Consumption Factor Optimization for Multihop Relaying over Nakagami- m Fading channels

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Abstract—In this paper, the energy efficiency of multihop relaying over Nakagami- m fading channels is investigated. The “consumption factor”, adopted as a metric to evaluate the energy efficiency, is derived for both amplify-and-forward and decode-and-forward relaying strategies. Then, based on the obtained expressions, we propose a power allocation strategy maximizing the consumption factor. In addition, a sub-optimal, low complexity, power allocation algorithm is proposed and analyzed, and the obtained power allocation scheme is compared in terms of energy efficiency to other power allocation schemes from the literature. Analytical and simulation results confirm the accuracy of our derivations, and assess the performance gains of the proposed approach.

Index Terms—Amplify-and-Forward, Capacity, Consumption factor, Decode-and-Forward, Energy efficiency, Optimization, Outage probability, Multihop relaying, Nakagami- m Fading.

I. INTRODUCTION

The problem of energy efficiency is one of the current biggest challenges towards green radio communications. In fact, as a result of the introduction of new attractive technologies (new devices, mobile streaming video, online game development, and social networking), there has been a massive increase of the number of cellular network subscribers during the last two decades. Therefore, there has been an exponential expansion of the cellular network market, followed by an increase in the number of base stations that have dominant energy requirements. Consequently, energy consumption by cellular networks and wireless service providers is nowadays a serious concern. In addition, from the users side, electromagnetic radiation is at its limit in many contexts, while for battery-powered devices, transmit and circuit energy consumption has to be minimized for better battery lifetime and performance.

On the other hand, the concept of multihop communications (where the source communicates with the destination via many

intermediate nodes) has been revisited and adapted to mitigate wireless channel impairments and ensure broader coverage [1]. In [2], the authors have shown that, in addition to extending coverage, overcoming shadowing and reducing the transmit power, multihop communications can increase the capacity of the network at a low additional cost.

The performance analysis and/or optimization of the energy-efficiency (EE) in the context of wireless networks have been investigated in the recent literature. In [3], the authors presented several route selection methods in multihop communications and evaluated their performance in terms of spectral efficiency (SE) and EE. The instantaneous trade-off between the total energy consumption-per-bit and the end-to-end rate under spatial reuse in wireless multihop networks was derived and analyzed in [4]. The authors of [5] investigated the basic trade-offs between energy consumption, hop distance, and robustness against fading. Also, other recent works present optimal power allocation (PA) strategies maximizing the EE for different scenarios in the context of cooperative communications. For instance, in [6], three optimal sets of PA, minimizing the energy consumption for a given rate, are derived. The authors of [7] propose an energy efficient relay selection and a PA scheme for a two-way relay channel with an arbitrary number of relays and, in [8], the expression of the average EE is derived for amplify-and-forward (AF) and a PA scheme that maximizes this energy efficiency is derived for the case when all nodes are constrained to use the same transmit power.

In this work, the energy efficiency of multihop communications is analyzed using the “consumption factor” (CF) introduced in [9] as a metric. We first provide an explicit analytical expression of CF, i.e., the average number of bits transmitted per unit of energy consumed by the end-to-end multihop communication system over Nakagami- m fading links. We consider both non regenerative AF and regenerative decode-and-forward (DF) relays. The obtained CF expressions are then used to derive a CF-optimal PA strategy maximizing the EE for both relaying techniques. In addition, a sub-optimal, low complexity, power allocation algorithm is proposed and analyzed. Then, we compare different power allocation strategies in terms of EE.

The rest of the paper is organized as follows. The system model is presented in Section II. In Section III, we present the derivation of the CF for both AF and DF cases, and the power allocation for CF optimization is derived in Section IV. Numerical results are presented in Section V. Section VI concludes the paper.

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This work was funded in part by King Abdullah University of Science and Technology (KAUST).

The work of M.-S. Alouini was supported by the Qatar National Research Fund (a member of Qatar Foundation) under NPRP Grant NPRP 5-250-2-087. The statements made herein are solely the responsibility of the authors.

