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Highlights

- DNS on the separated-transitional-reattached flow past the airfoil.
- The effects of the spanwise domain size are analyzed from various aspects.
- The spanwise domain size has significant effects on the various turbulent statistics.
- Small spanwise domain size tends to underpredict the spanwise variations.
- Examination through only a few quantities may lead to misleading conclusion.

Assessment of spanwise domain size effect on the transitional flow past an airfoil

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Abstract

In most large-eddy and direct numerical simulations of flow past an isolated airfoil, the flow is assumed periodic in the spanwise direction. The size of the spanwise domain is an important geometrical parameter determining whether the turbulent flow is fully developed, and whether the separation and transition patterns are accurately modeled. In the present study, we investigate the incompressible flow past an isolated NACA0012 airfoil at the angle of attack of 5 degrees and Reynolds number 5×10^4 . The spanwise domain size L_z , represented by the aspect ratio $AR=L_z/C$ where C is the airfoil chord length, is varied in the range 0.1 – 0.8. The effect of varying the normalized spanwise domain size AR is examined via direct numerical simulation (DNS) on several aspects of the turbulent flow quantities including the time-averaged and time-dependent behavior as well as the spanwise variation of the selected statistical quantities. DNS results reveal that different aspect ratios result in close predictions of the time-averaged aerodynamic quantities, and the velocity field except for a slight difference in the separation bubble. Smaller aspect ratios tend to underpredict the turbulent fluctuations near the separation point but overpredict them inside the separation bubble. Large differences are observed for multiple statistical quantities near the reattachment point, especially

the turbulent kinetic energy budget terms. The leading edge separation is notably three-dimensional for simulation at $AR=0.8$, while remaining quasi-2D for smaller aspect ratios. The spanwise two-point correlation coefficient shows significant dependence on the position of the probe and the velocity component analyzed: small aspect ratios do not produce uncorrelated results for all the velocity components. The simulation results demonstrate that examining only a few statistical quantities may result in a misleading conclusion regarding the sufficiency of the spanwise domain size. Reliable metrics to establish the sufficiency of spanwise domain size require thorough analysis of the turbulent statistics, and are necessary for three-dimensional simulation of turbulent flow in similar configurations.

Keywords: Direct numerical simulation, airfoil, spanwise, aspect ratio, separation, transition

2010 MSC: 76D10, 76F65

1. Introduction

The computational fluid dynamics literature has numerous examples of two-dimensional Navier-Stokes simulations of flow past bluff and aerodynamically-shaped bodies. Usually, 2D simulations require fewer computational resources than corresponding 3D ones, and 2D results are physically relevant when the three-dimensionality effect is not of great concern as in cases where the spanwise size of the bluff body is sufficiently large, no spanwise forcing exists or simply when the Reynolds number is not large enough to amplify spanwise disturbances. 2D simulation results may also be good approximations of real flows in terms of the spanwise-average quantities. However, at high Reynolds number, the flow is intrinsically three-dimensional in nature, and only

accurate 3D simulations can produce physically correct results. The spanwise size of the computational domain affects the simulation results, and a systematic investigation of spanwise size effect on the simulation quality is warranted.

Comparisons between experimental and numerical studies have shown that for flow
 15 past bluff bodies, the spanwise coherence of the vortices may be artificially reinforced in 2D simulations [1] and thus the validity of such results is questionable. In 3D simulations the spanwise domain size, representing the separation distance between end plates in experiments or the length of the bluff body in simulations, is normalized and represented by the aspect ratio $AR=L_z/L_{ref}$, where L_z is the spanwise domain size and
 20 L_{ref} is a reference length (cylinder diameter D or airfoil chord length C , etc.). In experiments the bluff body is usually held by the two end plates in a wind or water tunnel. The flow past such a bluff body is affected by the growth of the boundary layer on the plates, indicated by the non-uniformity of the measured quantities along the spanwise direction and oblique shedding of vortices. In numerical simulations the no-slip
 25 boundary condition constraint due to the end plates is usually relaxed by imposing slip or periodic boundary condition in the spanwise direction, with the expectation that the result is a closer approximation of flow in an infinite domain.

The spanwise domain size is a key geometric parameter in modern direct numerical simulation (DNS) and large-eddy simulation (LES) studies of flow past bluff bodies. It
 30 not only determines the accuracy of fluctuating aerodynamic forces but also the turbulent statistics (e.g., [2, 3]). Earlier studies (e.g. [4]) have shown that even for low-Re flow, the stochastic fluctuations of aerodynamic forces are accompanied by both low- and high-frequency modulations, which are suppressed in 2D simulations or 3D simu-

lations with small aspect ratio where perfect periodic variations are normally observed.

35 For the example of a flow past a cylinder, the aerodynamic forces acting on the cylinder are significantly affected mainly due to the inaccurate prediction of the surface pressure [5]. These examples provide a motivation for a systematic investigation of spanwise domain size effect in numerical (and also experimental) studies of flow past bluff bodies and similar configurations. In the present work we will narrow our review and
40 investigation on the spanwise domain size effect for an isolated airfoil under certain parameters, rather than a comprehensive study that covers a large parameter space.

1.1. Flow past an isolated airfoil: experiment

Compared with the isolated cylinder, the airfoil is a streamlined geometry, and consequently the flow separation and transition patterns are different. For incoming
45 laminar flow past a nominally two-dimensional airfoil (i.e., without spanwise skewness), the separation and transition patterns are complex and are strongly dependent on the angle of attack (AoA) and the Reynolds number. The low-Re flow ($Re < 70,000$ by Lissaman [20] and $\sim 15,000$ -500,000 by Mueller & DeLaurier [21]) past an airfoil shows detachment close to the leading edge due to the adverse pressure gradient (APG)
50 field; the separated flow is unstable and transitions rapidly to turbulence. At high AoA the turbulent flow remains detached from the airfoil surface, whereas at lower AoA it may reattach to form a turbulent boundary layer and the well-known laminar separation bubble [22]. The turbulent flow downstream of the airfoil trailing edge is relatively
fully developed into the turbulent regime, hence turbulent fluctuations must be taken
55 into account and the spanwise domain size should be sufficiently large to resolve the largest flow structure in this direction.

Table 1: Summary of selected experimental studies on flow past an isolated airfoil or inclined flat plate. The abbreviations are: HW-Hot Wire; PP-Pressure Probe; PIV-Particle Image Velocimetry; LDA-Laser Doppler Anemometer; HFA-Hot Film Anemometry; PTV-Particle Tracking Velocimetry; SWV-Smoke Wire Visualization.

