Finite frequency traveltime sensitivity kernels for acoustic anisotropic media: Angle dependent bananas

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SUMMARY

Anisotropy is an inherent character of the Earth subsurface. It should be considered for modeling and inversion. The acoustic VTI wave equation approximates the wave behavior in anisotropic media, and especially its kinematic characteristics. To analyze which parts of the model would affect the traveltime for anisotropic traveltime inversion methods, especially for wave equation tomography (WET), we derive the sensitivity kernels for anisotropic media using the VTI acoustic wave equation. A Born scattering approximation is first derived using the Fourier domain acoustic wave equation as a function of perturbations in three anisotropy parameters. Using the instantaneous traveltime, which unwraps the phase, we compute the kernels. These kernels resemble those for isotropic media, with the η kernel directionally dependent. They also have a maximum sensitivity along the geometrical ray, which is more realistic compared to the cross-correlation based kernels. Focusing on diving waves, which is used more often, especially recently in waveform inversion, we show sensitivity kernels in anisotropic media for this case.

INTRODUCTION

Modeling wave propagation in the Earth is at the heart of any inversion algorithm. The accuracy of the retrieved final model depends on the accuracy of the forward modeling. Usually, the Earth is assumed acoustic isotropic, which reduces the computational cost and the number of unknowns to invert for. However, the Earth is at least inherently anisotropic, and unlike ignoring the elastic nature of the Earth, anisotropy influences the phase. Alkhalifah (2000) derived an equation for acoustic VTI anisotropic media, which approximates elasticity by defining the wavefield as a scalar. The acoustic anisotropic wave equation describes wave propagation dependent on three anisotropy parameters and it approximates well the wave propagation kinematics. Alkhalifah (2013) used wavefield perturbation for an efficient and practical implementation for VTI and TTI media. It treats separately the wavefields corresponding to the anisotropy parameters and could be used as a residual correction operation.

Since the objective of Seismic exploration is to build an Earth model, which produces synthetic data and traveltimes resembling those observed in the Field, the sensitivity of the synthetic data and the traveltimes to the model is paramount to this objective. This objective is met naturally through a process referred to as Full Waveform Inversion (FWI) (Lailly, 1983; Tarantola, 1984; Mora, 1988; Tarantola, 2005; Brenders and Pratt, 2007; Virieux and Operto, 2009). In the process, the misfit between the modeled data, which includes amplitude and phase, and the observed data is reduced. Nevertheless, the critical information necessary to resolve the Earth is embed-

ded in the phase portion of the data. FWI suffers from complex nonlinearity in the objective function (Virieux and Operto, 2009), which usually renders gradient methods useless if the initial model in the inversion is not close to the true model. These problems are magnified when we consider anisotropic media and start inverting for more than one parameter. For VTI anisotropic media, Plessix and Cao (2011) show, using a parametric study, which anisotropy parameters could be retrieved using FWI. Their focus were diving waves, and in that case a combination of NMO and horizontal velocities provided the most sensitivity, in other words, the anellipticity parameter η .

Wave Equation Tomography (WET) is a reduced form of FWI, where we focus on the phase and it is usually done based on measuring the misfit between a finite-frequency traveltime of the observed and synthetic data. In WET traveltimes are computed automatically using cross-correlation (Luo and Schuster, 1991) or as proposed by Choi and Alkhalifah (2011) using the unwrapped phase. WET could be used to get the near surface structures using diving waves.

The parts of the model that effects the traveltimes at a receiver location for a wave emanating from a source is given by the sensitivity kernel (Dahlen et al., 2002). It can be used to update the velocity model by smearing the misfit over the model area weighted by the sensitivity kernel. In Djebbi and Alkhalifah (2013), we analyzed the traveltime sensitivity kernels for isotropic media using both the cross-correlation and unwrapped phase approaches. We noticed that the unwrapped phase gives more realistic kernels with full banana shapes, which is usually obtained from phase only inversion. For anisotropy, traveltime sensitivity kernels could be derived to get an insight on the data sensitivity with respect to the anisotropic parameters. Zhou (2009) derived multimode surface wave sensitivity kernels for a radially anisotropic earth media. She derived sensitivity kernels for the stiffness tensor coefficients. A generalized Born approximation is used to describe the scattering (Chapman and Coates, 1994; Červený, 2000), then the Rytov approximation is used to retrieve the traveltime information. In this abstract, we derive Born approximations for the acoustic anisotropic wave equation with respect to perturbations in the anisotropic parameters for anisotropic media. Specifically, the scattering equations are used to get the traveltime sensitivity kernels for the η parameter, NMO velocity and vertical velocity. Cross-correlation and unwrapped phase are used to compare the two approaches for WET. We start with a simple homogeneous model to show the properties of the kernels and its dependence on source-receiver orientation. Then, we show how sensitivity kernels for diving waves look like in anisotropic media.

