The Diversity-Multiplexing Tradeoff of Secret-Key Agreement over Multiple-Antenna Channels

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Abstract—We study the problem of secret-key agreement between two legitimate parties, Alice and Bob, in presence of an eavesdropper Eve. There is a public channel with unlimited capacity that is available to the legitimate parties and is also observed by Eve. Our focus is on Rayleigh fading quasi-static channels. The legitimate receiver and the eavesdropper are assumed to have perfect channel knowledge of their channels. We study the system in the high-power regime. First, we define the secret-key diversity gain and the secret-key multiplexing gain. Second, we establish the secret-key diversity multiplexing tradeoff (DMT) under no channel state information (CSI) at the transmitter (CSI-T). The eavesdropper is shown to “steal” only transmit antennas. We show that, likewise the DMT without secrecy constraint, the secret-key DMT is the same either with or without full channel state information at the transmitter. This insensitivity of secret-key DMT toward CSI-T features a fundamental difference between secret-key agreement and the wiretap channel, in which secret DMT depends heavily on CSI-T. Finally, we present several secret-key DMT-achieving schemes in case of full CSI-T. We argue that secret DMT-achieving schemes are also key DMT-achieving. Moreover, we show formally that artificial noise (AN), likewise zero-forcing (ZF), is DMT-achieving. We also show that the public feedback channel improves the outage performance without having any effect on the DMT.

Index Terms—Secret-key agreement, outage probability, high-power, diversity multiplexing tradeoff, artificial noise, conceptual wiretap channel.

I. INTRODUCTION

The increasing need for data transmission by mobile terminals is raising more and more concerns about privacy and security of data communicated over the wireless medium. Addressing these concerns efficiently is paramount in the new communication systems design. Traditionally, protection of the transmitted data relies on public-key cryptography, secret-key distribution and symmetric encryption, which is deemed secure based on the assumption of limited computational abilities of a wiretapper. Henceforth, communication is not provably secure. Such techniques are implemented in the higher layers of the protocol stack with little or absent awareness of the

nature of the physical medium. In contrary to this paradigm, information-theoretic security emerged as a solution that guarantees a certain level of security, with no computational restrictions placed on the eavesdropper. Its fundamental idea is to guarantee some secrecy at the physical layer by means of appropriate channel coding techniques. In his seminal paper [1], Shannon introduced the notion of perfect secrecy. His model assumes the existence of a shared key between the legitimate parties that is unknown to the eavesdropper. However, sharing a secret key is not necessary to achieve secrecy. Wyner [2] and Csiszár and Körner [3], later, proved in seminal papers the existence of channel coding guaranteeing not only robustness to transmission errors but also a desired level of data confidentiality, without relying a shared secret-key. In addition to the possibility of transmitting a message securely to a legitimate destination, secret-key agreement arises as one of the promising applications of physical security, in which the objective is to distill a secret-key shared between the legitimate parties. In [4], the achievability of a positive key rate was proved for memoryless binary channels when the destination and the eavesdropper channels are conditionally independent (given the source input symbols), even if the destination channel is not more capable than the eavesdropper channel. The notion of secret sharing is formalized in [5] based on the concept of common randomness between the source and the destination. In the same context, [5] suggests two different system models, called the “source model with wiretapper” (SW) and the “channel model with wiretapper” (CW). The SW model represents a situation in which three parties, two legitimate users and an eavesdropper, observe the realizations of a discrete memoryless source, which is assumed to be outside the control of all parties. The CW model differs in that the source of randomness is controlled by one of the legitimate parties, similar to the basic wiretap channel model [2] with an additional public feedback channel. Extension of both models to the case of secret sharing among multiple users, with the possibility of some terminals acting as helpers has been investigated in [6], [7].

A. Related work

The CW model for secret sharing with conditionally independent destination and eavesdropper channels and continuous channel alphabets is considered in [8], in which the secret-key capacity of such channel is derived. In particular, [8] establishes the secret-key capacity of the ergodic fast-fading multiple-input multiple-output (MIMO) wiretap channel and the corresponding optimal transmit strategy is derived. It is

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shown that for such a channel, the source does not require the instantaneous channel state information (CSI) of the legitimate receiver and the eavesdropper. For quasi-static wireless channels, the secret-key capacity is zero in the absence of CSI [9]. In [9], with full CSI knowledge at all the terminals, closed-form expressions for the low-power and high-power regimes are established.

The use of multiple antennas in fading wireless channels not only increases robustness but also transmission rates. One of the important measures that simultaneously investigates both type of gains is the diversity-multiplexing tradeoff (DMT), introduced in [10]. The DMT is a high power performance analysis that characterizes the fundamental tradeoff between the diversity gain and the multiplexing gain. The diversity gain describes the decay rate of the probability of error, and the multiplexing gain represents the rate of increase of the transmission rate in the high power regime. The DMT analysis and the outage probability are closely related as it is shown that the outage event generally dominates the probability of error in the high-power regime. An outage approach for the wiretap channel is investigated in [11], [12] and [13]. Outage probability for a target secrecy rate is also studied in [14] in the full CSI case where optimal power allocation policies that minimize the outage probability are investigated. The MIMO wiretap channel is investigated in [15] from a DMT point of view. It is shown that in the absence of CSI at the transmitter (CSI-T), the degrees of freedom in the main channel are decreased by the degrees of freedom in the source-eavesdropper channel, and thus the secret DMT depends on the remaining degrees of freedom. Full CSI-T is proven to impact the secret DMT in which case the eavesdropper *steals* only the transmitter antennas. A zero-forcing scheme is also proposed in [15] and shown to achieve the secret DMT in the case of full CSI-T. In [16], a shorter version of the present paper, we studied secret-key agreement with public discussion over Rayleigh fading quasi-static MIMO channels from a DMT perspective. The notions of secret-key multiplexing gain and secret-key diversity gain were defined. The secret-key DMT was established and it was shown that the eavesdropper is stealing antennas only from the source. This behavior was proved to be insensitive to CSI-T.

B. Contribution of the paper

In this paper, we consider secret-key agreement over quasi-static fading channels as in [16]. We define the secret-key multiplexing gain and the secret-key diversity order. First, we establish the secret-key DMT in case of no CSI-T and provide a detailed proof of the secret-key DMT achievability based on the idea of a conceptual wiretap channel. In this case, we show that the secret-key DMT is equivalent to the DMT of a MIMO peaceful system in which the number of effective transmit antennas is reduced by the number of eavesdropper antennas. In the CSI-T case, we show that, unlike secret DMT, CSI-T does not enhance the secret-key DMT and we present several secret-key DMT-achieving schemes. For instance, we argue that secret DMT-achieving schemes are also secret-key DMT-

achieving. In particular, a zero-forcing scheme, shown to be secret DMT-achieving in [15], is secret-key DMT. Moreover, we prove analytically, what has been observed numerically in [15], that artificial noise (AN) is both secret-key and secret DMT-achieving. Finally, we combine a given secret DMT-achieving scheme with the available public channel through a scheme that we refer to as *augmented scheme*. We show that an *augmented* scheme improves the outage performance of secret DMT-achieving scheme while still achieving the secret-key DMT. Numerical results supporting the results are presented.

The remainder of this paper is organized as follows. Section II describes the channel model for secret-key agreement. In Section III, we investigate the secret-key DMT in the no CSI-T case and then extend the arguments in the case of available CSI-T in Section IV. We conclude in Section V.

Notations: Throughout this paper, the symbol † indicates the conjugate transpose of a matrix. I_n and $0_{n \times m}$ denotes respectively the $n \times n$ identity matrix and the $n \times m$ matrix of zeros. For conciseness, we drop the subscripts whenever the matrix dimensions are clear from the context. Random quantities are denoted with capital letters. We use the symbol $\mathbb{I}(\cdot; \cdot)$ to denote the mutual information between two random variables. We denote the entropy and differential entropy of a random quantity with $\mathbb{H}(\cdot)$ and $h(\cdot)$, respectively. The symbols $|\cdot|$ and $\det(\cdot)$ are used to denote the determinant of a matrix and Tr denotes the trace operator. We use the notation $A \succeq 0$ to denote that A is a positive semi-definite matrix. The expression $f_1(SNR) \doteq f_2(SNR)$ is defined as $\lim_{SNR \rightarrow \infty} \frac{\log f_1(SNR)}{\log SNR} = \lim_{SNR \rightarrow \infty} \frac{\log f_2(SNR)}{\log SNR}$. Inequalities are defined analogously.