Source	Airfoil	Method	Re/10 ⁴	AoA	AR
Boutilier & Yarusevych [6]	NACA0018	HW,PP	10	0-15	0.5-2.5
Boutilier & Yarusevych [7]	NACA0018	HW,PP	5-25	0-21	2.0
Boutilier & Yarusevych [8]	NACA0018	HW,PP	10	0-15	3.0
Buchmann <i>et al.</i> [9]	NACA0015	PIV	3	18	5.0
Burgmann <i>et al.</i> [10]	SD7003	PIV	2	4-8	1.5
Ghaemi & Scarano [11]	NACA0012	PIV	38.6	0	1.0
Hain <i>et al.</i> [12]	SD7003	PIV	6.6	4	1.25
Hu & Yang [13]	GA(W)-1	PP,PIV	7	6-14	3.0
Lam [14]	Flat plate	LDA	3	30	11
Nakano <i>et al.</i> [15]	NACA0018	PIV	16	0-9	2.38
Packard <i>et al.</i> [16]	NACA64 ₃ -618	PIV,HFA	6.4	-1-20	1.0,2.37
Suzuki <i>et al.</i> [17]	NACA0012	PTV	0.1	15	2.86
Yarusevych <i>et al.</i> [18]	NACA0025	HW,SWV	5.5-20	0-10	3.03
Zhang <i>et al.</i> [19]	SD7003	PIV	2,6	4	1.25

We summarize the key parameters in selected experimental studies of flow past an airfoil in Table 1. The incoming flow is normally of low turbulent intensity and transition is expected to occur above the airfoil suction surface. In experiments the two end plates supporting the airfoil in the wind tunnel affect the flow in the middle

of the tunnel because of the boundary layers formed on them, and hence these end plates should be placed far away from each other to minimize any interference. It is seen in the table that the aspect ratio of the airfoil is normally larger than 1.0, which is larger than most simulation studies. Boutilier & Yarusevych [6] investigated the effect
65 of the end plates on flow past a NACA0018 airfoil at $Re=10^5$ and $AoA=0-15$ degrees, and found that the existence of the end plates improves the mean spanwise uniformity. The separation distance between the two end plates, i.e., the aspect ratio used in this paper, does not affect the frequency of the vortex shedding, and a smaller AR reduces the spanwise coherence of the wake vortices. Based on their experimental results,
70 the authors recommended that $AR \geq 7.0C \sin(AoA)$ should be used to minimize the end plates effect on the mean quantities at the mid-span plane of the airfoil where measurements are taken. Burgmann *et al.* [10] studied the flow past a SD7003 airfoil at $Re=2 \times 10^4$ and $AoA=4-8$ degrees, and found that the thickness of the boundary layer on the water tunnel walls is about $0.05C$ at the position of the airfoil, which is
75 roughly $2C$ from the inflow plane, and is believed to have no effect on the flow field at the airfoil mid-span. In the experiment the aspect ratio is 1.5 and the horseshoe vortices do not affect the flow field at the mid-span. Hain *et al.* [12] performed comparative experiments at $AR=1.25$ and 2.5 for flow past the SD7003 airfoil at $Re=6.6 \times 10^4$. Both experiments show that there is no strong mean spanwise velocity at the airfoil
80 mid-span, reflecting the negligible effect due to the end plates.

We conclude from the above discussion and the summary in Table 1 that the end plates effect is a crucial concern in experimental studies of flow past an isolated airfoil. Generally, for flow at Reynolds number higher than 10^4 , the boundary layer thickness

is small and has only negligible effect on the flow field at the airfoil mid-span, e.g.,
 85 through the formation of horseshoe vortices at the ends of the airfoil. However, the
 end plates do affect the spanwise uniformity of the flow field, in time-averaged and
 time-dependent senses, thus the aspect ratio of the airfoil has to be chosen with care.

1.2. Flow past an isolated airfoil: simulation

Numerical simulations of flow past an isolated airfoil, especially DNS and LES, are
 90 performed to investigate the physics of the separation, transition and development of
 the flow over the airfoil. Usually the end plate effect, due to its physical complexities,
 is not modeled. Instead, the usual choice is the application of periodic boundary con-
 dition in the spanwise direction to model an airfoil with large aspect ratio in an infinite
 medium. In these simulations, it is required that the spanwise size be sufficiently large
 95 to ensure that the fully developed turbulent flow is stochastic and becomes uncorrelated
 over a sufficiently large distance in this direction. The minimum required spanwise size
 could be fixed based on the experience of previous numerical investigations with sim-
 ilar parameters and configurations, or the adequacy of the spanwise domain size may
 be verified in *a posteriori* estimations such as two-point correlation along the spanwise
 100 direction. However, due to the large parameter space and the complexities of the fluid
 flow in such simulations, so far there is no consensus on the determination of the as-
 pect ratio based on certain empirical or analytical relationships. Although it is safe to
 choose a sufficiently large aspect ratio, the number of grid points in the spanwise direc-
 tion would proportionally grow to have well-resolved turbulent structures in DNS and
 105 LES. Consequently, a careful determination of the airfoil aspect ratio may help balance
 computational resource requirements against simulation quality. Table 2 summarizes

Table 2: Summary of selected computational studies on flow past an isolated airfoil or inclined flat plate.

The abbreviations are: LES-Large-Eddy Simulation; DNS-Direct Numerical Simulation; URANS-Unsteady

Reynolds-Averaged Navier-Stokes; DES-Detached Eddy Simulation.

Source	Airfoil	Method	Re/10 ⁴	AoA	AR
Incompressible flow simulations					
Breuer & Jovičić [23]	Flat plate	LES	2	18	1.0
Catalano & Tognaccini [24]	SD7003	LES	6	≤9	0.1
Eisenbach & Friedrich [2]	NACA4415	LES	10	18	0.5,1.0
Hoarau <i>et al.</i> [25]	NACA0012	DNS	0.08-1	20	4.0
Jovičić <i>et al.</i> [26]	NACA4415	LES	10	18	1.0
Kim <i>et al.</i> [27]	NACA0018	LES	16	0-9	10.0
Kitsios <i>et al.</i> [3]	NACA0015	LES	3	18	0.66
Lehmkuhl <i>et al.</i> [28]	NACA0012	DNS,LES	5	5,8	0.2
Liu <i>et al.</i> [29]	Joukowsky airfoil	DNS	12.6/20	0	1.0
Rodríguez <i>et al.</i> [30]	NACA0012	DNS	5	9.25,12	0.2
Savaliya <i>et al.</i> [31]	Eppler61	DNS	4.6	0-20	0.5
Schmidt & Thiele [32]	Aérospatiale A-airfoil	DES,URANS	200	13.3	0.2
Shur <i>et al.</i> [33]	NACA0012	URANS	10	90	1.0-4.0
Wissink & Rodi [34]	T106 cascade	DNS	7.2	-	0.2
Yang <i>et al.</i> [35]	Flat plate	DNS	0.1	20-30	6.0
You & Moin [36]	NACA0015	LES	89.6	16.6,20	0.2
Yuan <i>et al.</i> [37]	SD7003	LES	6	0-10	0.012
Zaki <i>et al.</i> [38]	NACA65 cascade	DNS	13.85	-	0.2
Compressible flow simulations					
Almutairi <i>et al.</i> [39]	NACA0012	LES	5	8.5-11	0.2,0.5
Almutairi & AlQadi [40]	NACA0012	LES	5	9.25-9.6	0.5
Deck [41]	RA16SC1	DES	170	-	0.06
Deng <i>et al.</i> [42]	NACA0012	DNS	10	4	0.1
Gross & Fasel [1]	NACA64 ₃ -618	DNS	6.42	8.64	0.2
Jones <i>et al.</i> [43]	NACA0012	DNS	5	5	0.2
Kojima <i>et al.</i> [44]	NACA0002,NACA0012	LES	2.3	0-9	0.2
Marsden <i>et al.</i> [45]	NACA0012	LES	50	0	0.05
Mary & Sagaut [46]	Aérospatiale A-airfoil	LES	210	13.3	0.005-0.03
Shan <i>et al.</i> [47]	NACA0012	DNS	10	4	0.1
Shan <i>et al.</i> [48]	NACA0012	DNS	10	6	0.1
Visbal [49]	SD7003	LES	1-6	4	0.2

selected simulations for flow past an airfoil or inclined flat plate. Compared with their experimental counterparts, the aspect ratio employed in the numerical simulation is usually smaller than 1.0, mostly in the range of 0.1-0.2.

