The Influence of Gaussian Signaling Approximation on Error Performance in Cellular Networks

Laila Hesham Afify, Hesham ElSawy, Tareq Y. Al-Naffouri, and Mohamed-Slim Alouini

Abstract—Stochastic geometry analysis for cellular networks is mostly limited to outage probability and ergodic rate, which abstracts many important wireless communication aspects. Recently, a novel technique based on the Equivalent-in-Distribution (EiD) approach is proposed to extend the analysis to capture these metrics and analyze bit error probability (BEP) and symbol error probability (SEP). However, the EiD approach considerably increases the complexity of the analysis. In this paper, we propose an approximate yet accurate framework, that is also able to capture fine wireless communication details similar to the EiD approach, but with simpler analysis. The proposed methodology is verified against the exact EiD analysis in both downlink and uplink cellular networks scenarios.

Keywords—Aggregate interference, cellular networks, stochastic geometry, equivalent-in-distribution, symbol error probability.

I. INTRODUCTION

Recent studies show that the spatial locations of base stations (BSs) in cellular networks exhibit random patterns rather than deterministic grids [1]. Such randomness involves numerous uncertainties to the network model, which makes modeling and understanding the signal-to-interference-plus-ratio (SINR) very challenging. In this context, stochastic geometry provides an elegant mathematical framework that naturally accounts for the involved uncertainties and enables spatially averaged SINR characterization [2]. The most common and simple approach used in the literature for SINR characterization is to account only for the coherent sum of the interferers' signal powers at the test receiver. Although this technique is useful for understanding the general network behavior, it abstracts important wireless communication details (e.g., modulation scheme) and limits the analysis to outage probability and ergodic rate characterization [3]–[6]. Recently, a new paradigm, presented in [7]–[9], uses an *Equivalent-in-Distribution* (EiD) approach to capture more system details and extend the stochastic geometry analysis for cellular networks to tangible error performance metrics (e.g., symbol error probability). However, the analysis associated with the EiD approach is involved as it statistically accounts for the transmitted symbol from each interferer¹. Further, the EiD approach requires base-band signal analysis to calculate the characteristic function of the probability density function (CF) of the complex interference amplitude, which is non trivial to compute in advanced system models [8].

This paper presents an approximate framework that is able to capture detailed communication system aspects as the EiD, but with much simpler analysis. The main idea, which is

inspired by the work in [10], [11], is to approximate the interferers' transmitted symbols by Gaussian codebooks. This approximation alleviates the baseband analysis of the EiD and only requires the computation of the Laplace Transform of the probability density functions (LT) of the interference power. Hence, it facilitates the analysis steps. Further, it results in less computationally intensive expressions than the EiD approach. The accuracy of the proposed approximation is verified against the exact EiD approach. It is worth mentioning that the proposed approximation aligns the error performance analysis with the outage and ergodic rate calculation in the literature, as all require the LT of the interference power only.

Both the EiD approach and the proposed framework aim to characterize the average symbol error probability (ASEP). The main problem is that the ASEP expressions available in the literature are only legitimate for AWGN or Gaussian interference channels [12, chapter 8], which is not the case in cellular networks [1]. This gives rise to the need to mathematically express the aggregate interference signal strength as a conditionally zero-mean complex Gaussian random variable. This representation is the key that merges stochastic geometry analysis with the rich literature available on the performance analysis in AWGN, which renders the ASEP expressions for AWGN applicable. Then, the obtained expressions are deconditioned to obtain the de facto ASEP performance. Both the EiD and the proposed analysis rely on the conditionally Gaussian interference representation. For the sake of a simple and self-contained presentation, we will briefly describe the EiD approach before we present our model and show numerical results. Throughout the paper, we use the following notations: $\stackrel{d}{=}$ means equivalent in distribution, $\|\cdot\|$ is the Euclidian norm, $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ is the Gamma function, and $\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$ is the complementary error function [13].

II. THE EiD APPROACH

This section highlights the outline of the EiD approach, further details may be found in [7]. In the EiD approach, the complex interference signal amplitude $\mathcal{I}_s$ is first characterized by its CF, which is then exploited to obtain the conditional Gaussian representation of $\mathcal{I}_s$. This is achieved via an infinite sum of randomly scaled zero-mean Gaussian random variables $\mathcal{I}_s \stackrel{d}{=} \sum_{q=0}^\infty \sqrt{\mathcal{B}_q} G_q$, in which the scale $\mathcal{B}_q$ has the moment generating function $\mathcal{M}_{\mathcal{B}_q}(s) = \exp(s^q)$ and the variance σ_q^2 of the Gaussian random variable G_q is selected such that the CF of $\sum_{q=0}^\infty \sqrt{\mathcal{B}_q} G_q$ matches the CF of $\mathcal{I}_s$. Matching CFs directly implies EiD. Conditioned on $\mathcal{B}_q$, the summation $\sum_{q=0}^\infty \sqrt{\mathcal{B}_q} G_q$ has Gaussian distribution. The Gaussian representation of $\mathcal{I}_s$ enables the legitimate use of performance expressions available in the AWGN channel literature. However, these expressions have to be deconditioned over the infinite series of random variables $\mathcal{B}_q$. Further, as the EiD approach accounts for every transmitted symbol by

The authors are with King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah Province, Saudi Arabia, email: {laila.afify, hesham.elsawy, tareq.alnaffouri, slim.alouini}@kaust.edu.sa.

