Analysis of the multi-component pseudo-pure-mode qP-wave inversion in vertical transverse isotropic (VTI) media

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SUMMARY

Multi-parameter inversion in anisotropic media suffers from the inherent trade-off between the anisotropic parameters, even under the acoustic assumption. Multi-component data, often acquired nowadays in ocean bottom acquisition and land data, provide additional information capable of resolving anisotropic parameters under the *acoustic* approximation assumption. Based on Born scattering approximation, we develop formulas capable of characterizing the radiation patterns for the acoustic pseudo-pure mode *P*-waves. Though commonly reserved for the elastic fields, we use displacement fields to constrain the acoustic vertical transverse isotropic (VTI) representation of the medium. Using the asymptotic Green's functions and a horizontal reflector we derive the radiation patterns for perturbations in the anisotropic media. The radiation pattern for the anellipticity parameter η is identically zero for the horizontal displacement. This allows us to dedicate this component to invert for velocity and δ . Computing the traveltime sensitivity kernels based on the unwrapped phase confirms the radiation patterns observations, and provide the model wavenumber behavior of the update.

INTRODUCTION

The real earth is inherently anisotropic and the accuracy of the modeling and inversion is heavily affected when ignoring the anisotropy influence. Many trials to approximate anisotropy with simple parameters have been developed. Alkhalifah (2000) derived a wave equation for the acoustic vertical transverse isotropic (VTI) medium which approximates anisotropy with a single mode by zeroing the vertical shear wave speed in the dispersion relation. The VTI acoustic wave equation describes VTI anisotropy through 3 parameters and it approximates well the wave propagation kinematics. Duveneck et al. (2008) presented a system of two second-order equations that describes the kinematics of VTI anisotropy as well. Cheng and Kang (2012, 2013) derived a 2nd order system of equations for the displacement vector components. They used a similarity transform by projecting the elastic wavefield on the isotropic polarization direction. The pseudo-acoustic system was successfully applied for reverse time migration (RTM) (Zhang et al., 2011) although the existence of shear waves artifacts.

Including anisotropy in seismic inversion causes ambiguity and/or trade-off between the different parameters (Plessix and Cao, 2011; Gholami et al., 2013; Alkhalifah and Plessix, 2014). Plessix and Cao (2011) used a numerical eigenvalue decomposition of the Hessian matrix to conclude that, for surface recorded diving waves, a combination of the NMO and horizontal velocities is the appropriate in inversion. Gholami et al. (2013) and Alkhalifah and Plessix (2014) analyzed the trade-off analytically based on the radiation pattern for the anisotropy pa-

rameters. The sensitivity to anisotropy parameters can be studied in a more qualitative way based on the derivation of the anisotropic sensitivity kernels (Sieminski et al., 2009; Zhou, 2009; Djebbi and Alkhalifah, 2013). The sensitivity kernels or Fréchet derivatives provide a map in the model space depicting the regions contributing to the data misfit at a specific receiver (Woodward, 1992; Dahlen et al., 2000). The sensitivity kernels are useful to analyze the effect of anisotropy parameters for a specific parametrization and specific acquisition geometry. Born approximation is generally used to compute a wavefield perturbation corresponding to one model parameter, then we extract the traveltime perturbation using cross-correlation (Dahlen et al., 2000) or the unwrapped phase (Djebbi and Alkhalifah, 2013, 2014).

Analysis of the traveltime sensitivity kernels for the acoustic VTI wave equation demonstrates the behavior of the anisotropy parameters perturbation and its dependency on the experimental set-up (Djebbi and Alkhalifah, 2013). Since the pseudo-pure-mode qP-wave system of equations (Cheng and Kang, 2012, 2013) was used efficiently for RTM, we propose in this abstract a study of the radiation patterns and the sensitivity kernels for seismic inversion methods. The system describes the multi-component displacement field propagation in acoustic anisotropic media. We investigate the prospect of using displacement component for seismic data inversion.

First, we derive the Born approximation for perturbations in the anisotropy parameters. We choose a parametrization based on the NMO velocity v , η and δ parameters. The Born approximation is then used to derive the radiation patterns for the displacement field in 2-D VTI media. Moreover, we compute the traveltime sensitivity kernels using the unwrapped phase (Djebbi and Alkhalifah, 2014) for the same medium. In the numerical examples, we show the derived radiation patterns for the horizontal and vertical displacement components. Then, we analyze the properties of the sensitivity kernels and the effect of the source-receiver orientation using a simple 2-D homogeneous model.