II. SYSTEM MODEL AND NOTATION

We consider a source R_1 and a destination R_{N+1} , at a fixed distance D , communicating through $(N - 1)$ AF or DF half-duplex cascaded relays distributed arbitrarily in the source–destination line.

Each node uses only the information received from its immediate predecessor. All links are considered to be independent and not necessarily identically distributed Nakagami- m fading. The i -th hop link is of length d_i (the relays are not necessarily uniformly distributed), has ν_i as pathloss exponent, m_i as fading parameter, and α_i as instantaneous fading coefficient (with arbitrary $\mathbb{E}[|\alpha_i|^2] = \bar{\alpha}_i^2$, cf. subsection V-B for details). Throughout the analysis, and without any loss of generality, fading parameters m_i are assumed to be integer, and the noise over all channels is zero-mean additive white gaussian (AWGN) with the same variance N_0 . Interference cancellation is out of scope here and is assumed to be perfect.

Each node is in one of two possible states: transmission or reception. The power consumed by node R_i during a transmission phase is given by $P_i^t/\varepsilon + P_i^{ct}$, where P_i^t is the transmit power used by R_i , ε is the power amplifier's efficiency ($\varepsilon \in [0, 1]$), and P_i^{ct} is the circuit power consumption (PC) in transmission mode. Similarly, we denote by P_i^{cr} and the PCs of R_i during the reception mode.

At the beginning of the communication, the source node sends the first packet, and the other nodes receive then relay to the next node. Therefore, it takes N slots before all the nodes are operational. But, subsequently, and since the source is not idle while other nodes are relaying (and with the assumption of perfect interference cancellation), the destination receives one packet every time slot.

We denote by $\gamma_i = \frac{|\alpha_i|^2 P_i^t}{N_0 d_i^{\nu_i}}$ and $\bar{\gamma}_i = \mathbb{E}[\gamma_i]$ the instantaneous and average SNRs of the i^{th} hop, respectively, where $\mathbb{E}[\cdot]$ denotes the mathematical expectation.

We adopt the following notation in the rest of the paper. $U(a, b, z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} e^t dt$ denotes the confluent hypergeometric function [10]. $\Gamma(v) = \int_0^\infty x^{v-1} e^{-x} dx$ and $\gamma(v, w) = \int_0^w x^{v-1} e^{-x} dx$ denote the Gamma and lower incomplete Gamma functions, respectively.

III. CONSUMPTION FACTOR DERIVATION

CF defines the energy efficiency as the maximum achievable rate (given by Shannon's capacity) per unit of energy consumed to transmit (considering the transmit power itself and the circuit powers). It can be expressed in our context as

$$CF = \frac{B \log_2(1 + \gamma_{e2e})}{P_{\text{tot}}}, \quad (1)$$

where B is the total channel bandwidth and γ_{e2e} is the end-to-end SNR from R_1 to R_N , and P_{tot} is the total power necessary to transmit the data from the source to the destination, including all transmit and circuit powers.

A. Amplify-and-Forward

For AF, the end-to-end SNR is derived in [1] as

$$\gamma_{e2e}^{\text{AF}} = \left[\prod_{i=1}^N \left(1 + \frac{1}{\gamma_i} \right) - 1 \right]^{-1}, \quad (2)$$

and the total consumed power for an end-to-end transmission is

$$P_{\text{tot}}^{\text{AF}} = \frac{1}{\varepsilon} \sum_{i=1}^N P_i^t + P_c + N P_c^{\text{AF}}, \quad (3)$$

where P_c represents all the circuit powers (during transmission and reception modes) from R_0 to R_N , i.e., $P_c = \sum_{i=1}^N P_i^{\text{ct}} + \sum_{i=2}^{N+1} P_i^{\text{cr}}$. P_c^{AF} is the additional power consumed by one node to amplify the received signal before forwarding. Therefore, the average CF is given by

$$\overline{CF}^{\text{AF}} = b_N \int_0^\infty \dots \int_0^\infty CF^{\text{AF}}(N) \prod_{i=1}^N \left(\gamma_i^{m_i-1} e^{-\frac{m_i \gamma_i}{\bar{\gamma}_i}} \right) d\gamma_i, \quad (4)$$

where

$$b_N = \prod_{i=1}^N \frac{1}{\Gamma(m_i)} \left(\frac{m_i}{\bar{\gamma}_i} \right)^{m_i}, \quad (5)$$

and

$$CF^{\text{AF}}(N) = \frac{B \log_2(1 + \gamma_{e2e}^{\text{AF}})}{P_{\text{tot}}^{\text{AF}}}. \quad (6)$$