II. SYSTEM MODEL AND SECRET-KEY DMT

As illustrated in Fig. 1, we consider the CW model for secret-key agreement between a transmitter and a legitimate receiver in a MIMO wireless network in the presence of an eavesdropper. We assume that the transmitter (Alice), the legitimate receiver (Bob) and the eavesdropper (Eve) are equipped with m_S , m_D and m_E antennas, respectively. Transmissions take place over quasi-static fading channels; that is, the MIMO channel matrices $H_D \in \mathbb{C}^{m_D \times m_S}$ and $H_E \in \mathbb{C}^{m_E \times m_S}$ are fixed for the whole duration of the communication. For each channel use, the observation at the legitimate receiver and at the eavesdropper are given by

$$\begin{aligned} Y_D &= H_D X + N_D \\ Y_E &= H_E X + N_E, \end{aligned} \quad (1)$$

where

- X is the $m_S \times 1$ complex-valued transmitted codeword vector by the source,
- Y_D (resp. Y_E) is the $m_D \times 1$ (resp. $m_E \times 1$) complex-valued received codeword vector at the destination (resp. at the eavesdropper),
- H_D (resp. H_E) is the $m_D \times m_S$ (resp. $m_E \times m_S$) channel matrix from the source to the destination (resp. the eavesdropper) with independent identically distributed

Fig. 1. System model of the channel model for secret-key agreement with public discussion.

(i.i.d.) zero-mean unit variance circular-symmetric complex Gaussian entries,

- N_D (resp. N_E) is the $m_D \times 1$ (resp. $m_E \times m_S$) noise vector with i.i.d zero-mean unit variance circular-symmetric complex Gaussian entries.

The transmitter is constrained in its total power, that is equivalent to a trace constraint on the input covariance matrix $K_X = E[XX^\dagger]$, given by

$$\text{Tr}(K_X) \leq m_S \text{ SNR}. \quad (2)$$

Note that as the noise variances are normalized to unity, SNR can be seen as the average signal-to-noise ratio per transmit antenna. In addition to the wireless channel, the transmitter and the receiver have access to an interactive, authenticated public channel with unlimited capacity, over which they can exchange unlimited number of public messages. Here, interactive means that the channel is two-way and can be used multiple times, unlimited capacity means that it is noiseless and has infinite capacity and public and authenticated mean that the eavesdropper can perfectly observe all communications over this channel but cannot tamper with the messages transmitted. For a precise description of a key-distillation strategy, we refer the reader to [5]. Concisely, the transmitter sends his codeword (i.e., n input symbols) over the noisy wireless channel. During this process, both legitimate parties can exchange messages over the public channel. At the end, the source generates its secret-key K and the destination generates its secret-key L , where K and L take values from the same finite set κ . We denote the messages sent over the public channel by the random variable F . R_k is an achievable secret-key rate through the channel (X, Y_D, Y_E) , if for every $\epsilon > 0$, there exists a permissible secret-sharing strategy of the form described above such that

- 1) $P\{K \neq L\} < \epsilon$,
- 2) $\frac{1}{n} \mathbb{I}(K; Y_E^n, F) < \epsilon$,
- 3) $\frac{1}{n} \mathbb{H}(K) > R_k - \epsilon$,

$$4) \frac{1}{n} \log |\kappa| < \frac{1}{n} \mathbb{H}(K) + \epsilon,$$

for sufficiently large n . The key capacity is the supremum of achievable secret-key rates. Condition (1) means the legitimate parties should agree on a common key with high probability. Condition (2) requires that a negligible rate of information about the key should be leaked to the eavesdropper. Condition (3) implies that the key rate is equal to R_k and condition (4) requires the key distribution to be almost uniform. Note that secret-key agreement schemes can be used in conjunction with wiretap codes to enhance the security of confidential messages. An example is to use the secret-key generated at the end of the process as a one-time pad encryption as [17, Equation 4.4]. This provides weak secrecy guarantees if the secret-key satisfies weak secrecy constraints or strong guarantees if the secret-key generates strong secrecy constraints. We note that while our work studies weak secret-keys, there is a systematic way of creating strong secret-keys from weakly secure keys [18]–[20].

In this work, we investigate the high SNR behavior of the probability of error with a target secret-key rate $R_k^{(T)}(\text{SNR})$ that scales with SNR. We define the secret-key multiplexing gain as

$$\lim_{\text{SNR} \rightarrow \infty} \frac{R_k^{(T)}(\text{SNR})}{\log \text{SNR}} \triangleq r_k.$$

The secret-key multiplexing gain indicates how fast the target secret-key rate scales with increasing SNR. We also define the secret-key diversity gain d_k as

$$\lim_{\text{SNR} \rightarrow \infty} \frac{\log P_e(\text{SNR})}{\log \text{SNR}} \triangleq -d_k,$$

where $P_e(\text{SNR})$ denotes the probability of error under secret-key constraint.

From the definition of the CW model for secret-key agreement, the probability of error $P_e(\text{SNR})$ of a system targeting a constant (i.e., does not vary with CSI) key rate $R_k^{(T)}(\text{SNR})$ that scales with SNR is, in general, due to three events:

- E1: reliability constraint violation, i.e., condition (1) in the definition of achievable key rate is not satisfied.
- E2: secret-key secrecy constraint violation, i.e., condition (2) is not satisfied.
- E3: uniformity constraint violation, i.e., condition (4) is not satisfied.

Condition (3) will be satisfied as the codebook size complies with the target key rate. Then,

$$P_e(\text{SNR}) = P(E1 \cup E2 \cup E3). \quad (3)$$

On the other hand, for given channel realization matrices H_D and H_E , the observations of the legitimate receiver and the eavesdropper are conditionally independent given the transmitted signal, i.e., $Y_D \rightarrow X \rightarrow Y_E$ forms a Markov chain. Under the assumption that the channel matrices H_D and H_E are known to all three terminals, [8, Theorem 1] proves that the secret key rate

$$R_k = [\mathbb{I}(X; Y_D) - \mathbb{I}(Y_D; Y_E)] \quad (4)$$

$$= \mathbb{I}(X; Y_D | Y_E) \quad (5)$$

is achievable for any input distribution $p(X)$. The secret-key capacity is given by [8]

$$C_K = \max_{\text{Tr}(K_X) \leq m_S SNR} [\mathbb{I}(X; Y_D) - \mathbb{I}(Y_D; Y_E)]. \quad (6)$$

For a particular key agreement scheme with input distribution $p(X)$, the probability of error is lower bounded by

$$P_e(SNR) \geq P(E1 \cup E2) \quad (7)$$

$$\geq P(\mathbb{I}(X; Y_D) - \mathbb{I}(Y_D; Y_E) < R_k^{(T)}(SNR)) \quad (8)$$

$$\triangleq P(\text{secret-key rate outage}),$$

where the mutual information is evaluated for the chosen input distribution $p(X)$, $R_k^{(T)}(SNR)$ is the target key rate and the probability of secret-key rate outage is the probability that the target key rate is not supported by the channel for the corresponding $p(X)$. As we show in the sequel, when a secret-key rate outage event happens, then we can have a secrecy leakage event or a key disagreement event.

In the following, we study the no CSI-T case. We evaluate the probability of secret-key rate outage to establish a lower bound on $P_e(SNR)$, i.e., to obtain an upper bound on the secret-key diversity gain d_k , then we will proceed to achieve the upper bound. Similarly, we explore the CSI-T case in Section IV.