THEORY

Since anisotropy is an inherent character of the subsurface, considering its effects in modeling and inversion algorithms is essential. However, including anisotropy without understanding the physical effect of anisotropy parameters will result in biased and poor inverted models. We aim to analyze the behavior of the pseudo-pure-mode qP-waves using their radiation patterns and traveltime sensitivity kernels.

Pseudo-pure-mode qP-wave equations

The VTI acoustic wave equation was initially derived by Alkhalifah (2000) by zeroing the shear wave velocity in the dispersion

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relation. The resulting 4th order equation can be written in a simpler form by splitting it into a system of 2nd order equations (Duveneck et al., 2008). Another variant of the acoustic wave equation is the pseudo-pure-mode system of equations, which describes the propagation of the displacement components in acoustic anisotropic medium. Cheng and Kang (2012, 2013) derived this system using a similarity transform by projecting the elastic wavefield on the isotropic polarization direction. For a VTI medium, the equations are given as,

$$\begin{cases} \rho \partial_{tt}^2 u_x = C_{11} \partial_{xx}^2 u_x + C_{66} \partial_{yy}^2 u_x + C_{44} \partial_{zz}^2 u_x \\ \quad + (C_{11} - C_{66}) \partial_{xx}^2 u_y + (C_{13} + C_{44}) \partial_{xx}^2 u_z, \\ \rho \partial_{tt}^2 u_y = (C_{11} - C_{66}) \partial_{yy}^2 u_x + C_{66} \partial_{xx}^2 u_y + C_{11} \partial_{yy}^2 u_y \\ \quad + C_{44} \partial_{zz}^2 u_y + (C_{13} + C_{44}) \partial_{yy}^2 u_z, \\ \rho \partial_{tt}^2 u_z = (C_{13} + C_{44}) \partial_{zz}^2 u_x + (C_{13} + C_{44}) \partial_{zz}^2 u_y \\ \quad + C_{44} \partial_{xx}^2 u_z + C_{44} \partial_{yy}^2 u_z + C_{33} \partial_{zz}^2 u_z, \end{cases} \quad (1)$$

where $\mathbf{u} = [u_x, u_y, u_z]$ is the particle displacement field, ρ is the density and C_{ij} are the stiffness coefficients.

Our objective is the analysis of the pseudo-pure-modes interaction with the anisotropic medium and their potential application in seismic inversion, mainly in full waveform inversion (FWI). Thus, we consider only qP-wave by zeroing C_{44} and C_{66} . We limit our derivation to the 2-D case for simplicity and we consider source function components f_x and f_z . We use the NMO velocity v , the anellipticity parameter η and Thomsen parameter δ (Thomsen, 1986) to describe the anisotropy. The system of equations in the frequency domain is,

$$\begin{cases} \frac{\omega^2}{v^2} u_x + (1 + 2\eta) \partial_{xx}^2 u_x + \frac{1}{\sqrt{1+2\delta}} \partial_{xx}^2 u_z = f_x, \\ \frac{\omega^2}{v^2} u_z + \frac{1}{\sqrt{1+2\delta}} \partial_{zz}^2 u_x + \frac{1}{(1+2\delta)} \partial_{zz}^2 u_z = f_z. \end{cases} \quad (2)$$

We use the same notation for the displacement field in the frequency domain for simplicity.

Radiation patterns

The radiation patterns give the seismic wave amplitude variation as a function of the scattering angles. It is a useful tool to study how a scattering point in the subsurface acts for a specific source-receiver setup. The main objective of analyzing the radiation pattern is the optimization of an inversion algorithm to invert for specific parts of the data.

The derivation of the radiation patterns is based on the Born approximation for wavefield perturbation. We perturb the anisotropy parameters in equation 2 of the form $\frac{1}{v^2} = \frac{1}{v_0^2} (1 + \alpha)$, $\eta = \eta_0 + d\eta$ and $\delta = \delta_0 + d\delta$. The anisotropy parameters perturbation induce a perturbation in the wavefield components: $u_x = u_{x0} + du_x$ and $u_z = u_{z0} + du_z$. The perturbation equations obtained by keeping only the first-order terms and considering isotropic background medium ($\eta_0 = \delta_0 = 0$) are given as,

$$\begin{cases} \frac{\omega^2}{v_0^2} du_x + \partial_{xx}^2 du_x + \partial_{xx}^2 du_z = -\alpha \frac{\omega^2}{v_0^2} u_{x0} - 2d\eta \partial_{xx}^2 u_{x0} + d\delta \partial_{xx}^2 u_{z0}, \\ \frac{\omega^2}{v_0^2} du_z + \partial_{zz}^2 du_x + \partial_{zz}^2 du_z = -\alpha \frac{\omega^2}{v_0^2} u_{z0} + d\delta \partial_{zz}^2 u_{x0} + 2d\delta \partial_{zz}^2 u_{z0}. \end{cases} \quad (3)$$