After a few manipulations (cf. Appendix), we get

$$\begin{aligned} \overline{CF}^{\text{AF}} &= c_N^{\text{AF}} \sum_{j=1}^\infty \frac{1}{j} \prod_{i=1}^N \frac{\Gamma(j + m_i)}{m_i!} U\left(j + m_i, 1 + m_i, \frac{m_i}{\bar{\gamma}_i}\right) \\ &= c_N^{\text{AF}} \sum_{j=1}^\infty \frac{1}{j} \prod_{i=1}^N \frac{\Gamma(j + m_i)}{m_i!} U\left(j + m_i, 1 + m_i, \frac{m_i N_0 (d_i)^{\nu_i}}{\mathbb{E}[|\alpha_i|^2] P_i^t}\right), \end{aligned} \quad (7)$$

with

$$c_N^{\text{AF}} = \frac{B}{\ln(2) \left[\frac{1}{\varepsilon} \sum_{i=1}^N P_i^t + P_c + N P_c^{\text{AF}} \right]}. \quad (8)$$

Note that the infinite sum in (26) converges quickly for $j \geq 10$.

B. Decode-and-Forward

For DF, each relay digitally decodes and re-encodes the received signal from the preceding terminal before retransmitting it to the next node. Therefore, from a decoding point of view, the equivalent end-to-end SNR is the minimum of the SNRs of individual hops, i.e.,

$$\gamma_{e2e}^{\text{DF}} = \min_{i=1 \dots N} \gamma_i, \quad (9)$$

and

$$P_{\text{tot}}^{\text{DF}} = \frac{1}{\varepsilon} \sum_{i=1}^N P_i^t + P_c + N P_c^{\text{DF}}, \quad (10)$$

P_c^{DF} being the additional power consumed by one node to decode the received signal before forwarding.

On the other hand, the cumulative distribution function (CDF) of γ_{e2e}^{DF} is given by

$$\begin{aligned} F_{\gamma_{e2e}^{\text{DF}}}(x) &= \Pr\left(\min_{i=1 \dots N} \gamma_i \leq x\right) \\ &= 1 - \Pr\left(\min_{i=1 \dots N} \gamma_i \geq x\right) \\ &= 1 - \prod_{i=1}^N \Pr(\gamma_i \geq x) \\ &= 1 - \prod_{i=1}^N (1 - F_{\gamma_i}(x)), \end{aligned} \quad (11)$$

where $F_{\gamma_i}(\cdot)$ is the CDF of the individual SNR of the i -th hop, given by

$$F_{\gamma_i}(x) = \frac{1}{\Gamma(m_i)} \gamma\left(m_i, \frac{m_i x}{\bar{\gamma}_i}\right). \quad (12)$$

The average CF for DF is then

$$\overline{\text{CF}}^{\text{DF}} = c_N^{\text{DF}} \mathbb{E}[\ln(1 + \gamma_{e2e}^{\text{DF}})], \quad (13)$$

where

$$c_N^{\text{DF}} = \frac{B}{\ln(2) \left[\frac{1}{\varepsilon} \sum_{i=1}^N P_i^t + P_c + NP_c^{\text{DF}} \right]}. \quad (14)$$

Recalling that for non-negative random variables [11]

$$\mathbb{E}[X] = \int_0^\infty \text{Prob}(X \geq x) dx, \quad (15)$$

we can write

$$\Pr(\ln(1 + \gamma_{e2e}^{\text{DF}}) \geq x) = 1 - F_{e2e}^{\text{DF}}(e^x - 1). \quad (16)$$

Using (15) and (12), we get

$$\begin{aligned} \overline{\text{CF}}^{\text{DF}} &= c_N^{\text{DF}} \int_0^\infty \prod_{i=1}^N \left(1 - \frac{\gamma\left(m_i, \frac{m_i(e^x - 1)}{\bar{\gamma}_i}\right)}{\Gamma(m_i)} \right) dx \\ &= c_N^{\text{DF}} \int_0^\infty \prod_{i=1}^N \left(1 - \frac{\gamma\left(m_i, \frac{m_i N_0 (d_i)^{\nu_i} (e^x - 1)}{\mathbb{E}(|\alpha_i|^2) P_i^t}\right)}{\Gamma(m_i)} \right) dx. \end{aligned} \quad (17)$$

which can be computed numerically through the Gauss-Laguerre quadrature [10].