*T. Y. Al-Naffouri is also affiliated with King Fahd University of Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia.

¹An interferer can be a BS in the downlink or user equipment (UE) in the uplink.

each interferer, the expression for the CF of $\mathcal{I}_s$ contains $\mathbb{E}[f(\mathcal{Z})]$, where $f(\cdot)$ is a function that depends on the network parameters (e.g., path loss exponent, MIMO setup) and $\mathcal{Z}$ is a random variable denoting the symbol transmitted by a generic interferer. The proposed model neither has infinite random variable series to decondition on (i.e., $\mathcal{B}_q$) nor does it need to evaluate $\mathbb{E}[f(\mathcal{Z})]$, which significantly simplifies the analysis.

III. SYSTEM MODEL

We consider a single-tier² cellular network, where the BSs are modeled via a homogeneous PPP Ψ_B with intensity λ . The locations of the UEs are modeled via an independent PPP Ψ_U with intensity λ_U . Each of the BSs and the UEs is equipped with a single antenna. Average radio signal strength based association is adopted. The signal power decays with the distance according to the power-law $\|x\|^{-\eta}$, where η is an environment dependent path-loss exponent. We assume i.i.d. Nakagmai- m channel gains, where m is assumed to be an integer. All BSs transmit with a constant power P in the downlink, while UEs use truncated channel inversion power control in the uplink. That is, each UE adjusts its transmit power P_u such that its signal is received with an average value of ρ at its serving BS. UEs that cannot invert their channels are kept silent. Saturation conditions are assumed where all BSs will always have UEs to serve and all BSs and UEs always have saturated buffers.

According to Slivinyak's Theorem [2], there is no loss in generality to study the ASEP for a test link in which the test receiver is located at the origin, o . At the test transmitter side, data is mapped to a bi-dimensional constellation $\mathbf{S}$ with $\mathcal{K}$ equiprobable symbols denoted as $s_o^{(k)}$, $k = 1, 2, \dots, \mathcal{K}$, such that, $\mathbb{E}[|s_o^{(k)}|^2] = 1$. Interferers' symbols are abstracted by a Gaussian signal s_i with unit power spectral density. The Gaussian interfering symbols are the core assumption that discriminates the proposed framework from the EiD approach, which accounts for the interferers' transmitted symbols. At the test receiver side, the intended symbol is recovered via a Maximum-likelihood receiver (MLR) with perfect Channel State Information (CSI). Further, it is assumed that the test receiver has perfect intended link CSI and is unaware of the inter-cell interference. In the downlink, the test receiver is the UE in which the received complex signal is represented as

$$y = \sqrt{Pg_o s_o} \|x_o\|^{-b} + \sum_{x_i \in \Psi_B \setminus x_o} \sqrt{Pg_i s_i} \|x_i\|^{-b} + n, \quad (1)$$

where g_o and g_i are independent gamma distributed intended and interfering channel power gains (i.e., Nakagami-fading), with integer shape parameters m_o and m , and scales Ω_o, Ω respectively. $b = \frac{\eta}{2}$, and $n \sim \mathcal{CN}(0, N_o)$ denotes the circularly symmetric complex Gaussian noise at the receiver. Note that $f_{\|x_o\|}(x) = 2\pi\lambda x e^{-\pi\lambda x^2}$, $x > 0$. In the uplink, the test receiver is the BS in which the received complex signal is represented as

$$y = \sqrt{\rho g_o s_o} + \sum_{x_i \in \hat{\Psi}_u} \sqrt{P_{u_i} g_i s_i} \|x_i\|^{-b} + n. \quad (2)$$

Note that the set of interfering UEs in the uplink $\hat{\Psi}_u \subset \Psi_u$ has intensity λ because we assume saturation conditions and only one UE can transmit at a given channel at a given time

instant per BS. Further, P_{u_i} is the random variable representing the adaptive transmit power of the i^{th} UE under the assumed truncated channel inversion power control [14].