The right hand side of the two equations 3 can be written as two virtual source components $-f_{x0}$ and $-f_{z0}$.

In order to derive an expression for the two wavefield perturbations du_x and du_z at the receiver location induced by the virtual sources of the equation 3, we need to consider the Green's functions components G_{in} for a unit source located at the receiver position and for the isotropic background medium. The Green's function component G_{in} is the i^{th} component caused by a point source oriented in the n^{th} cartesian direction. It verifies the system of equations:

$$\begin{cases} \frac{\omega^2}{v_0^2} G_{xn} + \partial_{xx}^2 G_{xn} + \partial_{xx}^2 G_{zn} = -\delta_{xn} \delta(\mathbf{x} - \mathbf{x}_r), \\ \frac{\omega^2}{v_0^2} G_{zn} + \partial_{zz}^2 G_{xn} + \partial_{zz}^2 G_{zn} = -\delta_{zn} \delta(\mathbf{x} - \mathbf{x}_r), \end{cases} \quad (4)$$

where δ_{xn} and δ_{zn} are the Kronecker delta functions and $n = x, z$.

Representation theorem (Červený, 2000; Aki and Richards, 2002) is given as:

$$du_n = \int f_{i0} G_{ni} d\mathbf{x} = \int f_{i0} G_{in} d\mathbf{x} \quad n, i = x, z. \quad (5)$$

Replacing the virtual sources $-f_{x0}$ and $-f_{z0}$ and applying the Green's theorem give:

$$\begin{cases} du_x(\mathbf{x}_s, \mathbf{x}_r, \omega) = \int \left[\alpha \frac{\omega^2}{v_0^2} (u_{x0} G_{xx} + u_{z0} G_{zx}) - 2d\eta \partial_x u_{x0} \partial_x G_{xx} \right. \\ \quad \left. + d\delta (-\partial_x u_{z0} \partial_x G_{xx} + \partial_z u_{x0} \partial_z G_{zx} + 2\partial_z u_{z0} \partial_z G_{zx}) \right] d\mathbf{x}, \\ du_z(\mathbf{x}_s, \mathbf{x}_r, \omega) = \int \left[\alpha \frac{\omega^2}{v_0^2} (u_{x0} G_{xz} + u_{z0} G_{zz}) - 2d\eta \partial_x u_{x0} \partial_x G_{xz} \right. \\ \quad \left. + d\delta (-\partial_x u_{z0} \partial_x G_{xz} + \partial_z u_{x0} \partial_z G_{zz} + 2\partial_z u_{z0} \partial_z G_{zz}) \right] d\mathbf{x}. \end{cases} \quad (6)$$

In real experiments, seismic sources have a vertical displacement component only. In this case, we have only two Green's functions components. Then, we use the notation G_x^r and G_z^r . The superscript r is used to indicate the receiver Green's functions. For the source wavefield components, we consider the Green's functions components G_x^s and G_z^s instead. Also, we omit the 0 subscript to simplify the notation. Equations 6 reduce to,

$$\begin{cases} dG_x(\mathbf{x}_s, \mathbf{x}_r, \omega) = \int \left[\alpha \frac{\omega^2}{v_0^2} G_z^s G_x^r + d\delta (\partial_z G_x^s \partial_z G_x^r + 2\partial_z G_z^s \partial_z G_x^r) \right] d\mathbf{x}, \\ dG_z(\mathbf{x}_s, \mathbf{x}_r, \omega) = \int \left[\alpha \frac{\omega^2}{v_0^2} (G_x^s G_x^r + G_z^s G_z^r) - 2d\eta \partial_x G_x^s \partial_x G_x^r \right. \\ \quad \left. + d\delta (-\partial_x G_z^s \partial_x G_x^r + \partial_z G_x^s \partial_z G_z^r + 2\partial_z G_z^s \partial_z G_z^r) \right] d\mathbf{x}. \end{cases} \quad (7)$$

We consider the asymptotic Green's functions to simplify the derivatives computation in equation 7. For a horizontal reflector the ray-parameters components are given by:

$$\mathbf{p}^s = \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix} \quad \mathbf{p}^r = \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}, \quad (8)$$

where θ is the scattering angle.