110 The sufficiency of the spanwise size in the turbulent flow simulations may be evaluated quantitatively by *a posteriori* spanwise two-point correlation analysis. The coefficient is defined as:

$$r_{\phi\phi}(\Delta z) = \frac{\overline{\phi'(z, t)\phi'(z + \Delta z, t)}}{\phi'_{\text{rms}}(z)\phi'_{\text{rms}}(z + \Delta z)}, \quad (1)$$

in which ϕ is some variable used in this evaluation and ϕ' is its turbulent fluctuating component, and Δz is the distance between the two probes placed along the spanwise direction. The coefficient is normally computed by averaging over combinations of probes with the same separation distance. As concluded by Fröhlich *et al.* [50], the spanwise domain size can be considered sufficiently large if ϕ' in the periodic direction is uncorrelated within a separation distance of half the domain size, i.e., $r = 0$ at $\Delta z = L_z/2$. Although in most cases this condition is not perfectly achieved (e.g., Breuer & Jovičić [23], Kitsios *et al.* [3]), it reflects the level to which the turbulent flow is developed and resolved in the spanwise direction. Several numerical simulations have proven that fluctuations in the spanwise direction become uncorrelated when the aspect ratio is sufficiently large [3, 23, 28, 30, 51, 52].

125 In addition to the above empirical and quantitative estimations of the necessary aspect ratio, there are a number of studies performing comparative simulations to investigate the aspect ratio effect for near-stalled or stalled flows. For a low Reynolds number $Re=800$, Hoarau *et al.* [25] confirmed that the characteristic spanwise wavelength is about $0.64C$ for the NACA0012 airfoil at $AoA=20$ degrees, while this value decreases

to $0.62C$ as the Reynolds number increases to 1200. Both wavelengths are in agree-
 130 ment with those in cylinder flow with an effective Reynolds number $Re \sin(AoA)$. The
 results show that the necessary aspect ratio could be smaller than $0.62C$ for Reynolds
 larger than 1200, but the specific value is still not clear and has to be determined. Al-
 mutairi *et al.* [39] simulated the flow past a NACA0012 airfoil at $AoA=9.25$ degrees
 and $Re=5 \times 10^4$ with two aspect ratios $AR=0.2$ and 0.5 . The results show that the
 135 small aspect ratio incorrectly predicts the mean flow field because the mean separated
 flow fails to reattach to the suction surface. The simulation using the larger aspect ratio
 accurately reveals the irregular low-frequency oscillations of the lift coefficient, and
 the spanwise two-point correlation of the velocity has decayed reasonably. It is con-
 cluded from the results that at least $AR=0.5$ is necessary for this massively separated
 140 flow since incorrect results are obtained for the smaller aspect ratio. Similar conclu-
 sions are also arrived by Kitsios *et al.* [3] for the NACA0015 airfoil at $Re=3 \times 10^4$
 and $AoA=18$ degrees. The spanwise two-point correlation of the turbulent fluctuat-
 ing velocity shows that the flow is still correlated for $AR=0.25$, while decorrelation is
 achieved for $AR=0.66$ regardless of the resolution in the cross-sectional plane. More-
 145 over, it seems that the small aspect ratio fails to accurately predict the eddy viscosity in
 the separation zone above the airfoil, which makes the results less reliable. Breuer &
 Jovičić [23] confirmed that for flow past an inclined flat plate, $AR=1$ is deemed to be
 sufficient for spanwise decorrelation. At a high Reynolds number $Re=10^5$, Eisenbach
 & Friedrich [2] found that the stalled flow past the NACA4415 airfoil is less affected
 150 by the aspect ratio as it increases from $AR=0.5$ to 1.0 ; the mean flow field and the
 turbulent statistics are only slightly changed.

The aspect ratio effect for the stalled or near-stalled low-Re flows is investigated through comparative simulations [2, 3, 23, 25, 39]. It is generally summarized from these studies that the aspect ratio should not be smaller than 0.5 for the full development of the massively separated flow. However, the aspect ratio is determined empirically for airfoils with low AoA where the laminar flow transitions to turbulent and reattaches to the airfoil surface, and AR around 0.2 is commonly chosen for both compressible and incompressible flow simulations.

1.3. Scope and objective of the present study

An examination of the literature, especially the DNS and LES studies, reveals that the aspect ratio of the airfoil has profound effect on both the time-averaged and time-dependent behavior of the global quantities and turbulent statistics. Although it is shown in Table 2 that a smaller aspect ratio around 0.2 is often considered acceptable for the low AoA airfoil [24, 28, 42–44, 47–49] with separated-reattached flow, a systematic investigation of the aspect ratio effect is still lacking. Low AoA transition and reattachment patterns of the turbulent flow are complex and indeed quite different from those of the massively separated flow at high AoA.

In order to identify the effect of the aspect ratio on the transitional flow past the airfoil, we perform DNS on the laminar inflow past the NACA0012 airfoil at $Re=5 \times 10^4$ and AoA=5 degrees with four aspect ratios, AR=0.1, 0.2, 0.4 and 0.8. The aspect ratio AR=0.1 is smaller than the one employed by most studies, while AR=0.8 is widely confirmed to be sufficiently large for the massively separated flow [2, 3, 25, 39]. The four aspect ratios cover a large range to enable conclusive comparisons between them and identify the AR effect on the transitional flow patterns. The objective of the present

study is to numerically determine the effect of the aspect ratio on the time-averaged and time-dependent flow behavior, turbulent statistics and spanwise variations of the flow field. Since the separation, transition and reattachment of the flow occur with the formation of the laminar separation bubble, the related flow dynamics in the local region are emphasized. We understand that the present study only provides applicable conclusions to the cases with similar parameters. However, considering the expensive resource cost for the DNS studies, it is practically impossible to perform a parametric study covering a large multitude parameter space (Re , AoA , airfoil) to make a universal conclusion regarding the underlying problem. The parameters chosen here are typical in a number of similar simulations. The results and conclusions presented in this work could be used as a reference or guidance in future simulations (and also experiments) to aid the determination of the appropriate aspect ratio.

The paper is organized as follows. Section 2 is a general description of the targeted problem, the employed numerical methods, the details of the simulation and evaluation of the simulation quality. Section 3 presents the main simulation results. The effects of the aspect ratio are presented from four aspects: the time-averaged results, the time-dependent unsteady results, the turbulent statistics and the spanwise variations of the related quantities and flow field. Some conclusions are drawn in section 4.