Special Case of Rayleigh Fading: For Rayleigh fading links, γ_i is exponentially distributed with parameter $1/\bar{\gamma}_i$. The distribution of γ_{e2e}^{DF} is also exponential with parameter $\lambda_{e2e} = \sum_{i=1}^N \frac{1}{\bar{\gamma}_i}$. In this case,

$$\overline{\text{CF}}^{\text{DF}} = c_N^{\text{DF}} \int_0^\infty \lambda_{e2e} \ln(1 + x) e^{-\lambda_{e2e} x} dx. \quad (18)$$

Using [10, (4.337-1)] and $E_1(x) = -\text{Ei}(-x)$, where $\text{Ei}(\cdot)$ is the exponential integral function, we get the expression of CF in a simple closed-form as

$$\overline{\text{CF}}^{\text{DF}} = c_N^{\text{DF}} e^{\lambda_{e2e}} E_1(\lambda_{e2e}). \quad (19)$$

IV. CONSUMPTION FACTOR OPTIMIZATION

In this section, we derive an energy-efficient, CF-optimal, transmit power allocation strategy for the analyzed multihop relaying setup.

A. Optimal Power Allocation

Given a total power constraint P_{tot} , the optimization problem can be formulated as follows

$$\begin{aligned} \min_{P_i^t} & -\overline{\text{CF}}^{\text{AF/DF}}(P_1^t, P_2^t, \dots, P_N^t) \\ \text{s.t.} & \sum_{i=1}^N P_i^t \leq P_{\text{max}}, \end{aligned} \quad (20)$$

where P_{max} is the maximum total transmit power for all nodes. Note that (20) is a convex optimization problem and has a single global minimum (it is easy to show that the second-order convexity conditions for the expressions of $\overline{\text{CF}}$, for both AF and DF, hold with respect to $P_1^t, \dots, P_N^t$; the proof is skipped here for space considerations.)

The Lagrangian of the problem is given by

$$\mathcal{L}(P^t, \mu) = -\overline{\text{CF}}^{\text{AF/DF}}(P_1^t, \dots, P_N^t) + \mu \left(\sum_{i=1}^N P_i^t - P_{\text{max}} \right), \quad (21)$$

where μ is the Lagrange multiplier corresponding to the inequality constraint. The Karush-Kuhn-Tucker (KKT) conditions can be expressed as

$$\begin{cases} -\frac{\partial \overline{\text{CF}}^{\text{AF/DF}}}{\partial P_i^t} + \mu = 0, & i = 1, \dots, N \\ \sum_{i=1}^N P_i^t \leq P_{\text{max}}. \end{cases} \quad (22)$$

Note that solving the equations system in (22) using Newton's method is quite complex as the expressions of the first and second derivatives of $\overline{\text{CF}}$ are not straightforward. Alternatively, in this work, we adopt the "Automatic Differentiation" (AD)¹ to compute the gradient of the objective function, then it is passed to MATLAB's fmincon ² as one of the parameters, while using the *interior-point* algorithm to further accelerate the calculation time.

Note that, in addition to the increasing computation complexity (with increasing N), the optimization problem in (20) has to be solved by a central unit aware of the channel statistics of all hops, which then broadcasts the optimal transmit powers to the relaying nodes. To avoid this, we further propose a low complexity decentralized algorithm yielding close-to-optimal transmit powers.

¹For MATLAB®, AD was implemented by R. D. Neidinger in 2008 through the *valder* class which implements AD by operator overloading: it computes the first order derivative or multivariable gradient vectors starting with a known simple *valder* and propagating it through elementary functions and operators.