The radiation patterns for the horizontal and vertical components of the displacement, considering the parametrization $\mathbf{r} = [v, \eta, \delta]$, are given by,

$$\mathbf{a}_x = \begin{pmatrix} 1 \\ 0 \\ 3 \cos^2 \theta \end{pmatrix} \quad \text{and} \quad \mathbf{a}_z = \begin{pmatrix} 2 \\ 2 \sin^2 \theta \\ \sin^2 \theta + 3 \cos^2 \theta \end{pmatrix}. \quad (9)$$

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As expected the radiation patterns for the NMO velocity are independent on scattering angles. However, since the source is vertical, η scattering has zero signature in the horizontal component. However, the η scattering is observed in the vertical component of the displacement perturbation.

Traveltime sensitivity kernels

The sensitivity kernels describes, in a more qualitative way, the dependence of the data on parameter perturbation by providing all the locations in the model that may have contributed to certain perturbations in our data. Following Djebbi and Alkhalifah (2013, 2014), we show the VTI acoustic traveltime sensitivity kernels for displacement components using the unwrapped phase. We focus on the finite frequency traveltimes of the wavefields as in seismic inversion it is more relevant to invert for the phase part. This is the direct result of the more linear behavior of the phase compared to the amplitude part.

The first-order Born approximation in frequency domain for VTI acoustic media equation 7 can be written as,

$$dG_i(\mathbf{x}_s, \mathbf{x}_r, \omega) = \int [K_{B_i}^v(\mathbf{x}_g, \mathbf{x}_s)dv + K_{B_i}^\eta(\mathbf{x}_g, \mathbf{x}_s)d\eta + K_{B_i}^\delta(\mathbf{x}_g, \mathbf{x}_s)d\delta] d\mathbf{x} \quad i = x, z. \quad (10)$$

$K_{B_i}^p$ is the wavefield kernel for the parameter p perturbation where $p = [v, \eta, \delta]$.

Our approach is based on the unwrapped phase to compute traveltime sensitivity information from the Born wavefield sensitivity kernel. Unwrapping the phase is achieved by taking the derivative of the wavefield with respect to frequency, dividing by the wavefield and finally taking the imaginary part (Shin et al., 2003; Choi and Alkhalifah, 2013). The traveltime sensitivity kernel for a displacement component i and anisotropy parameter p is given by,

$$K_{i_i}^p = \text{Im} \left(\frac{\partial (K_{B_i}^p(\mathbf{x}_g, \mathbf{x}_s))}{\partial \omega} \frac{1}{G_i(\mathbf{x}_s, \mathbf{x}_r, \omega)} \right) \quad \text{where } i = x, z. \quad (11)$$

Im is the imaginary part symbol. $G_i(\mathbf{x}_s, \mathbf{x}_r, \omega)$ is the Green's function component i ignited at the source location $\mathbf{x}_s$ and recorded at the receiver location $\mathbf{x}_r$.

NUMERICAL EXAMPLES

Radiation patterns

We show the radiation patterns for the displacement field components in Figure 1. The principal observation is the η parameter zero radiation for the horizontal displacement. The sensitivity to η is only observed for the vertical component of the displacement u_z (Figure 1(e)). The η radiation pattern for the vertical displacement is in agreement with the observations made by Gholami et al. (2013); Alkhalifah and Plessix (2014) for the acoustic pressure field. The maximum radiation is obtained for large scattering angles. For small angles it decreases to zero sensitivity at zero scattering angles. The zero sensitivity for small angles (vertical orientation of the source and the receiver) will be shown more qualitatively using the traveltime sensitivity kernels.

The NMO velocity radiation patterns is isotropic (no angle dependency). The same behavior is observed for the pressure field. Nonetheless, a difference in amplitude is observed between the different components. The vertical component radiation is two times larger than the horizontal one. This is the result of the vertical orientation of the source function. Note, in this analysis we ignore the radiation pattern of the source, and focus only on the perturbation.