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THEORY

Traveltime sensitivity kernels provides a sensitivity measure of model parameter to a change in the synthetic data. The single scatterer Born approximation is ideal to measure such kernels, and as a result, we start by deriving Born approximations for perturbations in the anisotropy parameters η , NMO velocity and vertical velocity.

The VTI acoustic wave equation

Alkhalifah (2000) derived a wave equation for acoustic VTI media by setting the vertical shear wave velocity to zero in the corresponding dispersion relation (Alkhalifah, 1998). Considering that v and v_v are the NMO and the vertical velocities, respectively, and the anisotropy parameter η is a function of Thomsen (Thomsen, 1986) parameters ε and δ , $\eta = \frac{\varepsilon - \delta}{1 + 2\delta}$, then, the acoustic wave equation for VTI media is given by

$$\frac{\partial^4 P}{\partial t^4} - (1 + 2\eta)v^2 \left(\frac{\partial^4 P}{\partial x^2 \partial t^2} + \frac{\partial^4 P}{\partial y^2 \partial t^2} \right) - v_v^2 \frac{\partial^4 P}{\partial z^2 \partial t^2} + 2\eta v^2 v_v^2 \left(\frac{\partial^4 P}{\partial x^2 \partial z^2} + \frac{\partial^4 P}{\partial y^2 \partial z^2} \right) = 0. \quad (1)$$

Setting $\eta = 0$, and $v = v_v$ yields the acoustic wave equation for isotropic media.

Equation 1 can be used to model wavefields in anisotropic media (Alkhalifah, 2000), which is the essential ingredient for performing FWI (Plessix and Cao, 2011). Unlike with the elastic wave equation, the acoustic one is dependent on only parameters that effect directly P-wave propagation.

Born approximation for VTI acoustic media

The wave equation 1 is parametrized using three anisotropic parameters v , v_v and η . Considering a reference model subjected to a small perturbation in these three parameters:

$$\eta = \eta_0 + \delta\eta, \quad \frac{1}{v^2} = \frac{1 + \alpha(\mathbf{x})}{v_0^2}, \quad \frac{1}{v_v^2} = \frac{1 + \beta(\mathbf{x})}{v_{v0}^2}. \quad (2)$$

Perturbations for NMO and vertical velocities are perturbations in slowness squared of the form $s^2 = s_0^2 + \delta s^2$, and thus, we get $\delta s^2 = \frac{\alpha(\mathbf{x})}{v_0^2}$ for the NMO velocity and $\delta s^2 = \frac{\beta(\mathbf{x})}{v_{v0}^2}$ for the vertical velocity.

As a response we get the perturbed Green's function

$$G(\mathbf{x}, \mathbf{x}_s) = G_0(\mathbf{x}, \mathbf{x}_s) + \delta G(\mathbf{x}, \mathbf{x}_s). \quad (3)$$

We first derive the Born approximation for perturbation in the η parameter. For neatness we divide the wave equation 1 by $(v^2 v_v^2)$, then apply time to frequency domain Fourier transform. The Green's function $G_0(\mathbf{x}, \mathbf{x}_s)$ for a single frequency satisfies the following Helmholtz form:

$$\begin{aligned} & \frac{\omega^4}{v^2 v_v^2} G_0(\mathbf{x}, \mathbf{x}_s) + \frac{\omega^2}{v_v^2} (1 + 2\eta_0) \left(\frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial x^2} + \frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial y^2} \right) \\ & + \frac{\omega^2}{v^2} \frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial z^2} + 2\eta_0 \left(\frac{\partial^4 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial x^2 \partial z^2} + \frac{\partial^4 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial y^2 \partial z^2} \right) \\ & = -\delta(\mathbf{x} - \mathbf{x}_s). \end{aligned} \quad (4)$$