III. NO CHANNEL STATE INFORMATION AT THE TRANSMITTER

In this section, we assume that the destination and the eavesdropper have perfect CSI of their respective channels from the source. The source, on the other hand, is assumed to have no knowledge of the channel matrices' realizations. The availability of a public channel with infinite capacity implies that this channel could be used to feed-back the source-destination channel matrix to the source. However, in our setting, we restrict our focus to the case where no such a CSI feedback is utilized and thus justifying our no CSI-T assumption. This can be regarded as a worst case scenario from a secret-key DMT perspective. Nevertheless, and as we show in Section IV, the secret-key DMT is insensitive to CSI-T so that even with full CSI-T, the secret-key DMT remains the same. In the setting of no CSI-T, the secret-key capacity is not known. However, for a chosen input distribution $p(X)$, the mutual information in (4) becomes

$$R_k = \mathbb{I}(X; Y_D) - \mathbb{I}(Y_D; Y_E) = h(Y_D | Y_E) - h(Y_D | X) \quad (9)$$

$$= h(Y_D | Y_E) - m_D \log(\pi e). \quad (10)$$

From [8, Lemma 1], in order to maximize R_k , without any loss of optimality, the input distribution can be taken Gaussian with a covariance matrix K_X in which case,

$$R_k = \log \det (K_{Y_D} - K_{Y_D Y_E} K_{Y_E}^{-1} K_{Y_E Y_D}), \quad (11)$$

where K_{Y_D} , K_{Y_E} , $K_{Y_D Y_E}$ and $K_{Y_E Y_D}$ are the covariance and cross-covariance matrices defined by: $K_{Y_D} = \mathbb{E}[Y_D Y_D^\dagger]$,

$K_{Y_E} = \mathbb{E}[Y_E Y_E^\dagger]$, $K_{Y_D Y_E} = \mathbb{E}[Y_D Y_E^\dagger]$ and $K_{Y_E Y_D} = K_{Y_D Y_E}^\dagger$. It is easy to check the following expressions: $K_{Y_D} = H_D K_X H_D^\dagger + I_{m_D}$, $K_{Y_D Y_E} = H_D K_X H_E^\dagger$, $K_{Y_E} = H_E K_X H_E^\dagger + I_{m_E}$ and $K_{Y_E Y_D} = H_E K_X H_D^\dagger$ so that (11) becomes

$$R_k = \log \det \left(I + H_D K_X H_D^\dagger - H_D K_X H_E^\dagger \left(H_E K_X H_E^\dagger + I_{m_E} \right)^{-1} H_E K_X H_D^\dagger \right) \quad (12)$$

$$= \log \det \left(I + H_D K_X \left[I - H_E^\dagger \left(H_E K_X H_E^\dagger + I_{m_E} \right)^{-1} H_E K_X \right] H_D^\dagger \right). \quad (13)$$

Optimizing over all covariance matrices that satisfy the power constraint, the secret-key rate outage probability is expressed as

$$P_{out} \left(R_k^{(T)}(SNR) \right) = \inf_{K_X \succeq 0, \text{Tr}(K_X) \leq m_S SNR} P \left(R_k < R_k^{(T)}(SNR) \right), \quad (14)$$

where the probability is taken over the random channel matrices H_D and H_E .

Lemma 1. The secret-key rate function $R_k(K_X)$ in (13) is an increasing function on the set of covariance matrices (positive definite/semi-definite matrices).

Proof: Suppose K_X is non-singular. Using the matrix inversion lemma, for any non-singular covariance matrix K_X , we write

$$K_X \left[I - H_E^\dagger (H_E K_X H_E^\dagger + I_{m_E})^{-1} H_E K_X \right] = (K_X^{-1} + H_E^\dagger H_E)^{-1}. \quad (15)$$

Let K_1 and K_2 be non-singular covariance matrices such that $K_1 \preceq K_2$ (i.e., $K_2 - K_1$ is positive semi-definite). Then, it follows that $K_1^{-1} \succeq K_2^{-1}$ and $K_1^{-1} + H_E^\dagger H_E \succeq K_2^{-1} + H_E^\dagger H_E$ which also leads to $(K_1^{-1} + H_E^\dagger H_E)^{-1} \preceq (K_2^{-1} + H_E^\dagger H_E)^{-1}$ and $H_D (K_1^{-1} + H_E^\dagger H_E)^{-1} H_D^\dagger \preceq H_D (K_2^{-1} + H_E^\dagger H_E)^{-1} H_D^\dagger$. Combining the last expression with (15) and the fact that $\log \det(\cdot)$ is an increasing function on the cone of positive-definite Hermitian matrices, we obtain $R_k(K_1) \leq R_k(K_2)$.

The case where K_X is singular can be handled by substituting K_X by $K_X + \epsilon I$. The above derivation still holds, then it suffices to let ϵ go to zero in order to prove the result. ■

If we simply pick $K_X = SNR I$, we get an upper bound on the key outage probability. On the other hand, K_X satisfies the power constraint $\text{Tr}(K_X) \leq m_S SNR$, hence $m_S SNR I - K_X \succeq 0$ and $R_k(K_X) \leq R_x(m_S SNR I)$ by virtue of Lemma 1. Accordingly, the secret-key outage probability satisfies

$$P \left(R_k(m_S SNR I) < R_k^{(T)} \right) \leq P_{out}(R_k^{(T)}) \leq P \left(R_k(SNR I) < R_k^{(T)} \right). \quad (16)$$

At high SNR,

$$\lim_{SNR \rightarrow \infty} \frac{P(R_k(SNR I) < R_k^{(T)})}{\log SNR} = \lim_{SNR \rightarrow \infty} \frac{P(R_k(m_S SNR I) < R_k^{(T)})}{\log(m_S SNR)} \quad (17)$$

$$= \lim_{SNR \rightarrow \infty} \frac{P(R_k(m_S SNR I) < R_k^{(T)})}{\log(SNR)}. \quad (18)$$

Therefore, the bounds are tight on the scale of interest. Hence, we have

$$P_{out}(R_k^{(T)}) \doteq P(R_k(SNR I) < R_k^{(T)}). \quad (19)$$

This shows that transmitting independent signals at equal power at each antenna is optimal at high SNR, which complies with the intuition that in absence of CSI, the source has no preference on one direction over the other for its transmission. With this choice of input covariance matrix, the achievable secret-key rate in (13) becomes ¹

$$\bar{R}_k = \log \det(I + SNR H_D \left[I - SNR H_E^\dagger (I_{m_E} + SNR H_E H_E^\dagger)^{-1} H_E \right] H_D^\dagger) \quad (20a)$$

$$= \log \det \left(I + SNR H_D (I + SNR H_E^\dagger H_E)^{-1} H_D^\dagger \right) \quad (20b)$$

$$= \log \frac{\det(I + SNR H_D^\dagger H_D + SNR H_E^\dagger H_E)}{\det(I + SNR H_E^\dagger H_E)}, \quad (20c)$$

where (20c) is obtained using the identity $|I + UV^{-1}U^\dagger| = \frac{|V + U^\dagger U|}{|V|}$.

To establish the key DMT, we first evaluate the secret-key rate outage probability for a given target key rate $R_k^{(T)}(SNR) = r_k \log SNR$.

Lemma 2. *The secret-key outage probability at high SNR is given by:*

$$P_{out}(r_k \log SNR) \doteq \begin{cases} SNR^{-d_{m_S - m_E, m_D}(r_k)} & \text{if } m_E < m_S \\ 1 & \text{else,} \end{cases} \quad (21)$$

where $d_{n,m}$ is the DMT of a MIMO channel with n transmit and m receive antennas.

Proof: For the case $m_E \geq m_S$, it is not hard to check

from (20c) that

$$\lim_{SNR \rightarrow \infty} \bar{R}_k(SNR) = \log \frac{\det(H_D^\dagger H_D + H_E^\dagger H_E)}{\det(H_E^\dagger H_E)}. \quad (22)$$

Indeed, $\bar{R}_k(SNR)$ can be written as

$$\bar{R}_k(SNR) = \log \frac{\det\left(\frac{1}{SNR} I_{m_S} + H_D^\dagger H_D + H_E^\dagger H_E\right)}{\det\left(\frac{1}{SNR} I_{m_S} + H_E^\dagger H_E\right)}. \quad (23)$$

As $H_E^\dagger H_E$ is invertible with probability 1 (w.p.1), using the continuity of the function $\Gamma \mapsto \log \det(\Gamma)$ and taking the limit as $SNR \rightarrow \infty$, we obtain (22). In this case, the secret-key rate levels off and does not increase with SNR, hence the key outage probability does not decrease with SNR and the key DMT reduces to the single point (0,0).