The radiation patterns for the δ parameter have a maximum radiation for small scattering angles. The δ radiation pattern shows that for the near vertical wave propagation, the sensitivity is large. The sensitivity for the horizontal component converges to zero for large scattering angles, however the vertical one is different than zero even for large angles.

These observations demonstrate an existing trade-off between the NMO velocity v and δ parameter which will result in ambiguity, however, the zero sensitivity for horizontal displacement component on η allow for smaller number of parameters to invert for and thus a better inversion compared to the acoustic VTI pressure field.

Figure 1: Radiation patterns for the horizontal and vertical components u_x and u_z of the displacement field. The first row is the u_x radiation patterns and the second row is u_z radiation patterns. (a,d) NMO velocity, (b,e) η parameter and (c,f) δ parameter.

Traveltime sensitivity kernels

We present the sensitivity kernels for the VTI acoustic pseudo-pure-modes. We consider a perturbation from an isotropic homogeneous background medium ($v_0 = 2.0 \text{ km/s}$, $\eta_0 = 0$ and $\delta_0 = 0$). The Green's functions for the homogeneous case are computed analytically.

For perturbations in the NMO velocity the sensitivity kernels are angle independent as shown in Figures 2(a-c) and 3(a-c) for three different source receiver configurations. All three kernels look similar. Cross-section slices shown in Figures 4(a) and 4(d) match perfectly, but with a difference in the kernel amplitude for the different displacement components.

The sensitivity of the horizontal component of displacement to

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Figure 2: Single frequency traveltime sensitivity kernels for the horizontal component u_x of the displacement. η sensitivity kernels are identically zero and are not shown.

Figure 3: Single frequency traveltime sensitivity kernels for the vertical component u_z of the displacement.

the η parameter is identically zero. Under the first-order Born approximation, any changes in η will result in no change of the horizontal component of the displacement. For the vertical displacement component, the sensitivity is angular dependent. The maximum sensitivity is observed for large scattering angles. The results are in good agreement with Alkhalifah and Plessix (2014) results for the acoustic pressure field radiation patterns. Focusing on the η perturbation sensitivity kernel in Figure 3(d) for a horizontal offset between source and receiver (conventional acquisition), we realize the same observation as in Djebbi and Alkhalifah (2013), that the reflection part of sensitivity kernel is weak up to 45 degrees. Thus, the high resolution information usually extracted from the reflection part of the data, will be void for η scattering information at depth. Thus, we are only able to resolve smooth η information at depth, usually available from the geometrical characteristics of the wavefield.

For perturbation in δ , the sensitivity kernel is angle dependent

Figure 4: Traveltime sensitivity kernels cross-sections. The first row is the cross-section of the horizontal component and the second row is the cross-section of the vertical component kernels. The black curve is a slice of the vertical kernel and the red dashed curve is the the slice of the horizontal kernel.

as shown in Figure 2(d-f) and 3(g-i). The vertical direction shows more energy for the horizontal displacement in the sensitivity kernel near the ray path and less so away from the ray path. The opposite behavior is realized for a source-receiver offset in the horizontal direction. For the vertical displacement component, δ sensitivity kernel is different than zero for the horizontal orientation. This result is seen from the radiation patterns and from slices of the vertically and horizontally oriented kernels in Figure 4(c).

CONCLUSIONS

The analysis of the dependence of anisotropy parameters on scattering angles is a crucial step to identify the best set of parameters to invert for in seismic inversion. We derived the radiation patterns and the traveltime sensitivity kernels for the acoustic pseudo-pure mode system of equations.

The VTI anisotropy is expressed in terms of NMO velocity, η and δ parameters and the Born scattering approximation is derived for a perturbation in the anisotropy parameters. Considering the asymptotic Green's functions, we derived equations for the radiation patterns for the horizontal and vertical displacement components. The NMO velocity radiation patterns show isotropic behavior with larger amplitude for the vertical component. The η parameter radiation pattern is identically zero for the horizontal displacement component, however, for the vertical component it is angular dependent with a maximum radiation for large scattering angles. The δ parameter shows an opposite behavior with a maximum radiation for small scattering angles. Deriving the traveltime sensitivity kernels for a homogeneous background medium, confirms the observations of the radiation patterns. A trade-off between the NMO velocity and δ parameter exist and is observed in both the radiation patterns and the sensitivity kernels.

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EDITED REFERENCES

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