²MATLAB's® *fmincon* is a powerful method for solving constrained optimization problems. However, it is not fast enough for a considerable number of hops in our context; especially for the AF case.

B. Low Complexity Suboptimal Power Allocation

The idea is to assume that each node knows only the statistics of the following hop. The first node solves the optimization problem (20), assuming that the rest of the links have the same channel statistics as the first hop. This does not mean that all links have the same statistics in the system model. Therefore, this optimization can be done with only one variable, which naturally consumes much less time compared to the optimization problem with N unknown. The calculated optimal value is the operating transmit power of the first node. Then, the first node transmits its corresponding term in the expression of CF (calculated with the transmit power and the channel statistic) to the next node. Once the second node receives the information from the first, it formulates a new optimization problem using the information obtained from the first node and assuming that all the following hops have the same channel statistics as the second hop. The process continues until the last node. Note that this optimization procedure is done only once for the whole transmission since it is based on the average statistics of the channels; not the instantaneous variations of the SNR.

Practically, at the n^{th} node R_n , first, the following optimization problem is solved

$$\begin{aligned} \max_x \quad & \overline{\text{CF}}^{\text{AF/DF}}(x) \\ \text{s.t.} \quad & (N - n + 1)x \leq P_{\max} - P_{\max, n-1}, \end{aligned} \quad (23)$$

where $P_{\max, n-1} = \sum_{i=1}^{n-1} P_i^t$ for $n = 1, \dots, N$ and $P_{\max, 0} = 0$.

For AF, the expression of $\overline{\text{CF}}^{\text{AF}}(x)$ is given by

$$\begin{aligned} \overline{\text{CF}}^{\text{AF}}(x) = & c_N^{\text{AF}}(x) \sum_{j=1}^J \frac{1}{j} \mathcal{T}_{n-1}^{\text{AF}}(j) \times \\ & \left(\frac{\Gamma(j + m_n)}{\Gamma(m_n)} \text{U}\left(j + m_n, 1 + m_n, \frac{m_n N_0 d_n^\nu}{\alpha_n^2 x}\right) \right)^{N-n+1}, \end{aligned} \quad (24)$$

where we consider only the first J terms for the infinite sum in (26), and

$$c_N^{\text{AF}}(x) = \frac{B/\ln(2)}{\frac{1}{\varepsilon} \left((N - n + 1)x + \sum_{i=1}^{n-1} P_i^t \right) + P_c + N P_c^{\text{AF}}}, \quad (25)$$

and

$$\mathcal{T}_{n-1}^{\text{AF}}(j) = \prod_{i=1}^{n-1} \frac{\Gamma(j + m_i)}{\Gamma(m_i)} \text{U}\left(j + m_i, 1 + m_i, \frac{m_i}{\bar{\gamma}_i}\right), \quad \mathcal{T}_0^{\text{AF}}(k) = 1. \quad (26)$$

For DF, using a Laguerre-Gauss quadrature with K terms, the expression of $\overline{\text{CF}}^{\text{DF}}(x)$ is given by

$$\begin{aligned} \overline{\text{CF}}^{\text{DF}}(x) \approx & c_N^{\text{DF}}(x) \sum_{k=1}^K w_k e^{x_k} \mathcal{T}_{n-1}^{\text{DF}}(k) \times \\ & \left(1 - \frac{\gamma\left(m_n, \frac{m_n N_0 d_n^\nu (e^{x_k} - 1)}{\alpha_n^2 x}\right)}{\Gamma(m_n)} \right)^{N-n+1}, \end{aligned} \quad (27)$$

where

$$c_N^{\text{DF}}(x) = \frac{B/\ln(2)}{\frac{1}{\varepsilon} \left((N - n + 1)x + \sum_{i=1}^{n-1} P_i^t \right) + P_c + N P_c^{\text{DF}}}, \quad (28)$$

and

$$\mathcal{T}_{n-1}^{\text{DF}}(k) = \prod_{i=1}^{n-1} \left(1 - \frac{\gamma\left(m_i, \frac{m_i (e^{x_k} - 1)}{\bar{\gamma}_i}\right)}{\Gamma(m_i)} \right), \quad \mathcal{T}_0^{\text{DF}}(k) = 1. \quad (29)$$