For the case $m_E < m_S$, $H_E H_E^\dagger$ is invertible w.p.1. From (20a), we can write

$$\bar{R}_k(SNR) = \log \det \left(I + SNR H_D \left[I - H_E^\dagger \left(\frac{1}{SNR} I_{m_E} + H_E H_E^\dagger \right)^{-1} H_E \right] H_D^\dagger \right). \quad (24)$$

Defining

$$\bar{R}_\infty(SNR) = \log \det \left(I + SNR H_D \left[I - H_E^\dagger (H_E H_E^\dagger)^{-1} H_E \right] H_D^\dagger \right), \quad (25)$$

and expressing the singular value decomposition (SVD) of H_E as $H_E = U_W [S_W \ 0_{m_S - m_E}] [\tilde{V}_W \hat{V}_W]^\dagger$, [8] shows that $\bar{R}_\infty(SNR)$ can be expressed as

$$\bar{R}_\infty(SNR) = \log \det \left(I + SNR H_D \hat{V}_W \hat{V}_W^\dagger H_D^\dagger \right). \quad (26)$$

It is also shown in [8] that

$$\begin{aligned} 0 &\leq \liminf_{SNR \rightarrow \infty} [\bar{R}_k(SNR) - \bar{R}_\infty(SNR)] \\ &\leq \limsup_{SNR \rightarrow \infty} [\bar{R}_k(SNR) - \bar{R}_\infty(SNR)] \\ &\leq (m_D - j) \log \left(1 + \text{Tr} \left([H_E H_D^\dagger]^\dagger H_E H_D^\dagger \right) \right), \end{aligned} \quad (27)$$

where j is the number of positive eigenvalues of $H_D \hat{V}_W \hat{V}_W^\dagger H_D^\dagger$. Since $H_D \hat{V}_W \hat{V}_W^\dagger H_D^\dagger$ is positive semi-definite, j is equal to its rank. Since the elements of H_D are i.i.d. and are independent of the elements of H_E , then $j = \text{rank}(H_D \hat{V}_W (H_D \hat{V}_W)^\dagger) = \text{rank}(H_D \hat{V}_W) = \min(m_D, m_S - m_E)$. Hence, in case $m_D = \min(m_D, m_S - m_E)$, we obtain immediately

$$\lim_{SNR \rightarrow \infty} \bar{R}_k(SNR) = \lim_{SNR \rightarrow \infty} \bar{R}_\infty(SNR). \quad (28)$$

Otherwise, since the limit of the difference in (27) is finite, we have

$$\bar{R}_k(SNR) \doteq \bar{R}_\infty(SNR). \quad (29)$$

¹The notation $\bar{R}_k$ is used to indicate the key rate evaluated for $K_X = SNR I$.

Hence, (19) becomes

$$P_{out}(R) \doteq P[\bar{R}_k(SNR) < r_k \log SNR] \quad (30)$$

$$\doteq P[\bar{R}_\infty(SNR) < r_k \log SNR] \quad (31)$$

$$= P\left[\log \det(I + SNR H_D \hat{V}_W \hat{V}_W^\dagger H_D^\dagger) < r_k \log SNR\right]. \quad (32)$$

Conditioning on the elements of H_E , $\hat{V}_W$ is a deterministic quantity that depends on H_E . Let $\Psi = H_D \hat{V}_W$, since the columns of $\hat{V}_W$ are orthonormal, then, we can check that Ψ is an $(m_D, m_S - m_E)$ matrix whose entries are i.i.d. Gaussian entries with zero mean and unit variance as in [15]. The system is equivalent to an $(m_S - m_E) \times m_D$ MIMO system without secrecy constraints whose DMT is well known [10].

$$P\left[\log \det(I + SNR H_D \hat{V}_W \hat{V}_W^\dagger H_D^\dagger) < r_k \log SNR | H_E\right] \\ = P\left[\log \det(I + SNR \Psi \Psi^\dagger) < r_k \log SNR | H_E\right] \quad (33)$$

$$\doteq SNR^{-d_{m_S - m_E, m_D}(r_k)}. \quad (34)$$

Averaging over the realizations of H_E as in [15], we obtain the result

$$P_{out}(r_k \log SNR) \doteq SNR^{-d_{m_S - m_E, m_D}(r_k)}. \quad (35)$$

■

The key outage probability provides an upper bound on the key DMT. The achievability of this upper bound is proved in the following proposition.

Proposition 1. For the multiple-antenna CW channel model, if $m_E < m_S$, the secret-key diversity-multiplexing tradeoff is given by the piecewise linear function joining the points $(l, d_k(l))$, where $l = 0, 1, \dots, \min(m_S - m_E, m_D)$ and $d_k(l) = (m_S - m_E - l)(m_D - l)$.

If $m_E \geq m_S$: then the secret-key diversity multiplexing tradeoff reduces to the single point $(0, 0)$.

Proof: We already obtained a lower bound on the probability of error. To achieve the lower bound, we use the same scheme proposed in the achievability proof of [8, theorem 1]. The scheme is based on the idea of a conceptual wiretap channel from the destination back to the source which leverages the results of wiretap codes as well as the existence of a public channel with unlimited capacity.

We recall briefly the steps of the scheme. First, the source generates randomness by sending a sequence of i.i.d. codewords X^n over the wiretap channel such that $E[|X|^2] \leq m_S SNR$ (so that the power constraint is satisfied). Then, the destination generates, at the target key rate, a sequence U^n randomly and independent of (X^n, Y_D^n, Y_E^n) , observes the realization Y_D^n and sends $U^n + Y_D^n$ back to the source through the public channel. This creates a conceptual wiretap channel with input symbol U , for which the source plays now the role of a legitimate receiver and observes $(U + Y_D, X)$ and the eavesdropper observes $(U + Y_D, Y_E)$.

In this scheme, the error events are the same error events in

a system with secrecy constraints, which is studied in [15].

$$P_e(SNR) = P(\text{secrecy not achieved} \\ \text{or main channel decoding error}) \quad (36)$$

$$\leq P(\text{secrecy not achieved}) \\ + P(\text{main channel decoding error}), \quad (37)$$

where, as in [15], we have

$$P(\text{secrecy not achieved}) = P(\mathbb{I}(U; (U + Y_D, Y_E)) > R^d) \quad (38)$$

$$P(\text{main channel decoding error}) \\ \doteq P(\mathbb{I}(U; (U + Y_D, X)) < R^T), \quad (39)$$

where $R^T = R_k^{(T)} + R^d$ denotes the total rate communicated through the conceptual wiretap channel and R^d is the dummy codewords rate. Since our target key rate scales as $R_k^{(T)} = r_k \log(SNR)$, we have $R^T(SNR) = r_k \log(SNR) + R^d(SNR)$. In order to choose the rates R^T and R^d , we evaluate the mutual informations involved. Let X Gaussian with $K_X = SNR I$.