2. Numerical setup

2.1. Problem definition

The schematic of the computational domain is shown in Figure 1. The NACA0012 airfoil is rescaled to unit chord length C and extended to include a sharp trailing edge

Figure 1: Schematic of the computational domain.

located at $(x/C, y/C) = (1.0, 0.0)$. The airfoil is assumed to be straight in the spanwise (z-) direction. The angle of attack is 5 degrees. The whole computational domain is discretized by a C-type mesh starting and ending at the outflow boundary and wrapping
 200 around the airfoil. The mesh used for all simulations is 2048 in the streamwise direction wrapping around the airfoil, 256 in the wall-normal direction, and 32 to 256 in the spanwise direction depending on the aspect ratio. The mesh is clustered around the airfoil surface to ensure that the first layer mesh size in the wall-normal direction follows $\Delta\eta_{\max}^+ \approx 0.9$. In the streamwise and spanwise directions the mesh sizes are around
 205 $\Delta\xi_{\max}^+ \approx 5.6$ and $\Delta z_{\max}^+ \approx 10.8$. The mesh is sufficiently fine considering that the above maximum values follow the well accepted criterion $10 \leq \Delta\xi_{\max}^+ \leq 20$, $\Delta\eta_{\max}^+ < 1$, $5 \leq \Delta z_{\max}^+ \leq 10$ [53]. The wake length W and the domain radius R are respectively 10C and 6C, which are confirmed to be sufficiently large for such simulations [43, 51, 52].

2.2. Solution method

210 The governing equations for the present flow simulations are the three-dimensional incompressible Navier-Stokes equations in non-dimensional form:

$$\frac{\partial u_k}{\partial x_k} = 0, \quad (2)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_k u_i}{\partial x_k} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{1}{\text{Re}} \frac{\partial^2 u_i}{\partial x_k \partial x_k}, \quad (3)$$

where $\text{Re} = \rho U_0 C / \nu$ is the Reynolds number; $(u_1, u_2, u_3) \equiv (u, v, w)$ are the velocity components; p is the pressure; $(x_1, x_2, x_3) \equiv (x, y, z)$ are the Cartesian coordinates representing the streamwise, transverse and spanwise directions. A uniform flow 215 $(u, v, w) = (U_0, 0, 0)$ is imposed at the inflow boundary. At the outflow plane the convective outflow condition $\partial \mathbf{u} / \partial t + U_B \partial \mathbf{u} / \partial x = 0$ is implemented, in which U_B is the bulk velocity. A no-slip and no-penetration boundary condition for velocity is prescribed on the airfoil surface. Periodic boundary condition is employed in the spanwise direction.

220 A generalized curvilinear coordinate transformation is applied to map the physical domain (x, y, z) to the computational domain (ξ, η, z) , in which the governing equations are spatially discretized on a collocated mesh. The spatial discretization is an energy conservative fourth-order scheme [54] for the convective terms, and second-order central difference for the pressure gradient and viscous terms. In order to minimize the 225 aliasing error, the convective term is written and discretized in the skew-symmetric form [54, 55]. The discretized governing equations are solved by a semi-implicit fractional step method [56]. The convective and viscous terms are treated explicitly and implicitly, respectively. The modified-Helmholtz equation resulting from the implicit treatment of viscous terms in the predictor step, and the pressure Poisson equation are

Figure 2: Ratio between of the local mesh size and the Kolmogorov lengthscale h/η_k obtained at $AR=0.4$.

both solved using the multigrid solvers with line-relaxed Gauss-Seidel smoothers. The code has been validated and used in previous work [51, 52]. In the present study, the time step size is fixed at $2.5 \times 10^{-4}C/U_0$ for all simulations, corresponding to a maximum CFL number smaller than 1.0. The 3D simulation is initialized with a precursor low-resolution simulation in which the 3D structures have been stimulated. The production simulation first runs for $50C/U_0$ for the fully development of the transitional flow, and then an additional $50C/U_0$ for data acquisition and statistics.

2.3. Assessment of simulation quality

The quality of the simulation is largely dependent on the temporal and spatial resolutions and the duration of the whole simulation. For the present DNS of the separated-reattached flow past the airfoil, the spatial resolution has to be sufficiently fine to guarantee that the small-scale turbulent structures are well resolved. Furthermore, the duration of the simulation should be adequate that the initial transient is washed out and a statistical steady-state is achieved.

The spatial resolution is evaluated by the ratio between the mesh size (h) and the Kolmogorov lengthscale (η_k), as presented in Figure 2. The two quantities are defined

Figure 3: Time histories of the instantaneous velocity u/U_0 monitored at $(x/C, y/C, z/C) = (1.200, 0.000, AR/2)$. From top to bottom: $AR=0.1, 0.2, 0.4$ and 0.8 .

as:

$$h = (\Delta\xi\Delta\eta\Delta z)^{1/3}, \eta_k = (v^3/\epsilon)^{1/4}, \epsilon = 2\nu\overline{S'_{ij}S'_{ij}}, \quad (4)$$

where S'_{ij} is the symmetric part of $\nabla\mathbf{u}'$. Since the same two-dimensional mesh is used, and the mesh spacing is constant in the spanwise direction in all simulations, the distributions of h/η_k are numerically close for different aspect ratios and hence we only present this for the $AR=0.4$ case. In the center of the laminar separation bubble $0.5 \leq x/C \leq 0.6$, the mesh size is at most 2-3 times that of the Kolmogorov length scale, while the ratio decreases to around and even smaller than 1.0 for the reattached turbulent flow and the near wake flow. Although this ratio is not a guarantee, nonetheless, we believe the mesh is fine enough to resolve the essential small-scale structures of the transitional and reattached turbulent flow based on our experience and the conclusions of previous works [57, 58].

Figure 4: Time histories of the mean lift and drag coefficients as functions of integrating time. The coefficients are defined in Eq.(5).

Lehmkuhl *et al.* [28] concluded that the simulation duration should be at least $60C/U_0$ for the well resolved first- and second-order statistics in different zones for a DNS study of the same parameters. As discussed in section 2.2, our simulation lasts for $100C/U_0$, with the last $50C/U_0$ used for statistics. Figure 3 shows the streamwise velocity monitored by a probe in the near wake during the time interval when flow statistics are measured. Any trace of an initial transient is absent at this stage and a statistical steady-state has been achieved for all cases. We further examine the influence of the simulation duration by measuring the time-dependent averaging of the lift and drag coefficients, defined as:

$$C_L(t) = \frac{1}{t - t_0} \int_{t_0}^t \frac{\text{Lift}}{\frac{1}{2}\rho U_0^2} dt, \quad C_D(t) = \frac{1}{t - t_0} \int_{t_0}^t \frac{\text{Drag}}{\frac{1}{2}\rho U_0^2} dt, \quad t_0 < t \leq t_1, \quad (5)$$

where $t_0 = 0$ and $t_1 = 50C/U_0$ are the start and end of the production simulation as shown in Figure 3; $\rho = 1$ is the constant density of the fluid. The two coefficients are anticipated to converge to constant values as the integrating time becomes sufficiently large. Figure 4 gives the variations of the two quantities as a function of the integrating time t . There are large fluctuations of the force coefficients when t is small, while

Figure 5: Instantaneous vortical structures represented by the isosurface of $Q = 30$ and colored by the streamwise velocity. The flow fields are obtained at $AR=0.2$.

remaining relatively steady for $tU_0/C \geq 20$. The amplitude of the low-frequency oscillation is about 0.2% of the converged quantity around $tU_0/C = 50$ for both coefficients; the simulation duration of the present study is much larger than most DNS and LES studies [51] and is considered to be adequate for the present study.