After solving the optimization problem in (23), the computed optimal value of x is the operating transmit power P_n^t for the n^{th} node. This node transmits to the next node $P_{\max, n} = P_{\max, n-1} + P_n^t$ and $\mathcal{T}_n^{\text{AF}}(j)$ or $\mathcal{T}_n^{\text{DF}}(k)$ for $j = 1, 2, \dots, J$ or $k = 1, 2, \dots, K$. Note that, recursively, we can write

$$\mathcal{T}_n^{\text{AF}}(j) = \mathcal{T}_{n-1}^{\text{AF}}(j) \frac{\Gamma(j + m_n)}{m_n!} \text{U}\left(j + m_n, 1 + m_n, \frac{m_n}{\bar{\gamma}_n}\right), \quad (30)$$

and

$$\mathcal{T}_n^{\text{DF}}(k) = \mathcal{T}_{n-1}^{\text{DF}}(k) \left(1 - \frac{\gamma\left(m_n, \frac{m_n (e^{x_k} - 1)}{\bar{\gamma}_n}\right)}{\Gamma(m_n)} \right). \quad (31)$$

V. PERFORMANCE RESULTS AND COMPARISONS

A. Comparison Framework

For performance comparison purposes, and in order to complete the analysis, we first define another power allocation strategy based on the optimization of the end-to-end capacity. The end-to-end capacity of the analyzed system was already implicitly derived in Sec. III as it is the numerator in the CF expression. Explicitly, for AF, it can be expressed as

$$C^{\text{AF}} = \frac{B}{\ln(2)} \sum_{j=1}^J \frac{1}{j} \prod_{i=1}^N \frac{\Gamma(j + m_i)}{m_i!} \text{U}\left(j + m_i, 1 + m_i, \frac{m_i}{\bar{\gamma}_i}\right), \quad (32)$$

and, for DF, with analogy to (17) and using a K -order Gauss-Laguerre quadrature, it can be written as

$$C^{\text{DF}} \approx \frac{B}{\ln(2)} \sum_{k=1}^K w_k e^{x_k} \prod_{i=1}^N \left(1 - \frac{\gamma\left(m_i, \frac{m_i (e^{x_k} - 1)}{\bar{\gamma}_i}\right)}{\Gamma(m_i)} \right), \quad (33)$$

where x_k and w_k are the sample points and the weight factors of the Laguerre polynomial [10]. Based on these expressions, we can derive a capacity-optimal power allocation scheme as follows

$$\begin{aligned} \min_{P_i^t} \quad & -C^{\text{AF/DF}}(P_1^t, P_2^t, \dots, P_N^t) \\ \text{s.t.} \quad & \sum_{i=1}^N P_i^t \leq P_{\max}. \end{aligned} \quad (34)$$

Again, this is a convex optimization problem (cf. IV-A), and it is solved by following the same approach adopted to solve (20). We thus discuss the performance of the proposed CF-based approach in (20), i.e., the CF optimizing PA (denoted as CFoPA), and we compare it to the results obtained with other PAs strategies, namely: *i*) a first low complexity CF-based suboptimal uniform PA (CFsoUPA) where all nodes are constrained to transmit with the same power, computed to maximize the CF [8], *ii*) a second low complexity CF-based suboptimal PA (CFsoPA) presented in IV-B, *iii*) a capacity-optimal PA (CoPA) defined in (34), and finally *iv*) a uniform PA (UPA) where the total power is just equally divided between transmitting nodes, i.e., $P_i^t = P_{\max}/N$, as a reference case.

B. Numerical Results

We consider that the total distance between the source and the destination is normalized to unity, and that the relays are positioned uniformly between the source and the destination. Therefore, the average SNR over each single hop is given by $\bar{\gamma}_i = \frac{\bar{\alpha}_i^2 P_i^t}{N_0 d_i^{\nu_i}}$, where $d_i = D/N$. For numerical results, the values of $\bar{\alpha}_i^2$ are randomly generated uniformly between 0 and 2 (to avoid dominant links) with a unit mean for all values of N (to allow a fair comparison between all considered schemes), and the pathloss exponent is $\nu=4$. The system bandwidth and the noise power are normalized. The power amplifier's efficiency is $\epsilon=0.35$; unless it is clearly specified otherwise. The total circuit power for AF relays is $P_c + N P_c^{\text{AF}} = 0.3N$, and for DF and $P_c + N P_c^{\text{DF}} = 0.4N$.