The perturbed version of equation 4 for the Green's function perturbation $\delta G(\mathbf{x}, \mathbf{x}_s)$ is given by

$$\begin{aligned} \mathcal{L}_0 \delta G(\mathbf{x}, \mathbf{x}_s) = & -2\delta\eta \frac{\omega^2}{v_v^2} \left(\frac{\partial^2 G(\mathbf{x}, \mathbf{x}_s)}{\partial x^2} + \frac{\partial^2 G(\mathbf{x}, \mathbf{x}_s)}{\partial y^2} \right) \\ & - 2\delta\eta \left(\frac{\partial^4 G(\mathbf{x}, \mathbf{x}_s)}{\partial x^2 \partial z^2} + \frac{\partial^4 G(\mathbf{x}, \mathbf{x}_s)}{\partial y^2 \partial z^2} \right), \end{aligned} \quad (5)$$

where the operator $\mathcal{L}_0$ is the Helmholtz operator for the VTI acoustic wave equation given by

$$\begin{aligned} \mathcal{L}_0 = & \frac{\omega^4}{v^2 v_v^2} + \frac{\omega^2}{v_v^2} (1 + 2\eta_0) \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \\ & + \frac{\omega^2}{v^2} \frac{\partial^2}{\partial z^2} + 2\eta_0 \left(\frac{\partial^4}{\partial x^2 \partial z^2} + \frac{\partial^4}{\partial y^2 \partial z^2} \right). \end{aligned} \quad (6)$$

Contending with the Born-like approximation, we set $G = G_0$ in the right hand side of equation 5 and obtain

$$\begin{aligned} \mathcal{L}_0 \delta G(\mathbf{x}, \mathbf{x}_s) = & -2\delta\eta \frac{\omega^2}{v_v^2} \left(\frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial x^2} + \frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial y^2} \right) \\ & - 2\delta\eta \left(\frac{\partial^4 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial x^2 \partial z^2} + \frac{\partial^4 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial y^2 \partial z^2} \right). \end{aligned} \quad (7)$$

Using the representation theorem (Červený, 2000; Zhou, 2009), the Green's function perturbation recorded at the receiver position $\delta G(\mathbf{x}_g, \mathbf{x}_s)$ is given by the following formula:

$$\begin{aligned} \delta G(\mathbf{x}_g, \mathbf{x}_s) = & \int_D 2G_0(\mathbf{x}, \mathbf{x}_g) \delta\eta \left[\frac{\omega^2}{v_v^2} \left(\frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial x^2} \right. \right. \\ & \left. \left. + \frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial y^2} \right) + \frac{\partial^4 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial x^2 \partial z^2} + \frac{\partial^4 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial y^2 \partial z^2} \right] d\mathbf{x}^3. \end{aligned} \quad (8)$$

$G_0(\mathbf{x}, \mathbf{x}_g)$ is the Green's function at the receiver position $\mathbf{x}_g$ due to a source at position $\mathbf{x}$.

Using Green's theorem (Červený, 2000), we consider smoothly varying anisotropy parameters, so that their contribution to the derivatives is negligible, and simplify the Born representation for for perturbations in η as follows:

$$\begin{aligned} \delta G(\mathbf{x}_g, \mathbf{x}_s) \approx & \int_D 2\delta\eta \left[-\frac{\omega^2}{v_v^2} \frac{\partial G_0(\mathbf{x}, \mathbf{x}_g)}{\partial x} \frac{\partial G_0(\mathbf{x}, \mathbf{x}_s)}{\partial x} \right. \\ & - \frac{\omega^2}{v_v^2} \frac{\partial G_0(\mathbf{x}, \mathbf{x}_g)}{\partial y} \frac{\partial G_0(\mathbf{x}, \mathbf{x}_s)}{\partial y} + \frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_g)}{\partial x \partial z} \frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial x \partial z} \\ & \left. + \frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_g)}{\partial y \partial z} \frac{\partial^2 G_0(\mathbf{x}, \mathbf{x}_s)}{\partial y \partial z} \right] d\mathbf{x}^3. \end{aligned} \quad (9)$$

The same way we derived Born approximations for the two other anisotropy parameters. Equations 10 and 11 represents the final expressions for NMO and vertical velocity, respectively.