275 3. Results and discussion

3.1. General description of the unsteady flow

We first present a qualitative description of the separated-reattached flow patterns for the AR=0.2 case. The results obtained at other aspect ratios are, of course, different in details but qualitatively the characteristics are similar. Figure 5 shows five snapshots
280 of the instantaneous vortical structures above the airfoil and in the near wake taken at equally spaced time intervals. The vortical structures are represented by the iso-surface, colored by the streamwise velocity, of the so-called “ Q -criterion” defined as $Q = 0.5(\|\boldsymbol{\Omega}\|^2 - \|\boldsymbol{S}\|^2)$, in which $\boldsymbol{S}$ and $\boldsymbol{\Omega}$ are respectively the symmetric and anti-symmetric parts of $\nabla\mathbf{u}$. Evident in these snapshots is the development from quasi-2D
285 ($x/C \leq 0.45$) to fully 3D turbulent flow.

The incoming laminar flow impinges the airfoil surface at the leading edge, then moves along the surface eventually succumbing to the adverse pressure gradient. The attached flow separates from the airfoil surface and forms a quasi-2D separated shear layer, i.e., develops in the x - y plane but is uniform in the spanwise direction. This separated shear layer is unstable to the Kelvin-Helmholtz modes, and develops the quasi-2D
290 rollers around $x/C = 0.4$. Since the 2D vortex shedding is itself unstable to spanwise perturbations [43], corrugated structures develop and large-scale 3D structures are initiated at around $x/C=0.5$ -0.6. A strong flow reversal develops close to the airfoil surface in this region. As the flow develops downstream, the large-scale structures break into
295 smaller ones, and the separated flow reattaches to the surface, indicated by the forward flow with positive streamwise velocity close to the surface. The 3D turbulent boundary layer flow is observed downstream of approximately $x/C = 0.6$ with no notable flow

reversals.

Jones *et al.* [43] carried out DNS and linear stability analysis on compressible flow
 300 past the NACA-0012 airfoil at the same Reynolds number and angle of attack. They
 proposed a mechanism that the perturbation is amplified in the braid region and con-
 vects upstream by the reversed flow, and then is convected downstream with the next
 developing vortex and gets further amplified. Shan *et al.* [47] arrived at a similar con-
 clusion that the perturbation, first observed close to the trailing edge, convects upstream
 305 and is amplified in the separation bubble. Recent works [59, 60] show that the unstable
 modes trigger the vorticity roll-up, while not directly leads to flow transition. In the
 present simulations, the initial condition is provided by a precursor simulation which
 has already developed into 3D. Hence, the production simulation is conducted without
 any prescribed perturbation. The initialization and amplification of the perturbation in
 310 the precursor simulation are not considered relevant to the overall objective of quanti-
 fying the effect of the aspect ratio. It is emphasized that the duration of diagnostics and
 collecting statistics should be long enough and initial transients must be washed out.

3.2. Time-averaged results

In this section, the effect of the aspect ratio on the time-averaged statistical quanti-
 315 ties are assessed. The mean lift and drag coefficients, and the separation and reattach-
 ment positions are given in Table 3. The results are compared with those of Lehmkühl
et al. [28], and the XFOIL [61] results of low-Mach compressible flow ($Ma=0.1$) at
 the same Reynolds number and angle of attack. The results of Lehmkühl *et al.* [28]
 are transformed into our coordinate system for a direct comparison. The statistical
 320 quantities obtained at different aspect ratios are generally in agreement with those by

Table 3: Mean aerodynamic quantities. C_L and C_D are the mean lift and drag coefficients; x_{sep} and x_{reatt} are the mean separation and reattachment positions.

Source	AR	C_L	C_D	x_{sep}/C	x_{reatt}/C
XFOIL [61]	-	0.618	0.0240	0.120	0.512
Lehmkuhl <i>et al.</i> [28]	0.2	0.569	0.0291	0.175	0.568
Present	0.1	0.562	0.0282	0.141	0.580
	0.2	0.568	0.0285	0.138	0.578
	0.4	0.566	0.0285	0.138	0.579
	0.8	0.566	0.0285	0.143	0.576

Figure 6: Distributions of the mean pressure coefficient and skin friction coefficient.

Lehmkuhl *et al.* [28] with the exception of the earlier separation point. Figure 6 shows the surface distributions of the pressure coefficient and the skin friction coefficient. The skin friction coefficient predicted at $AR=0.8$ is notably smaller in magnitude than the smaller ARs at the suction peak around $x/C = 0.52$, resulting in a weaker separation bubble.

Figure 7 shows the mean streamwise velocity distribution around the airfoil, with a magnified view of the streamlines in the separation bubble. For this averaging over

Figure 7: Isolines of the mean streamwise velocity $\bar{u}/U_0$. The black, red, green and blue lines respectively denote $AR=0.1, 0.2, 0.4$ and 0.8 .

a duration of $50C/U_0$, the mean velocity distributions at different aspect ratios are in excellent agreement close to the airfoil, and the agreement slightly deteriorates in the wake relatively far from the airfoil. The mean velocity distribution in the separation bubble exhibits some difference: for $\bar{u}/U_0 = 0.0$, the isolines obtained at $AR \leq 0.4$ agree with each other, while the one for $AR=0.8$ slightly deviates from them. The velocity profiles at $AR \leq 0.4$ are in good agreement close to the reattachment point but are different at the center of the bubble at $x/C=0.40-0.45$. The mean velocity streamlines and the skin friction coefficient (plotted in Figure 6) for $AR=0.8$ stands out as being the most different compared with the other three aspect ratios.

Figure 8: Space-time diagrams of instantaneous spanwise-averaged skin friction coefficient. The solid contour is the isoline $C_f = 0$.

3.3. Time-dependent behavior

3.3.1. Convection of the separation bubble

As evident from Figure 5, the flow in the separation bubble is unsteady and shows coherent flow structures that are convected downstream along with the cross-flow in the form similar to a roller. The evolution of the separation bubble, especially the portion close to the reattachment point, is depicted by the space-time diagram of the instantaneous spanwise-averaged skin friction coefficient shown in Figure 8. The evolution, although shown only for a short time duration of $5C/U_0$, reveals the primary features. The separation bubble does not have a constant size and small recirculation bubbles occasionally form beneath it. Consistent with the findings in Figure 6, the flow reversal is strongest around $x/C = 0.52$, where the quasi-periodic pulsation of the separation bubble along the airfoil surface is also identified. Simulations with different aspect ratios give almost the same convection velocity, computed based on the slope of the stripes. The slight difference is attributed to the sampling error arising in the determination of the inclined strip (the dashed line in the figure). Dandois *et al.*

Figure 9: Streamwise variations of the maximum root-mean-square values of the fluctuating velocity components.