We first evaluate the mutual information of the main channel.

$$\mathbb{I}(U; U + Y_D, X) \\ = \mathbb{I}(U; X) + \mathbb{I}(U; U + Y_D | X) \\ = h(U) - h(U + Y_D | X, U) + h(U + Y_D | X) - h(U | X). \quad (40)$$

Because U is independent of (X, Y_D, Y_E) , we have $h(U + Y_D | X, U) = h(Y_D | X, U) = h(Y_D | X)$. On the other hand, the term $h(U + Y_D | X) - h(U | X)$ satisfies

$$h(U + Y_D | X) - h(U | X) \geq h(U + Y_D | X, Y_D) - h(U | X) \\ = h(U | X, Y_D) - h(U) = 0$$

$$h(U + Y_D | X) - h(U | X) \leq h(U + Y_D) - h(U).$$

Let U be Gaussian with covariance $\sigma_U^2 I$. Using [21, Theorem 8.6.5] and the independence between U and Y_D , the last inequality becomes

$$h(U + Y_D | X) - h(U | X) \\ \leq \log \det(\pi e K_U + K_{Y_D}) - \log \det(\pi e K_U) \quad (41)$$

$$= \log \frac{\det(K_U + K_{Y_D})}{\det K_U}. \quad (42)$$

Now, since the input symbol U has no power constraint, we can choose σ_U^2 large enough such that for small $\epsilon > 0$, we have

$$\log \frac{\det(K_U + K_{Y_D})}{\det K_U} \leq \epsilon.$$

Indeed, let $L = \min(m_D, m_E)$ and $\lambda_i, i = 1, \dots, L$ be the positive eigenvalues of $H_D H_D^\dagger$, then

$$\log \frac{\det(K_U + K_{Y_D})}{\det K_U} = \log \frac{\det(\sigma_U^2 I + SNR H_D H_D^\dagger + I)}{\det(\sigma_U^2 I)} \\ = \left(\frac{1 + \sigma_U^2}{\sigma_U^2}\right)^{m_D - L} \prod_{i=1}^L \frac{1 + \sigma_U^2 + \lambda_i SNR}{\sigma_U^2}.$$

Hence, we have

$$\log \frac{\det(K_U + K_{Y_D})}{\det K_U} = (m_D - L) \log(1 + \frac{1}{\sigma_U^2}) + \sum_{i=1}^L \log(1 + \frac{1}{\sigma_U^2} + \frac{\lambda_i SNR}{\sigma_U^2}). \quad (43)$$

Therefore, if we let σ_U^2 scale as $\sigma_U^2 = (1 + \lambda_{max}) SNR^{1+\delta}$ for $\delta > 0$ (recall that the destination knows its channel, hence it knows λ_{max}), then we can write for any $\epsilon > 0$ as small as desired and for high SNR

$$0 \leq h(U + Y_D|X) - h(U|X) \leq \log \frac{\det(K_U + K_{Y_D})}{\det K_U} \leq \epsilon.$$

Hence, for such choice of $K_U(\epsilon)$, we obtain

$$\begin{aligned} \mathbb{I}(U; U + Y_D, X) &\doteq h(U) - h(Y_D|X) + o(\epsilon) \\ &\doteq h(U) - h(Y_D|X). \end{aligned} \quad (44)$$

Now (39) becomes

$$\begin{aligned} P(\text{main channel decoding error}) &\doteq P(h(U) - h(Y_D|X) < R^T(SNR)) \\ &= P(\log \det(\pi e K_U) - m_D \log(\pi e) < R^T(SNR)) \end{aligned} \quad (45)$$

$$= P(\log \det(K_U) - m_D \log(SNR) < R^d). \quad (46)$$

$$= P(\log \det(K_U) < r_k \log(SNR) + R^d). \quad (47)$$

Equation (47) represents the first term of the upper bound in (36) and it will determine later the choice of the dummy-codewords rate. We evaluate the probability of secrecy not achieved. Following the same procedure as above (or simply replacing X by Y_E), and with the choice of K_U as explained above, we write similarly

$$\mathbb{I}(U; U + Y_D, Y_E) = h(U) - h(Y_D|Y_E) + o(\epsilon). \quad (48)$$

Applying [22, Lemmas 3 and 4], $[Y_D^T Y_E^T]^T$ is circular-symmetric complex Gaussian random vector. Hence, using [8, Lemma 1], we obtain

$$\begin{aligned} h(Y_D|Y_E) &= \log \det(K_{Y_D} - K_{Y_D Y_E} K_{Y_E}^{-1} K_{Y_E Y_D}) \\ &\quad + m_D \log(\pi e) \end{aligned} \quad (49)$$

$$= \bar{R}_k(SNR) + m_D \log(\pi e). \quad (50)$$

Hence,

$$\begin{aligned} P(\text{secrecy not achieved}) &\doteq P(h(U) - h(Y_D|Y_E) > R^d) \\ &= P(\log \det(\pi e K_U) - \bar{R}_k(SNR) - m_D \log(\pi e) > R^d) \end{aligned} \quad (51)$$

$$= P(\log \det(K_U) - \bar{R}_k(SNR) > R^d). \quad (52)$$

$$= P(\bar{R}_k(SNR) < \log \det(K_U) - R^d). \quad (53)$$

Equations (47) and (51) determine the upper bound on the total probability of error (36). Let $K_X = SNR I$, and if we let

$$R^d(SNR) = \log \det(K_U) - r_k \log(SNR), \quad (54)$$

for $r_k = 0, \dots, \min(m_S - m_E, m_D)$. Based on Lemma 2, we

obtain

$$\begin{aligned} P(\text{secrecy not achieved}) &\doteq SNR^{-d_{m_S - m_E, m_D}(r_k)}, \\ P(\text{main channel decoding error}) &\doteq 0. \end{aligned} \quad (55)$$

Overall, the upper bound on the probability of error (36) becomes

$$P_e(SNR) \leq SNR^{-d_{m_S - m_E, m_D}(r_k)}. \quad (56)$$

Therefore, we conclude that the secret-key DMT is equal to $d_{m_S - m_E, m_D}(r_k)$ if $m_S > m_E$. If $m_S \leq m_E$, the key outage probability does not scale with SNR and the key DMT reduces consequently to the single point (0,0). ■

Proposition 1 states that the eavesdropper steals m_E antennas from the source only, but does not affect the destination. The key DMT is clearly higher than the secret DMT for which it is shown in [15] that the eavesdropper steals m_E from both legitimate parties. This advantageous behavior of the key DMT is explained by the availability of the public channel with infinite capacity that compensates for the absence of CSI-T at the source.

When the degrees of freedom in the source-eavesdropper channel, $\min(m_S, m_E)$ is equal to m_S , then no degrees of freedom are left for the legitimate users and the secret-key DMT, as the key DMT, reduces to the point (0,0).

On the other hand, when $m_E < m_S$, the CW MIMO system becomes equivalent to an $(m_S - m_E) \times m_D$ MIMO system, from a DMT point of view.

Fig. 2. The DMT without secrecy constraint, secret DMT with no CSI-T and key DMT. The source, the destination and the eavesdropper have 2, 4 and 1 antennas, respectively.

In Fig. 2, the key DMT is shown for $m_S = 2, m_D = 4$ and $m_E = 1$. In this configuration, the DMT without secrecy constraints, the secret DMT with no CSI-T and the key DMT are shown to be respectively equal to $d_{2,4}(r)$, $d_{1,3}(r_s)$ and $d_{1,4}(r_k)$. In this example, secrecy and secret-key constraints impose both multiplexing gain and diversity gain losses compared the classical DMT. Nonetheless, the secret-key DMT outperforms the secret DMT in terms of diversity gain.

In Fig. 3, the key DMT is shown for $m_S = 5, m_D = 3$ and $m_E = 2$. In this configuration, the DMT without secrecy constraints, the secret DMT with no CSI-T and the key DMT

Fig. 3. The DMT without secrecy constraint, secret DMT with no CSI-T and key DMT. The source, the destination and the eavesdropper have 5, 3 and 2 antennas, respectively.

are shown to be respectively equal to $d_{5,3}(r)$, $d_{3,1}(r_s)$ and $d_{3,3}(r_k)$. We clearly see that the secrecy constraint, as in the previous example, imposes not only a diversity gain loss but also a multiplexing gain loss for the secret DMT with no CSI-T in comparison with the DMT without secrecy constraint. On the other hand, the key DMT experiences only a diversity gain loss but still achieves all multiplexing gains. In fact, as highlighted in [15], the secret DMT with no CSI-T always experiences multiplexing gain loss (m_E degrees of freedom) while the key DMT can achieve all multiplexing gains in case $m_S - m_E \geq m_D$ albeit it does experience a secret-key diversity gain loss.