1) *Effect of The Number of Hops*: Figs. 1 and 2 show the variation of $\bar{\text{CF}}$ with the number of hops for different PA techniques and different values of m . It can be seen that, for both AF and DF, and for all PA techniques, operating with a non optimal number of hops can considerably affect the performance in terms of CF, with losses up to about 50%. We note that the figure also confirms the performance of the simple CFsoPA proposed in IV-B which is practically the same as the optimal CFoPA, and slightly outperforming the sub-optimal CFsoUPA because the channels are not identically distributed ($\bar{\alpha}_i^2$ are arbitrarily generated). It can be seen that there is a number of hops between the source and the destination maximizing the CF because, for small N , the additional energy consumed in the circuits is not considerable, therefore it does not result in a notable increase of CF because of an increase in capacity. Differently, for high values of N , circuits power are very considerable.

2) *Effect of The Power Budget*: Fig. 3 and 4 show $\bar{\text{CF}}$ as a function of the total power budget for different PA techniques. It can be observed that CFoPA and CoPA, and similarly UPA and CFsoUPA, yield similar results for high to average power constraints. The performance of non CF-optimizing methods then decreases for high power budgets. This can be explained by the fact that CoPA is operating with the maximum available transmit power with no constraints on the transmit rate, the constraint in (34) is hence always satisfied at the boundary of the feasible region, i.e., $\sum_{i=1}^N P_i^t = P_{\max}$.

Fig. 1. Variation of $\bar{\text{CF}}^{\text{AF}}$ with the number of hops for AF and different PA schemes. $P_{\max}^{\text{AF}} = 0 \text{ dB}$.

Fig. 2. Variation of $\bar{\text{CF}}^{\text{DF}}$ with the number of hops for DF and different PA schemes. $P_{\max}^{\text{DF}} = 0 \text{ dB}$.

For relatively low power budgets, the power term in the denominator of the expression of CF is limited even at the maximum transmit power allocation. The CF optimization in (20) is therefore a maximization of the capacity (numerator in the expression of CF), using the entire transmit power budget. Beyond a given power budget threshold, referred to as the “critical power budget”, the behavior of the consumption factor changes. With CF-optimizing algorithms, we can see that CF is constant at a maximum value. This could be explained by the fact that, when the total budget is excessive, the optimization is not solved at the boundary, i.e. the nodes are not consuming the total power budget in order to moderate the denominator in the CF expression. On the other hand, for capacity-maximizing algorithms, CF is decreasing because the optimization problem allocates the total available power budget in order to increase the capacity. Therefore, high power

Fig. 3. Variation of $\overline{CF}^{AF}$ according to the total transmit power budget for different PA schemes ($N = 2$).

Fig. 5. Tradeoff between the C^{AF} (Capacity) and the $\overline{CF}^{AF}$, $\alpha_1^2 = \alpha_2^2 = 1$, $N = 2$ and $P_{max} = 0$ dB.

Fig. 4. Variation of $\overline{CF}^{DF}$ according to the total transmit power budget, for different PA schemes ($N = 2$).

Fig. 6. Tradeoff between the C^{DF} (Capacity) and the $\overline{CF}^{DF}$, $\alpha_1^2 = \alpha_2^2 = 1$, $N = 2$ and $P_{max} = 0$ dB.

budgets result in a capacity–EE trade-off. Figs. 5 and 6 illustrate this trade-off for both AF and DF.