[62] comprehensively summarized the convection velocity of shear layer vortices of different geometrical configurations, including flow separation over a backward facing step or cavity, at the flat plate leading edge, around a fence, etc. The computed convection velocity is $0.5-0.6U_0$ for almost all configurations. In their LES study of flow past a ramp, the convection velocity is $0.5U_0$, close to the computed value in our DNS. They concluded that this convection velocity is expected for two-dimensional structures arising due to the Kelvin-Helmholtz instability. Although the flow separates from the airfoil naturally as a result of the adverse pressure gradient rather than the forced separation as in a backward facing step, the convection velocity is close to those configurations due to the similar Kelvin-Helmholtz mechanism that occurs in the separated shear layer.

3.3.2. Turbulent fluctuations

The effect of the aspect ratio on the statistics of turbulent flow quantities is presented in this section. Figure 9 shows the variations of the y-direction maximum $(\mathbf{u}')_{\text{rms}}$ along the streamwise direction. The incoming laminar flow is steady at the airfoil lead-

Figure 10: Profiles of the turbulent Reynolds stress components.

ing edge, and the unsteadiness grows almost linearly in the streamwise direction until around the separation point. The fluctuating velocity components grow rapidly from $x/C = 0.40$ to approximately the reattachment point $x/C = 0.58$, and then decrease almost monotonically in the reattached turbulent flow and in the near wake flow. At the trailing edge $x/C = 1.0$ there is a slight increase of $(u')_{\text{rms}}$ due to the interaction between the suction surface boundary layer flow and the counterclockwise rotating vortex emerging from the pressure surface of the airfoil. The streamwise velocity component $(u')_{\text{rms}}$ grows faster than the other two components and its peak value is larger. The fluctuating velocity components obtained at different aspect ratios exhibit notable differences approximately within the separation bubble $x/C=0.13-0.58$. Considering the $AR=0.8$ case as the benchmark solution, smaller aspect ratios tend to underpredict $(u')_{\text{rms}}$, especially in the region $x/C \leq 0.45$ where the underprediction is about

40%. The $(v')_{\text{rms}}$ is overpredicted around the reattachment point by about 15%, and
 380 the overprediction is smaller in magnitude in the wake. Although there is no spanwise
 perturbation imposed on the flow field, the fluctuating spanwise velocity is of the same
 order as the other two components. The differences in $(w')_{\text{rms}}$ between different aspect
 ratios is somewhat small for $\text{AR} \geq 0.2$, while $\text{AR}=0.1$ notably underpredicts the value
 starting from the reattachment point.

385 The turbulent Reynolds stress components $\overline{u'u'}/U_0^2$ and $\overline{-u'v'}/U_0^2$, as a measure of
 the turbulent intensity, are shown in Figure 10. The profiles of the Reynolds stress
 components are presented at ten stations in the streamwise direction, from downstream
 of the mean separation point $x/C = 0.3$ to the near wake $x/C = 1.2$. During the initial
 stage of transition at $x/C = 0.3$ and 0.4 , both Reynolds stresses are small as $O(10^{-3})$
 390 and are underpredicted by $\text{AR}=0.1$ and 0.2 , while large aspect ratios $\text{AR}=0.4$ and 0.8
 produce results that are in agreement especially at $x/C = 0.4$. It is noted that the
 threshold value $\overline{-u'v'}/U_0^2 = 0.001$ is commonly used as an indicator for the laminar-
 turbulent transition onset both in numerical and experimental studies (e.g. [13, 63–65]).
 Here we observe that the transition takes place in the separated shear layer roughly at
 395 $x/C = 0.4$ as predicted by $\text{AR}=0.4$ and 0.8 , while for smaller aspect ratios the transi-
 tion is predicted to occur at a further downstream position. The stresses increase in the
 streamwise direction and reach the peak values at $x/C = 0.6$ close to the reattachment
 point, and then gradually decrease in the near wake. Generally the different aspect ra-
 400 tios give close predictions on both components, while the stresses obtained at $\text{AR} \leq 0.4$
 are always overpredicted compared with the results obtained at $\text{AR}=0.8$. The discrep-
 ancies are the most significant in the region around the reattachment point $x/C=0.5-0.8$

Figure 11: Isolines of the mean turbulent kinetic energy. The black, red, green and blue lines respectively denote AR=0.1, 0.2, 0.4 and 0.8.

and is remarkably far from the airfoil surface, sometimes up to 50% in magnitude. As the turbulent flow is relatively fully developed downstream of $x/C = 0.9$, the profiles predicted by different aspect ratios better agree with each other with maximum discrepancy of about 25% for $\overline{u'u'}/U_0^2$ and even smaller for $-\overline{u'v'}/U_0^2$.

3.3.3. Turbulent kinetic energy budget

In this section, we further investigate the turbulent flow field by analyzing the turbulent kinetic energy budget and the emergence and development of the mean turbulent kinetic energy (T.K.E., $k=0.5(\overline{u'u'} + \overline{v'v'} + \overline{w'w'})/U_0^2$) due to different sources. We first examine the turbulent kinetic energy distribution at the airfoil mid-span in Figure 11. The T.K.E. is highest above the airfoil at around the reattachment position $x/C = 0.58$ with its peak value about 0.09 (not plotted in the figure, which is also close to the value 0.124 by Jones *et al.* [43] for compressible flow). The T.K.E. fields obtained at different aspect ratios exhibit large difference. It is seen from the isolines that the core region predicted by AR=0.8 is smaller than the other aspect ratios, and the four isolines deviate from each other at the core region.

Figure 12: Turbulent kinetic energy budget normalized by U_0^3/C . Solid line for AR=0.1; dashed line for AR=0.2; dotted line for AR=0.4; dash-dotted line for AR=0.8.

The turbulent kinetic energy budget is given by [66, 67]:

$$\frac{\partial k}{\partial t} = \underbrace{-\bar{u}_j \frac{\partial k}{\partial x_j}}_{C_k} - \underbrace{\frac{\partial \bar{u}'_j p'}{\partial x_j}}_{\Pi_k} - \underbrace{\frac{1}{2} \frac{\partial \bar{u}'_i u'_i u'_j}{\partial x_j}}_{T_k} + \underbrace{\nu \frac{\partial^2 k}{\partial x_j^2}}_{D_k} - \underbrace{\bar{u}'_i u'_j \frac{\partial \bar{u}_i}{\partial x_j}}_{P_k} - \underbrace{\nu \frac{\partial \bar{u}'_i}{\partial x_j} \frac{\partial \bar{u}'_i}{\partial x_j}}_{\epsilon_k} \quad (6)$$

where the terms on the RHS of Eq.(6) are identified as: C_k =convection, Π_k =velocity-pressure gradient, T_k =turbulent transport, D_k =viscous diffusion, P_k =production, ϵ_k =viscous
 420 dissipation. Since k is the mean turbulent kinetic energy, the terms are balanced and their summation is zero.