Remark 1. If the destination has access to the realization of the eavesdropper channel H_E , then no secret-key leakage can be guaranteed in the achievability proof. Indeed, the destination can adapt its dummy codewords rate to its channel to the eavesdropper and set $R^d = \log \det K_U - R_K(SNR)$. Then, (47) and (51) become

$$P(\text{main channel decoding error}) \doteq P(\log \det K_U < r_k \log(SNR) + R^d) \quad (57)$$

$$= P(R_k(SNR) < r_k \log(SNR)) \quad (58)$$

$$= P(\text{secret-key rate outage}) \quad (59)$$

$$\doteq SNR^{-d_{m_S - m_E, m_D}(r_k)} \quad (60)$$

$$P(\text{secrecy not achieved}) \doteq 0. \quad (61)$$

Furthermore, the use of the input covariance matrix $K_X = SNR I$ for the conceptual wiretap channel scheme is not restrictive. In fact, as long as the input covariance matrix K_X achieves the same key rate outage probability, it could be employed as will be discussed further in the following section.

IV. CHANNEL STATE INFORMATION AT THE TRANSMITTER

In the previous section, secret-key DMT was investigated with no CSI-T. In this section, we make the assumption that the source has full knowledge about its channel to the

destination as well as its channel to the eavesdropper. Though it is arguable that such assumptions are more of a theoretical interest than of a practical regard, investigating the system under this setup will help us understand the fundamental limits of secret-key DMT.

In the next subsections, we establish the secret-key DMT and we revisit some schemes that achieve the key DMT with CSI-T.

A. secret-key DMT with CSI-T

We recall the expression of the secret-key capacity (6), known from [9] and given by :

$$C_K = \max_{\text{Tr}(K_X) \leq m_S SNR} \mathbb{I}(X; Y_D | Y_E).$$

Having full CSI-T enables the source to determine the optimal covariance matrix achieving the key capacity, or equivalently minimizing the secret-key rate outage probability. Hence, for K_X attaining the maximum in (6), the secret-key rate outage probability is given by

$$P_{out}(r_k \log SNR) = P\left(\max_{K_X \geq 0, \text{Tr}(K_X) \leq m_S SNR} \mathbb{I}(X; Y_D | Y_E) < r_k \log(SNR)\right) \quad (62)$$

$$= \min_{\text{Tr}(K_X) \leq m_S SNR} P(\mathbb{I}(X; Y_D | Y_E) < r_k \log(SNR)) \quad (63)$$

$$\doteq SNR^{-d_{m_S - m_E, m_D}(r_k)}, \quad (64)$$

where the last equality follows from (19) and Lemma 2. This states that even though $K_X = SNR I$ would not necessarily be the optimal covariance matrix maximizing the secret-key capacity, from a key DMT perspective, splitting the available power equally among the different source antennas is optimal. Thus, the key outage probability is the same in case of CSI-T. Therefore, the same upper bound on the secret-key diversity still holds in the CSI-T case. Achieving this upper bound is straightforward. The schemes based on the conceptual wiretap channel, as described in the proof of Proposition 1 or in Remark 1, can be used since they do not require CSI-T to achieve the key DMT.

Recall that for the MIMO wiretap channel, a uniform power allocation is not optimal from the secret DMT perspective [15]. This observation highlights again another difference between the key agreement setup and coding for secrecy problem. In fact, splitting the power equally does not achieve the secret outage probability, but rather results in the eavesdropper stealing both transmitter and receiver antennas. To achieve the maximum secret DMT in case of CSI-T, a more sophisticated input covariance matrix must be used as shown in [15]. In Fig. 4, we illustrate the latter idea. We compare the secrecy outage probability and the secret-key outage probability for a 2-antenna source, a 2-antenna destination and a single antenna eavesdropper, using the input covariance matrix $K_X = SNR I$ in both cases. The target multiplexing gain is also the same, and it is assumed to be equal to 0.8; thus the secret diversity gain is equal to 0.2, whereas the key diversity

gain is equal to 0.4. Fig. 4 confirms these results and highlights this disparity between the two scenarios.

Fig. 4. The source, the destination and the eavesdropper have 2, 2 and 1 antennas, respectively.

Remark 2. Based on the previous analysis, perfect knowledge of CSI-T does not enhance the DMT compared to the case of no CSI-T. The case of estimated CSI-T lies in between these both extremes, which are characterized by the same DMT performance. Thus, we conclude that estimated CSI-T achieves, likewise, the same DMT.

Remark 3. For a single input single output (SISO) system, one can always achieve a positive secret-key rate with perfect CSI-T. However, from a DMT perspective, the achievable multiplexing gain reduces to zero, as the secret-key capacity does not scale with the input power for a SISO system.

B. Other Schemes achieving the key DMT

In the CSI-T case, the secret DMT coincides with the key DMT. This is relevant since a wiretap code over the wiretap channel (X, Y_D, Y_E) is a key-agreement strategy for the CW model. Any scheme achieving the secret DMT could hence be employed to achieve the same key DMT without relying on the existence of the public channel.

A wiretap code over the main channel with the optimum covariance matrix maximizing the secrecy capacity was used to prove the achievability of the secret DMT in [15].

A Zero-forcing scheme is also suggested and proved to achieve the secret DMT. It consists in transmitting information in the nullspace of the eavesdropper, which guarantees not only secrecy but also a good outage performance. Let U represent the secret information, which is a length $m_S - m_E$ and A be an $m_S \times m_S - m_E$ matrix whose columns form an orthonormal basis of the nullspace of H_E . Then, the transmitted signal can be expressed as $X = AU$. The achievable secrecy rate is shown in [15] to be

$$R^{ZF} = \log \det \left(I_{m_D} + \frac{m_S}{m_S - m_E} \text{SNR} H_D A A^\dagger H_D^\dagger \right). \quad (65)$$

Instead of using the nullspace of the eavesdropper channel matrix, we can exploit the nullspace of the destination matrix

in case $m_D > m_S$, using an artificial noise scheme. Artificial noise is presented in [23]. It consists in transmitting information in the range space of the destination channel matrix H_D along with transmitting extra noise in the nullspace of H_D . It is a semi-blind scheme since it requires only the knowledge of H_D to construct the signaling codebook. Let T be an $m_S \times (m_S - m_E)$ matrix, whose columns form an orthonormal basis for the nullspace of H_D , V an $m_S - m_D$ column vector with i.i.d. zero mean complex Gaussian entries that represent the extra noise, and S be $m_S - m_D$ column vector that carries source messages. Then, the transmitter sends $X = S + TV$. The achievable secrecy rate is shown in [15] to be

$$R_s^{AN} = \log |I_{m_D} + \frac{\text{SNR}}{2} H_D H_D^\dagger| - \log \frac{|K + \frac{\text{SNR}}{2} H_E H_E^\dagger|}{|K|}, \quad (66)$$

where $K = I + \frac{\text{SNR}}{2(m_S - m_D)} H_E T T^\dagger H_E^\dagger$. It is suggested in [15] through simulations, but without a formal proof, that the artificial noise scheme achieves the secret DMT. We formally establish its secret DMT optimality by leveraging our result for the key rate outage expression.

Proposition 2. The artificial noise scheme achieves the full secret DMT in case of CSI-T.

Proof: See Appendix A ■

DMT-optimality of ZF and AN over the wiretap channel with public discussion suggests that the public channel has in fact no value from a DMT point of view. However, as we show below, the public channel may be useful in improving the outage performance. An intuitive way of enhancing the performance of a secret DMT-achieving scheme is to use its corresponding covariance matrix structure to create randomness through the main channel and then communicate the key over the conceptual wiretap channel. We refer to the new scheme obtained that way as the augmented scheme. The outage performance of an augmented scheme is expected to be higher, and this idea is corroborated by the following lemma.

Lemma 3. For a given covariance matrix K_X , the achievable secret rate $R_s(K_X)$ is lower than the achievable key rate $R_k(K_X)$.