$$\begin{aligned} \delta G(\mathbf{x}_g, \mathbf{x}_s) \approx & \int_D \frac{\alpha(\mathbf{x})}{v_0^2} \left[\frac{\omega^4}{v_v^2} G_0(\mathbf{x}, \mathbf{x}_g) G_0(\mathbf{x}, \mathbf{x}_s) \right. \\ & \left. - \omega^2 \frac{\partial G_0(\mathbf{x}, \mathbf{x}_g)}{\partial z} \frac{\partial G_0(\mathbf{x}, \mathbf{x}_s)}{\partial z} \right] d\mathbf{x}^3. \end{aligned} \quad (10)$$

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$$\delta G(\mathbf{x}_g, \mathbf{x}_s) \approx \int_D \frac{\beta(\mathbf{x})}{v_0^2} \left[\frac{\omega^4}{v^2} G_0(\mathbf{x}, \mathbf{x}_g) G_0(\mathbf{x}, \mathbf{x}_s) - \omega^2 (1 + 2\eta) \frac{\partial G_0(\mathbf{x}, \mathbf{x}_g)}{\partial x} \frac{\partial G_0(\mathbf{x}, \mathbf{x}_s)}{\partial x} - \omega^2 (1 + 2\eta) \frac{\partial G_0(\mathbf{x}, \mathbf{x}_g)}{\partial y} \frac{\partial G_0(\mathbf{x}, \mathbf{x}_s)}{\partial y} \right] d\mathbf{x}^3. \quad (11)$$

If we set $\eta = 0$ and $v = v_v$ in the Born approximations 10 and 11, the additional terms vanish and we obtain a scaled version of the Born approximation for the isotropic case.

$$\delta G_{iso}(\mathbf{x}_g, \mathbf{x}_s) \approx \int_D \frac{\omega^2}{v^2} \frac{\alpha(\mathbf{x}) \omega^2}{v^2} G_0(\mathbf{x}, \mathbf{x}_g) G_0(\mathbf{x}, \mathbf{x}_s) d\mathbf{x}^3. \quad (12)$$

Traveltime sensitivity kernels

The sensitivity kernels provide us with regions in the model that may have contributed to a perturbation in our data. Following our previous developments (Djebbi and Alkhalifah, 2012) for isotropic sensitivity kernels we derive the VTI acoustic traveltime sensitivity kernels using the instantaneous traveltime.

The first-order Born approximation in frequency domain for VTI acoustic media could be written in the following form:

$$\delta G(\mathbf{x}_g, \mathbf{x}_s) = \int_D K_{U_B}(\mathbf{x}_g, \mathbf{x}_s) \delta p(\mathbf{x}) d\mathbf{x}^3, \quad (13)$$

where $\delta p(\mathbf{x})$ is the corresponding perturbation of the anisotropy parameter η , v or v_v . Also, $K_{U_B}(\mathbf{x}_g, \mathbf{x}_s)$ is the wavefield sensitivity kernel.

The traveltime perturbations could be computed from the wavefield perturbation by a time domain cross-correlation of the total wavefield $U(t)$ with the background wavefield $U_0(t)$ (Dahlen et al., 2002).

$$\delta T = \frac{\int_{t_1}^{t_2} \dot{U}(t) \delta U(t) dt}{\int_{t_1}^{t_2} \dot{U}(t) U(t) dt} = \int_V K_T(\mathbf{x}_g, \mathbf{x}_s) \delta p(\mathbf{x}) d\mathbf{x}^3, \quad (14)$$

where $\dot{U}(t)$ and $\ddot{U}(t)$ denote, respectively, the first and second derivative with respect to t . $K_T(\mathbf{x}_g, \mathbf{x}_s)$ is the traveltime sensitivity kernel.

Alternatively, we use the instantaneous traveltime, with unwrapped phase features, to get traveltime sensitivity information from the Born wavefield sensitivity kernel (Djebbi and Alkhalifah, 2012, 2013). Unwrapping the phase is achieved by taking the derivative of the wavefield with respect to the frequency, dividing by the wavefield and finally taking the imaginary part (Shin et al., 2003; Choi and Alkhalifah, 2011). Traveltime information is extracted using this method and could be used in several wave equation or traveltime tomography algorithms (Choi and Alkhalifah, 2011).

$$t = \text{Im} \left(\frac{\frac{\partial U(\omega)}{\partial \omega}}{U(\omega)} \right). \quad (15)$$

We consider monochromatic traveltime sensitivity kernels using the instantaneous traveltime 15. The resulting traveltime sensitivity kernel for VTI acoustic parameters perturbation is given by

$$K_T = \text{Im} \left(\frac{\frac{\partial (K_{U_B}(\mathbf{x}_g, \mathbf{x}_s))}{\partial \omega}}{U_0(\mathbf{x}_g, \mathbf{x}_s)} \right). \quad (16)$$

The band-limited kernels are obtained by a weighted summation over frequencies.