The distributions of the T.K.E. budget are shown in Figure 12 at several x/C locations covering the high- k region in Figure 11. At $x/C = 0.40$ and 0.45 , all the six budget terms are relatively small due to the local low- k flow. The T.K.E. at the edge
 425 of the high- k region is increased primarily by the production (P_k) and is balanced by the convection (C_k) and the velocity-pressure gradient (Π_k) terms. For flow very close to the airfoil surface, the T.K.E. is dominated by the velocity-pressure gradient (Π_k) term. Since $\partial \bar{u}/\partial y$ is much larger than other velocity derivatives and $-\bar{u}'v'$ is positive, the production term (P_k) is dominated by $P_{k,12} = -\bar{u}'v' \partial \bar{u}/\partial y$ and is negative due to the
 430 reversed mean flow ($\partial \bar{u}/\partial y < 0$). Large differences are observed between the T.K.E results obtained at different aspect ratios. It can be safely concluded from the profiles at $x/C = 0.45$ that small aspect ratio generally tends to overpredict the budget terms above the suction surface of the airfoil, especially at the edge of the high- k region and beyond. Considering that the terms are small in magnitude, the discrepancy measured
 435 in percentage is large.

The profiles from $x/C = 0.50$ to $x/C = 0.58$ focus on the T.K.E. budget in the high- k region (see Figure 11) from the center of the separation bubble to the reattachment

point, and also cover the peak of the mean C_f (as previously shown in Figure 6). All terms have increased to their maximum magnitude compared with their counterparts at the upstream positions, especially the turbulent transport (T_k) and dissipation (ϵ_k) terms which have notably increased within the separation bubble (see Figure 7). At the upper boundary of the separation bubble the local T.K.E. is mainly enhanced by the turbulent transport (T_k), and is convected downstream (C_k). In the separation bubble the T.K.E. is almost solely contributed by the production (P_k) but is balanced by multiple terms (C_k , Π_k , T_k and ϵ_k). For flow very close to the airfoil surface, the T.K.E. is increased by the pressure-gradient term (Π_k) and lowered by the dissipation (ϵ_k) and viscous diffusion (D_k) terms. The viscous diffusion term (D_k) is mostly active only in this region. It is observed that the profiles obtained at different aspect ratios are similar in trend from the airfoil surface to the far field potential flow. The main difference is that the smaller aspect ratios overpredict almost all terms. Assuming the results obtained at AR=0.8 is the benchmark solution, the budget terms in the separation bubble can be overpredicted by more than 50% in magnitude, especially for the convection (C_k) and pressure-velocity gradient (Π_k) terms. The discrepancy is much smaller close to the airfoil surface.

The T.K.E. budget profiles downstream of the mean reattachment point exhibit different patterns compared with those in the separation bubble. The magnitude of all terms reduces and gets progressively smaller as the flow develops downstream, while the shapes of the profiles are still maintained. Although the profiles obtained at different aspect ratios are similar mostly due to their smaller magnitude, there are still remarkable discrepancies except in the viscous diffusion (D_k) and dissipation (ϵ_k) terms. The

budget is dominated by the turbulent transport (T_k) and the production (P_k) terms, and much larger discrepancy is observed for the former term. The convection (C_k) and pressure-velocity (Π_k) terms greatly reduce but still take important roles, and the discrepancies between various aspect ratios are relatively smaller. As the flow develops
465 downstream, both the magnitude and discrepancy decrease and the aspect ratio are in reasonable agreement for the various T.K.E. budget terms.

It is seen in Figure 12 that the aspect ratio has a great effect on the T.K.E. budget terms especially in the separation zone and close to the reattachment point. The profiles of all terms are generally similar in shape but their magnitudes differ a lot depending
470 on the aspect ratio. It is also seen that although the budget terms are largely affected by the aspect ratio, the dissipation term (ϵ_k) is less affected and the profiles close to the airfoil surface are almost identical within the separation bubble. This implies that the Kolmogorov length scale computed by Eq.(4) ought to be nearly the same for different aspect ratios. Although the dissipation term in Eq.(4) is calculated in a different manner
475 compared with that in Eq.(6), it is confirmed that the isotropic part (ϵ_k) dominates the dissipation rate ($2\nu\overline{S'_{ij}S'_{ij}}$) even for the near-wall flow [68]. The distribution of h/η_k in Figure 2 judiciously confirms that the small-scale turbulent structures are well resolved for all aspect ratios. Another point to be noted here is that for the present separated-reattached flow, the T.K.E. is greatly determined by multiple budget terms (C_k , Π_k , D_k
480 and T_k) in addition to the production (P_k) and dissipation (ϵ_k) terms; the assumption that the production is balanced by dissipation in the eddy viscosity parameterization is not a good approximation.

Figure 13: Instantaneous skin friction coefficient distribution on the suction surface of the airfoil computed using the streamwise wall-tangent velocity.

3.4. Spanwise variation of the flow field

3.4.1. Flow separation and reattachment

The spanwise variation of any selected quantity is perhaps the most straightforward way to present the effect of the aspect ratio on the flow field. The transition from 2D to 3D, and the separation and reattachment of the flow are depicted in Figure 13 by the distribution of the instantaneous skin friction coefficient. It is noted that the span-

Figure 14: Time histories of the spanwise-averaged value and the maximum spanwise variation of the leading edge separation position.

Figure 15: Percentage time of the forward flow measured at the airfoil mid-span.

wise domain size is rescaled for clarity. The attached flow at the airfoil leading edge is quasi-2D; the separated flow becomes 3D as visualized by the nonuniform skin friction in the spanwise direction; the reattached flow further develops and the large-scale structures break up into small-scale structures until the airfoil trailing edge. The spanwise non-uniformity starts around $x/C = 0.4-0.5$ for $AR \leq 0.4$, while for $AR=0.8$ it is clearly observed even upstream of the mean separation point at $x/D \approx 0.14$. The time history of the leading edge separation position is plotted in Figure 14. $\langle x_{sep} \rangle$ is the

spanwise-averaged separation position; $x_{sep,max}$ and $x_{sep,min}$ are respectively the most downstream and most upstream instantaneous separation positions in the spanwise direction, reflecting the spanwise non-uniformity. The spanwise-averaged value varies within the range $x/C = 0.13-0.16$ and experiences a low-frequency fluctuation. The

500 spanwise non-uniformity increases with the aspect ratio. The maximum difference in the spanwise direction is negligible for $AR \leq 0.2$, while for $AR=0.4$ the value is of the order $O(10^{-3})$ and hence this variation is not discernible in Figure 13. At $AR=0.8$ the normalized maximum difference is around 0.02, which is more than one order larger than that of $AR=0.4$ and reflects the strong spanwise non-uniformity. The inaccurate

505 prediction of the leading edge separation line using a small aspect ratio has also been observed by Gross & Fasel [1]. They found a spanwise straight separation line in their DNS study using $AR=0.2$, while the comparative experiment with $AR=4.0$ shows a curved separation line [69]. Although in experiments the separation is affected by various noise sources and due to the confinement by the end plates, the results of the present

510 study confirms the inaccuracy of leading edge separation arising from an inadequacy of the spanwise domain size in numerical simulations.