Proof: Let K_X be a non-singular input covariance matrix, we have from (13) and (15),

$$R_k(K_X) = \log \det \left(I + H_D \left(K_X^{-1} + H_E^\dagger H_E \right)^{-1} H_D^\dagger \right) \quad (67)$$

$$= \log \frac{\det \left(K_X^{-1} + H_E^\dagger H_E + H_D^\dagger H_D \right)}{\det \left(K_X^{-1} + H_E^\dagger H_E \right)}, \quad (68)$$

where the last equality is obtained using the identity $|I + UV^{-1}U^\dagger| = \frac{|V + U^\dagger U|}{|V|}$. We also know from [15] that the achievable secrecy rate is

$$R_s(K_X) =$$

$$\left[\log \det \left(I + H_D K_X H_D^\dagger \right) - \log \det \left(I + H_E K_X H_E^\dagger \right) \right]^+.$$

We consider the case where $R_s(K_X) \geq 0$, otherwise the inequality is trivial. Using the identity $|I + AB| = |I + BA|$, we write

$$R_s(K_X) = \log \det \left(I + H_D K_X H_D^\dagger \right) - \log \det \left(I + H_E^\dagger H_E K_X \right) \quad (69)$$

$$= \log \det \left(I + H_D K_X H_D^\dagger \right) - \log \det \left(K_X^{-1} + H_E^\dagger H_E \right) \det(K_X) \quad (70)$$

$$= \log \frac{\det \left(I + H_D K_X H_D^\dagger \right)}{\det \left(K_X^{-1} + H_E^\dagger H_E \right) \det(K_X)}. \quad (71)$$

Hence, we obtain

$$R_k(K_X) - R_s(K_X) = \log \det \left(K_X^{-1} + H_E^\dagger H_E + H_D^\dagger H_D \right) - \log \frac{\det \left(I + H_D K_X H_D^\dagger \right)}{\det(K_X)} \quad (72)$$

$$= \log \det \left(I + K_X \left(H_E^\dagger H_E + H_D^\dagger H_D \right) \right) - \log \det \left(I + K_X H_D^\dagger H_D \right) \quad (73)$$

$$= \log \det \left(I + K_X^{\frac{1}{2}} \left(H_E^\dagger H_E + H_D^\dagger H_D \right) K_X^{\frac{1}{2}} \right) - \log \det \left(I + K_X^{\frac{1}{2}} H_D^\dagger H_D K_X^{\frac{1}{2}} \right) \quad (74)$$

$$\geq 0, \quad (75)$$

where $K_X^{\frac{1}{2}} \succeq 0$ denotes the square root of K_X , i.e., $K_X = K_X^{\frac{1}{2}} K_X^{\frac{1}{2}}$ and the last inequality is due to

$$I + K_X^{\frac{1}{2}} (H_E^\dagger H_E + H_D^\dagger H_D) K_X^{\frac{1}{2}} \succeq I + K_X^{\frac{1}{2}} H_D^\dagger H_D K_X^{\frac{1}{2}}.$$

The case where K_X is singular can be handled by substituting K by $K + \epsilon I$. The above derivation is hence applicable, then it suffices to let ϵ go to zero to obtain the desired result. ■

Lemma 3 also provides a simple way to check that for any scheme achieving secret DMT over the wiretap channel, the corresponding augmented scheme achieves as well the key DMT. Indeed, we note that

$$R_s(K_X) \leq R_k(K_X) \leq C_k, \quad (76)$$

where C_k denotes the key capacity. Assuming $P(R_s(K_X) < r_k \log \text{SNR}) \doteq \text{SNR}^{-d_{m_S - m_E, m_D}(r_k)}$, we also know from Lemma 2 that $P(C_k \leq r_k \log \text{SNR}) \doteq \text{SNR}^{-d_{m_S - m_D, m_E}(r_k)}$, then it follows immediately that $P(R_k(K_X) \leq r_k \log \text{SNR}) \doteq \text{SNR}^{-d_{m_S - m_D, m_E}(r_k)}$.

This reasoning applies to the artificial noise where we clearly see from Fig. 5 a performance enhancement of the corresponding augmented scheme.

However, no benefit is obtained by “augmenting” the zero-forcing scheme since $K_X^{ZF} = \frac{m_S}{m_S - m_D} \text{SNR} H_E^\perp$, where $H_E^\perp$

denotes the projection matrix onto nullspace of H_D ; which implies that $H_E K_X^{ZF} = 0$ resulting in $R_s(K_X^{ZF}) = R_k(K_X^{ZF})$.

Fig. 5. Outage performance of different schemes with full CSI-T knowledge. The source, the destination and the eavesdropper have 3, 2 and 1 antennas, respectively, $r_k=1.75$.

In Fig. 5, the outage performance of the described schemes achieving the key DMT is represented in case of full CSI. The schemes are the conceptual wiretap scheme with equally-split (uniform) power, the zero-forcing and the artificial noise schemes along with their corresponding ‘augmented’ schemes. The figure confirms that the schemes achieve a secret-key diversity 0.25 for a key multiplexing gain 1.75. Another observation is that transmitting the secret-key in the nullspace of the eavesdropper is the most attractive scheme for two reasons. First, it performs better in terms of outage probability compared to the artificial noise and a uniform power distribution schemes. Second, it does not require the public channel to achieve its performance, hence the cost of using the public channel can be saved.

V. CONCLUSION

In this paper, we have investigated the secret-key diversity multiplexing tradeoff of Rayleigh fading quasi-static MIMO channels. We have established the secret-key DMT for arbitrary number of antennas at the transmitter, the destination and the eavesdropper. First, we have studied the case of no-CSI-T. Our analysis shows that, in the high-power regime, a uniform power allocation achieves the maximum exponent of the probability of error in the considered regime. This means that distributing the power uniformly across the transmit antennas is optimum from a DMT perspective. We have also shown that the transmitter obtains secret-key diversity only if his number of antennas is strictly greater than the eavesdropper’s antennas, i.e., $m_S > m_E$. In this case, the eavesdropper steals m_E antennas from the source and the secret-key DMT is equivalent to that of a $(m_S - m_E) \times m_D$ MIMO system without secrecy constraints. Furthermore, we have outlined that the secret-key DMT is insensitive to CSI-T. This observation implies that coding over the wiretap channel, without the need of the public channel, is sufficient to achieve the full secret-key DMT, since in this case the secret DMT coincides with the secret-key DMT. In particular, we

have proved analytically that artificial noise, likewise zero-forcing, is secret DMT and key DMT-achieving. In the CSI-T case, it is shown that the public feedback channel improves the outage performance without having any effect on the DMT.

APPENDIX A

PROOF OF PROPOSITION 2

In the artificial noise scheme, with the assumption that the transmitter has full knowledge of both channels matrices, the achievable secrecy rate can be written as [15]

$$R_s^{AN} = \log |I_{m_D} + \frac{SNR}{2} H_D H_D^\dagger| - \log \frac{|K + \frac{SNR}{2} H_E H_E^\dagger|}{|K|},$$

where $K = I + \frac{SNR}{2(m_S - m_D)} H_E T T^\dagger H_E^\dagger$ and T is an $(m_S \times m_S - m_D)$ matrix whose columns form an orthonormal basis for $\text{Null}(H_D)$.

Let $H_D = U [\Delta \quad 0_{m_D \times m_S - m_D}] \begin{bmatrix} T_2^\dagger \\ T^\dagger \end{bmatrix}$ be the SVD of H_D

where U and $\begin{bmatrix} T_2 & T \end{bmatrix}$ are unitary matrices.