NUMERICAL RESULTS

In this section, we present the sensitivity kernels for VTI acoustic media. First, we consider a 2-D model with homogeneous anisotropy parameters ($\eta = 0.4$, $v = 2$ km/s, $v_v = 2$ km/s) to illustrate the kernels properties.

Figures 1(a)- 1(c) show the kernels for the η perturbation. Traveltime sensitivity for η depends on the source-receiver orientation. The sensitivity is maximum in the horizontal direction and zero in the vertical direction as shown in Figure 2. Comparing unwrapped phase based kernels (Figure 2(a)) and cross-correlation based ones (Figure 2(b)) a similar observation to the isotropic case (Djebbi and Alkhalifah, 2013) could be reiterated. The kernels using our approach have maximum sensitivity along the ray-path, however the Banana-Doughnut kernels with maximum sensitivity off the ray-path (Dahlen et al., 2002) are present in the cross-correlation kernels.

The second row of Figure 1 shows the kernels for NMO velocity. The kernels are similar to the velocity kernels in isotropic media. The additional term in equation 10 compared to the isotropic case has an ω^2 factor less, and thus, asymptotically they are similar.

For the vertical velocity, in the third row of Figure 1 a similar behavior as the NMO velocity is observed. A key observation

Figure 1: Traveltime sensitivity kernels for anisotropy parameter perturbations using unwrapped phase, (a-c) η parameter perturbation, (d-f) NMO velocity perturbation, (g-i) vertical velocity perturbation. The dependency on source-receiver orientation is shown.

here is that since η represents ratios of velocities, its sensitivity does not include the common isotropic element present in the other two parameters, and thus, is very angular dependent. We

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can expect the same behavior if we linearized the scattering with respect to δ instead of the NMO velocity. As a result, using the dimensionless anisotropic parameters in the inversion process allows us to isolate wavefield dependency with respect to angle. The same observation would have been realized if we use the elastic coefficients. Note, here we have not addressed the tradeoff as our objective is to invert for one parameter at a time.

Figure 2: Traveltime sensitivity cross-section for η parameter perturbation. Effect of source-receiver orientation is shown: continuous black curve: horizontal, Dashed blue curve: 45° , Dotted red curve: vertical. (a) Unwrapped phase, (b) Cross-correlation.

Since diving waves are currently the major source of information utilized by FWI (Plessix and Cao, 2011), we show in Figure 3 the sensitivity kernels for a linearly increasing velocity model with depth, $v(z) = v_0 + \alpha z$, where $v_0 = 2.0 \text{ km s}^{-1}$ and $\alpha = 1.0 \text{ s}^{-1}$. From Figures 3 and 4, the unwrapped phase kernels are full bananas, which is more consistent with full waveform observations, considering the phase component. The kernels resemble those for velocity perturbation in isotropic media using unwrapped phase (Djebbi and Alkhalifah, 2013).

CONCLUSIONS

We derive the traveltime sensitivity kernels corresponding to the acoustic anisotropic wave equation. To do that we constructed the Born scattering approximations with respect to perturbation in the anisotropy parameters. Then, the unwrapped phase is used to compute the sensitivity kernels. The shape of the kernels resembles those for the isotropic case. They have maximum sensitivity along the ray-path contrary to the cross-correlation kernels with off ray-path maximum sensitivity. The sensitivity depends on the source-receiver orientation for the η parameter perturbation.

For diving waves, the kernels have smooth full-banana shape. They could be used to formulate an anisotropic acoustic WET algorithm for diving waves.

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Figure 3: Diving wave traveltime sensitivity kernel for a constant gradient $v(z)$ velocity model. (a) η perturbation (b) NMO velocity perturbation (c) vertical velocity perturbation.

Figure 4: Diving wave traveltime sensitivity kernel cross-section for a constant gradient $v(z)$ velocity model. (a) η perturbation (b) NMO velocity perturbation (c) vertical velocity perturbation.

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EDITED REFERENCES

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