfectly the exact results which give our expression more value instead of using special functions. In the same fashion, two dominants terms asymptotic results provide better match than single dominant term.

Figure 4: The effect of turbulence conditions ((a) Strong turbulence and (b) Moderate conditions) on the average BER of DPSK over single FSO link, dual-branch FSO/FSO and RF/FSO SC with severe pointing error, $\xi = 1.2$

Moreover, Fig. 4 shows the impact of the turbulence conditions on the performance of FSO systems. Clearly, they lead to some degradation. Again, the asymptotic results including all terms in the summation provide an excellent match with the exact result. It is noticeable that for strong conditions the asymptotic expressions including single dominant terms is better than for the moderate conditions and that is due to the fact that the differences between $\{\frac{\xi_{1i}^2}{v_i r_i}, \frac{\beta_{1i}}{r_i \sigma_i}, \frac{\beta_{2i}}{r_i \lambda_i}\}$, $i \in \{a, b\}$, for moderate conditions is not significance.

Secondly, we evaluate another performance metric that is the ergodic capacity of dual-branch SC FSO/FSO and RF/FSO SC as shown in Fig. 5. Generally, SC systems provides higher data rate than a single link system. Also, we can notice that behavior of RF/FSO and FSO/FSO capacity are not in big difference in contrast with BER.

B. Dual-Hop Transmission Systems

We evaluate BER under DPSK modulation and ergodic capacity of FSO relay systems including dual-hop RF-FSO and FSO-FSO links as shown in Figs. 6 and 7. We had to compare the performance of a FSO link that suffers strong turbulence conditions in addition to severe pointing errors with relay assisted link of RF-FSO and FSO-FSO experiencing moderate turbulence conditions with $\xi \gg 1$. Clearly, relay links outperform the single link in addition to providing higher

Figure 5: Ergodic capacity of single FSO link, dual-branch FSO/FSO, RF/FSO SC in strong turbulence conditions and under severe pointing errors, $\xi = 1.2$.

Figure 6: Average BER of DPSK over (a) single FSO link (b) dual-hop FSO-FSO (c) RF-FSO.

Figure 7: Ergodic capacity of single FSO link and dual-hop FSO-FSO and RF-FSO.

data rate than a single FSO link. Moreover, the asymptotic analysis shows an excellent match with the exact results even

for low SNR values (i.e starting at 20 dB).

C. Two Data Streams Multiplexing

Fig. 8 shows the ergodic capacity of our proposed scheme with different values of ϵ . As observed, as ϵ increases, D_1 increases while D_2 decreases and vice versa. By this model, D_1 can get data rate that is close to the maximum capacity the system can provide and at the same time D_2 gets higher data rate than expected.

Figure 8: Capacity Multiplexing of two i.i.d FSO link with $\xi = 6.5$ in strong turbulence conditions

VI. CONCLUSION

Closed-form expressions for the CDF, the PDF, the MGF, and the moments of the maximum and the minimum of two modified double generalized gamma variates were derived. Furthermore, we derived analytical expressions for average bit error rate and ergodic capacity and then applied them to find the performance of both dual-branch hybrid FSO SC scheme and dual-hop hybrid FSO relay system. Monte-Carlo computer simulations were carried out to validate our results. Finally, based on our results for the maximum and minimum, we have proposed a data streams multiplexing scheme for two independent FSO links that compromises each link's data rate judiciously in order to utilize the network available capacity in a flexible fashion.

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