3) *Impact of Dissimilar Link Conditions:* From the above discussion, it is obvious that the difference between CFoPA and CFsoUPA, and similarly the difference between CoPA and UPA, will not be notable for identically distributed hops. In order to show the impact of the link conditions on the performance of the sub-optimal power strategies, we consider the particular case of a dual-hop relaying. Fig. 7 shows a comparison between CFoPA and CFsoUPA in terms of $\overline{CF}$ for both AF and DF when the first and the second hops experience different fading conditions, e.g., $\alpha_1^2 \neq \alpha_2^2$. We can observe that the loss in performance of the sub-optimal CFsoUPA increases when the difference $\Delta_\alpha = (\alpha_1^2 - \alpha_2^2)/\alpha_i^2$ is increasing (we

assume that $\alpha_1^2 > \alpha_2^2$ without loss of generality).

VI. CONCLUSION

In this paper, expressions of the average energy consumption factor of multihop relaying for both AF and DF are derived. A power allocation technique maximizing this CF metric was then proposed, and its performance (in terms of EE and capacity) was compared to other PA schemes from the literature.

The analysis of the numerical results shows an interesting trade-off between CF and the end-to-end capacity, and the existence of a critical operating transmit power budget. In addition, it was shown that the number of hops should be defined carefully taking into consideration both end-to-end power

Fig. 7. Effect of dissimilar fading on $\overline{CF}$ with $P_{\text{tot}}^{\text{AF}} = P_{\text{tot}}^{\text{DF}} = 0$ dB for $N = 2$.

constraints and individual circuit powers of the relaying nodes. Finally, and in all investigated scenarios, it was observed that DF presents a relative advantage compared to AF from the energy efficiency point of view.

Among the many possible extensions of the actual work, the analysis of the critical transmit power budget, and the derivation of the optimal number of hops over other types of fading and/or in other spectrum sharing contexts would be of high interest to complete the investigation of multihop relaying schemes.

APPENDIX DERIVATION OF $\overline{CF}^{\text{AF}}$ IN (26)

From the definition in (4), we have

$$\overline{CF}^{\text{AF}} = b_N \int_0^\infty \dots \int_0^\infty CF^{\text{AF}}(N) \prod_{i=1}^N \left(\gamma_i^{m_i-1} e^{-\frac{m_i \gamma_i}{\overline{\gamma}_i}} d\gamma_i \right), \quad (35)$$

where

$$\begin{aligned} CF^{\text{AF}}(N) &= \frac{B}{P_{\text{tot}}^{\text{AF}} \ln(2)} \ln \left(1 + \left[\prod_{i=1}^N \left(\frac{\gamma_i + 1}{\gamma_i} \right) - 1 \right]^{-1} \right) \\ &= -\frac{B}{P_{\text{tot}}^{\text{AF}} \ln(2)} \ln \left(\frac{\prod_{i=1}^N (\gamma_i + 1) - \prod_{i=1}^N \gamma_i}{\prod_{i=1}^N (\gamma_i + 1)} \right) \\ &= -\frac{B \ln(1 - f_N)}{P_{\text{tot}}^{\text{AF}} \ln(2)}, \end{aligned} \quad (36)$$

and $f_N = \prod_{i=1}^N \gamma_i / \prod_{i=1}^N (1 + \gamma_i)$. Using the Taylor series expansion $\ln(1 - x) = -\sum_{j=1}^{\infty} \frac{x^j}{j}$, for $|x| < 1$, and noting that

$$\begin{aligned} &\int_0^\infty \left(\frac{\gamma_i}{1 + \gamma_i} \right)^j \gamma_i^{m_i-1} e^{-\frac{m_i \gamma_i}{\overline{\gamma}_i}} d\gamma_i \\ &= \left(\frac{m_i}{\overline{\gamma}_i} \right)^{-m_i} \Gamma(j + m_i) \mathcal{U} \left(j + m_i, 1 + m_i, \frac{m_i}{\overline{\gamma}_i} \right), \end{aligned} \quad (37)$$

we can finally write

$$\overline{CF}^{\text{AF}} = \frac{B}{P_{\text{tot}}^{\text{AF}} \ln(2)} \sum_{j=1}^{\infty} \frac{1}{j} \prod_{i=1}^N \frac{\Gamma(j + m_i)}{\Gamma(m_i)} \mathcal{U} \left(j + m_i, 1 + m_i, \frac{m_i}{\overline{\gamma}_i} \right). \quad (38)$$

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