The secrecy rate outage probability is expressed as

$$\begin{aligned} P_{out}^{AN}(SNR) &= P \left(\log |I_{m_D} + \frac{SNR}{2} H_D H_D^\dagger| - \log \frac{|K + \frac{SNR}{2} H_E H_E^\dagger|}{|K|} \right. \\ &\quad \left. < r_s \log(SNR) \right) \end{aligned} \quad (77)$$

$$\begin{aligned} &= P \left(\log |I_{m_D} + \frac{SNR}{2} H_D H_D^\dagger| + \log |K| \right. \\ &\quad \left. - \log |K + \frac{SNR}{2} H_E H_E^\dagger| < r_s \log(SNR) \right) \end{aligned} \quad (78)$$

$$\begin{aligned} &= P \left(\log (|I_{m_D} + \frac{SNR}{2} H_D H_D^\dagger| |K|) \right. \\ &\quad \left. - \log |K + \frac{SNR}{2} H_E H_E^\dagger| < r_s \log(SNR) \right). \end{aligned} \quad (79)$$

We develop now each term separately. Using the fact that $H_d T = 0$ (since the columns of T span the nullspace of H_d), we write

$$\begin{aligned} A &= |I_{m_D} + \frac{SNR}{2} H_D H_D^\dagger| |K| \\ &= |I_{m_S} + \frac{SNR}{2} H_D^\dagger H_D| \left| I_{m_S} + \frac{SNR}{2(m_S - m_D)} T T^\dagger H_E^\dagger H_E \right| \\ &= \left| I_{m_S} + \frac{SNR}{2} H_D^\dagger H_D + \frac{SNR}{2(m_S - m_D)} T T^\dagger H_E^\dagger H_E \right|. \end{aligned} \quad (80)$$

Let $B = |K + \frac{SNR}{2} H_E H_E^\dagger|$. We carry out the proof on three steps. First, we prove that

$$A \doteq |I_{m_S} + SNR H_D^\dagger H_D + SNR H_E^\dagger H_E|.$$

Let $A_\infty = |I_{m_S} + \frac{SNR}{2} H_D^\dagger H_D + \frac{SNR}{2(m_S - m_D)} H_E^\dagger H_E|$. Since $\begin{bmatrix} T_2 & T \end{bmatrix}$ is unitary, then $T_2^\dagger T_2 + T^\dagger T = I$, which implies

$$\begin{aligned} A &= \left| I_{m_S} + \frac{SNR}{2} H_D^\dagger H_D + \frac{SNR}{2(m_S - m_D)} H_E^\dagger H_E \right. \\ &\quad \left. - \frac{SNR}{2(m_S - m_D)} T_2^\dagger T_2 H_E^\dagger H_E \right|. \end{aligned} \quad (81)$$

Then, we write

$$\begin{aligned} \log A_\infty - \log A &= -\log \left| I - \left(I + \frac{SNR}{2} H_D^\dagger H_D \right. \right. \\ &\quad \left. \left. + \frac{SNR}{2(m_S - m_D)} H_D^\dagger H_D \right)^{-1} \frac{SNR}{2(m_S - m_D)} T_2^\dagger T_2 H_E^\dagger H_E \right| \end{aligned} \quad (82)$$

$$\begin{aligned} &= -\log \left| I - \left(I + \frac{SNR}{2} H_{eq}^\dagger H_{eq} \right)^{-1} \right. \\ &\quad \left. \frac{SNR}{2(m_S - m_D)} T_2^\dagger T_2 H_E^\dagger H_E \right|, \end{aligned} \quad (83)$$

where $H_{eq} = \begin{bmatrix} H_D \\ \frac{1}{\sqrt{m_S - m_D}} H_E \end{bmatrix}$. From the SVD decomposition of H_D , we can easily check that $T_2^\dagger T_2 = H_D^\dagger (H_D H_D^\dagger)^{-1} H_D$. Indeed, $H_D^\dagger = T_2^\dagger \Delta U^\dagger$ which implies $T_2 = H_D^\dagger U \Delta^{-1}$ and $T_2^\dagger T_2 = H_D^\dagger U \Delta^{-2} U^\dagger H_D = H_D^\dagger (H_D H_D^\dagger)^{-1} H_D$. Then, (83) becomes

$$\begin{aligned} \log A_\infty - \log A &= -\log \left| I - \frac{1}{m_S - m_D} \left(\frac{1}{SNR/2} I + H_{eq}^\dagger H_{eq} \right)^{-1} \right. \\ &\quad \left. H_D^\dagger (H_D H_D^\dagger)^{-1} H_D H_E^\dagger H_E \right| \end{aligned} \quad (84)$$

$$\begin{aligned} &= -\log \left| I - \frac{1}{m_S - m_D} \left(\frac{1}{SNR/2} I + H_{eq}^\dagger H_{eq} \right)^{-1} \right. \\ &\quad \left. H_{eq}^\dagger C \right|, \end{aligned} \quad (85)$$

where the last equality is obtained by writing $H_D^\dagger = \begin{bmatrix} H_D^\dagger & \frac{1}{\sqrt{m_S - m_D}} H_E^\dagger \end{bmatrix} \begin{bmatrix} I_{m_D \times m_D} \\ 0_{m_E \times m_D} \end{bmatrix}$, and $C = \begin{bmatrix} I_{m_D \times m_D} \\ 0_{m_E \times m_D} \end{bmatrix} (H_D H_D^\dagger)^{-1} H_D H_E^\dagger H_E$. Using the limit property of a pseudo-inverse [24], we have

$$\lim_{SNR \rightarrow \infty} \left(\frac{1}{SNR/2} I + H_{eq}^\dagger H_{eq} \right)^{-1} H_{eq}^\dagger = H_{eq}^+, \quad (86)$$

where H_{eq}^+ denotes the pseudo-inverse of H_{eq} . Hence, (85), (86) and the continuity of the function $\Gamma \mapsto \log \det(\Gamma)$ imply that

$$\lim_{SNR \rightarrow \infty} (\log A_\infty - \log A) = -\log \left| I - \frac{1}{m_S - m_D} H_{eq}^+ C \right|. \quad (87)$$

Therefore,

$$A \doteq A_\infty \doteq \log \left| I_{m_S} + SNR H_D^\dagger H_D + SNR H_E^\dagger H_E \right|. \quad (88)$$

Second, we show now that $B \doteq |I_{m_k} + \frac{SNR}{2} H_E H_E^\dagger|$.

$$\begin{aligned} \log B &= \log \left| K + \frac{SNR}{2} H_E H_E^\dagger \right| \\ &= \log \left| I_{m_k} + \frac{SNR}{2} H_E (I_{m_S} + \frac{1}{2(m_S - m_D)} T T^\dagger) H_E^\dagger \right|. \end{aligned} \quad (89)$$

Since T has orthonormal columns, then $\text{Tr}(T^\dagger T) = \text{rank}(T^\dagger T) = m_S - m_D$, hence

$$I_{m_s} + \frac{1}{2(m_s - m_D)} TT^\dagger \preceq \frac{3}{2} I_{m_s} \text{ and}$$

$$\log B \leq \log \left| I_{m_k} + \frac{3SNR}{4} H_E H_E^\dagger \right|.$$

Similarly, $I_{m_s} + \frac{1}{2(m_s - m_D)} TT^\dagger \succeq I_{m_s}$ which implies that $\log B \geq \log \left| I_{m_k} + \frac{SNR}{2} H_E H_E^\dagger \right|$.

In short, we obtain:

$$\begin{aligned} \log \left| I_{m_k} + \frac{SNR}{2} H_E H_E^\dagger \right| &\leq \log B \\ &\leq \log \left| I_{m_k} + \frac{3SNR}{4} H_E H_E^\dagger \right|. \end{aligned} \quad (91)$$

which results in

$$B \doteq \left| I_{m_k} + SNR H_E H_E^\dagger \right|. \quad (92)$$

Finally, we leverage the results from the secret-key rate outage calculation.

$$\begin{aligned} P_{out}^{AN}(SNR) &= P(A - B < rs \log(SNR)) \end{aligned} \quad (93)$$

$$\begin{aligned} &\doteq P\left(\log \frac{|I_{m_s} + SNR H_D^\dagger H_D + SNR H_E^\dagger H_E|}{|I_{m_s} + SNR H_E^\dagger H_E|}\right. \\ &\quad \left.< rs \log(SNR)\right). \end{aligned} \quad (94)$$

From equation (20c), we see that the last expression is exactly the same expression used to calculate the secret-key rate outage probability. Hence, using the result of Lemma 2, we obtain

$$P_{out}^{AN}(SNR) \doteq SNR^{-d_{m_s - m_E; m_D}(r_s)}, \quad (95)$$

which concludes the proof.

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