It can be seen in Figure 13 that the streamwise and spanwise variations of the reattachment point are quite different depending on the aspect ratio. Figure 15 presents the streamwise variation of γ , the percentage of time that the near-wall flow moves

515 forwardly, i.e., from upstream to downstream direction, at the mid-span plane of the airfoil. A small γ indicates a locally dominant reverse flow. There are two noticeable differences between the distribution at $AR=0.8$ and those obtained at smaller aspect ratios $AR \leq 0.4$. The first is the delay in the onset of full separation. The separation of

Figure 16: Spanwise standard deviation of the streamwise velocity.

AR=0.8 first occurs at around $x/C=0.14$, while the flow fully detaches from the airfoil surface at $x/C=0.17$, both are further downstream than the results obtained at smaller aspect ratios. The second is that the separated flow is always seen at $x/C=0.40-0.55$. Although the peak C_f at AR=0.8 is smaller in magnitude (see Figure 6), the reverse flow accounts for the majority of the time.

3.4.2. Spanwise distortion of the flow

The transition from 2D to 3D flow is characterized by the corrugated structures of the quasi-2D shear layer, and the spanwise non-uniformity of the 3D flow can be reflected by the spanwise distortion of the flow field. Here we quantify the spanwise distortion by the mean spanwise standard variation of the streamwise velocity, defined as:

$$\sigma_u = \left[\frac{\int_0^{L_z} [u(x, y, z, t) - \langle u \rangle(x, y, t)]^2 dz}{U_0^2 L_z} \right]^{\frac{1}{2}}, \quad (7)$$

where $\langle u \rangle$ stands for the spanwise-averaged velocity. This quantity is zero for the quasi-2D flow and increases as the flow develops three-dimensionally. The distribution of σ_u within and downstream of the mean separation bubble is given in Figure 16 at several streamwise stations. The first probe is placed away from the local airfoil surface by

about $\Delta y = 0.005C$. The discrepancy is relatively larger for flow in the separation
535 bubble and close to the airfoil surface but decreases to zero for flow far away from
the airfoil. It increases in the streamwise direction from the interior of the separation
bubble ($x/C=0.423$) to its peak value at the mean reattachment point ($x/C=0.581$).
The value is small close to the airfoil surface and increases to its maximum at about
 $\Delta y = 0.01C$ away from it. The turbulent flow is more developed and small-scale
540 structures are formed downstream of the mean reattachment point (see Figure 13) and
the deviation is getting smaller. It is seen in the figure that although different aspect
ratios generally lead to similar predictions, the value of σ_u obtained at $AR=0.8$ is at
most 20% larger than those of small aspect ratios in the mean separation bubble. The
difference is smaller in the reattached turbulent boundary layer but is still around 10%.
545 The three-dimensional structures could not fully develop with small spanwise domain
size, and hence are not properly resolved in the simulations, which is consistent with
the findings in Figure 14.

3.4.3. Spanwise two-point correlation

The effect of the aspect ratio is evaluated by the spanwise two-point correlation of
550 the turbulent velocity component in Figure 17. Since the flow reattaches at the airfoil
surface at around $x/C=0.58$, six spanwise probes are chosen for the analysis with three
in the turbulent boundary layer close to the airfoil and the other three in the near wake.
The correlation based on Eq.(1) is performed for all three velocity components for a
comprehensive and comparative evaluation of the aspect ratio effect. It is seen in the
555 figure that large aspect ratio generally leads to better decorrelation at $\Delta z = L_z/2$. For
the probe just downstream of the mean reattachment point (subfigure (a)), the reat-

Figure 17: Spanwise two-point correlation coefficient of the fluctuating velocity components obtained at spanwise-oriented probes.

tached flow has not fully developed and the correlation is observed for the u' -velocity for all aspect ratios. Better decorrelation is achieved as the flow develops downstream, as shown in subfigure (b) and (c). Although AR=0.1 fails to produce uncorrelated results, the decorrelation is improved for $AR \geq 0.2$ for all velocity components. Comparing the profiles obtained at different aspect ratios, it is noted that a certain velocity component may achieve decorrelation even at AR=0.2, such as r_{uu} in subfigure (c), and the result compares well with those obtained at AR=0.4 and 0.8. For flow in the near wake, it is seen in subfigure (d) that AR=0.2 is sufficiently large for decorrelation except r_{ww} . The aspect ratio AR=0.4 does produce uncorrelated results especially for r_{vv} . The decorrelation is better for AR=0.8 although the result of r_{vv} is not perfect. Overall

large aspect ratio is still more preferable in that the decorrelation is generally achieved for all velocity components.

We may conclude here that for the present separated-reattached flow, the spanwise two-point correlation coefficient is largely determined by the position of the spanwise probe and the velocity component chosen for the evaluation. Decorrelations are generally obtained for large aspect ratios. Small aspect ratios may produce uncorrelated result for a certain velocity component but not all of them, thus the determination of the sufficiency of the aspect ratio can be misleading by just inspecting one velocity component on a few probes at certain positions.

4. Conclusions

Direct numerical simulation is performed for flow past a NACA0012 airfoil at 5 degrees angle of attack and Reynolds number 5×10^4 to investigate the effect of the airfoil aspect ratio on the flow separation and transition patterns. The simulations are based on an energy conservative fourth-order scheme. Our objective is to identify the aspect ratio effect on the flow pattern, by examining the time-averaged and time-dependent behavior, turbulent statistics and spanwise variations of selected statistical quantities. The simulations are carried out at aspect ratio $AR=0.1-0.8$ and the results are comparably analyzed.

The aspect ratio has different effect on the various statistical quantities. Generally a large aspect ratio permits the flow to fully develop in the spanwise direction, thus the three-dimensional flow patterns are probably closer to realistic circumstances, and the simulation results are more reliable especially for the high-fidelity LES and DNS

studies. The aspect ratio has negligible effect on the mean aerodynamic quantities such
 590 as the lift and drag coefficients, and the mean separation and reattachment positions,
 although it does slightly affect the mean velocity field within the separation bubble.
 The effect of the aspect ratio on the turbulent statistics is more notable. The turbulent
 Reynolds stress components and the turbulent kinetic energy budget terms are normally
 overpredicted by small aspect ratios, which may be significant when measured as a
 595 relative percentage difference, especially in the region near the reattachment point. The
 small aspect ratio simulations do not correctly predict the three-dimensional separation
 at the airfoil leading edge, and the spanwise variation of the velocity is also confined
 and underpredicted. The spanwise two-point correlation of the fluctuating velocity
 greatly depends on the position of the probe, and a simple evaluation on one velocity
 600 component could possibly be misleading.

The results presented in this study provides a useful reference that caution must
 be exercised in determining the aspect ratio of the bluff body in three-dimensional tur-
 bulent flow simulations. The sufficiency of the domain size may not be guaranteed
 solely based on the analysis of only a few characteristic quantities, and thorough inves-
 605 tigations are required to ensure high fidelity results. Special attention is suggested to
 be paid to the turbulent statistics and the spanwise variations of the selected statistical
 quantities since the time-averaged results are negligibly affected by the aspect ratio.

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