IV. PROPOSED ASEP ANALYSIS

Due to the Gaussian codebook assumption for the interfering signals (i.e., $s_i \sim \mathcal{CN}(0, 1)$), the aggregate interference in (1) and (2) are conditionally Gaussian (i.e., conditioned on $\|x_o\|$, P_{u_i} , g_i and $\|x_i\| \forall i$). Hence, the conditionally averaged SINR can be represented as

$$\Upsilon(g_o, \|x_o\|, P_{u_i}, g_i, \|x_i\|) = \frac{g_o}{\sum_i \text{SIR}_i^{-1} + \text{SNR}^{-1}}, \quad (3)$$

where $\text{SIR}_i = \frac{\mathcal{P}}{g_i \|x_i\|^{-\eta}}$ is the average received signal-to- i^{th} interference-ratio, with $\mathcal{P} = \|x_o\|^{-\eta}$ for the downlink, while $\mathcal{P} = \frac{\rho}{P_{u_i}}$ for the uplink. $\text{SNR} = \frac{\mathcal{P} \|x_o\|^{-\eta}}{N_o}$ for the downlink, and $\text{SNR} = \frac{\rho}{N_o}$ for the uplink, is the average received signal-to-noise-ratio. Since Υ is averaged over a conditionally zero-mean Gaussian aggregate interference signal, we can utilize the ASEP performance in AWGN channels expressions. In this case, following [12, chapter 8], for square quadrature amplitude modulation scheme, i.e., M -QAM, the conditional ASEP is written as

$$\text{ASEP}(M\text{-QAM}|\Upsilon) = w_1 \text{erfc}(\sqrt{\beta \Upsilon}) + w_2 \text{erfc}^2(\sqrt{\beta \Upsilon}), \quad (4)$$

where $w_1 = 2 \frac{\sqrt{M}-1}{\sqrt{M}}$, $w_2 = -\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)^2$, and $\beta = \frac{3}{2(M-1)}$. The unconditional ASEP is then obtained by averaging over the random variables $(g_o, \sum_i \text{SIR}_i^{-1})$ for the uplink, and $(g_o, \sum_i \text{SIR}_i^{-1}, \text{SNR}^{-1})$ for the downlink. For the deconditioning step, we follow [15] which shows that

$$\mathbb{E} \left[\text{erfc} \left(\sqrt{\frac{Y}{X+C}} \right) \right] = 1 - \frac{\Gamma(m_o + \frac{1}{2})}{\Gamma(m_o)} \frac{2}{\pi} \int_0^\infty \frac{1}{\sqrt{z}} e^{-z(1+m_o C)} {}_1F_1 \left(1 - m_o; \frac{3}{2}; z \right) \mathcal{L}_X(m_o z) dz, \quad (5)$$

and

$$\mathbb{E} \left[\text{erfc}^2 \left(\sqrt{\frac{Y}{X+C}} \right) \right] = 1 - \frac{4m_o}{\pi} \int_0^\infty e^{-zm_o C} \mathcal{L}_X(m_o z) \int_0^{\frac{\pi}{4}} {}_1F_1 \left(m_o + 1; 2; \frac{-z}{\sin^2 \vartheta} \right) \frac{d\vartheta}{\sin^2 \vartheta} dz, \quad (6)$$

where $Y \sim \text{Gamma}(m_o, \Omega_o)$ follows a Gamma distribution, X has an arbitrary distribution with the LT $\mathcal{L}_X(\cdot)$, C is constant, and ${}_1F_1(a; c; z) = \sum_{q=0}^\infty \frac{(a)_q z^q}{(c)_q q!}$ is the Kummer confluent hypergeometric function [13]. Note that (5) and (6) are only valid for integer m_o . Projecting back to the SINR expressions in (4), $Y = g_o$, $X = \sum_i \text{SIR}_i^{-1}$ and $C = \text{SNR}^{-1}$ which is constant³. Next, it is essential to characterize the LT of $\sum_i \text{SIR}_i^{-1}$ for both the downlink and uplink cases. Generally, the LT of the interference from a PPP can be written as [2]

$$\mathcal{L}_X(s) = \exp \left\{ -2\pi\lambda \int_t^\infty \mathbb{E} \left[1 - e^{-s \frac{\|x\|^{-\eta} g}{P}} \right] x dx \right\} \quad (7)$$

³In the downlink case, SNR is constant when conditioning on $\|x_o\|$, we will decondition on $\|x_o\|$ later.

²Extension to a multi-tier network is straightforward by following [3], [14].

such that (7) is obtained by utilizing the PGFL of the interfering PPP, with $t = \|x_o\|$ and $\tilde{\Psi} = \Psi_B$ for the downlink, whereas $t = \left(\frac{P_{u_i}}{\rho}\right)^{\frac{1}{\eta}}$ and $\tilde{\Psi} = \hat{\Psi}_u$ for the uplink. Using the expression in (7) for the uplink case is an *approximation* as the set of interfering UEs do not constitute a PPP. The accuracy of this approximation has been verified in [14]. Let $\mathcal{H}(\eta, m, s) = {}_2F_1\left(\frac{-2}{\eta}, m; 1 - \frac{2}{\eta}; \frac{-s\Omega}{m}\right) - 1$, where ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ is the Gauss hypergeometric function [13]. Let $\mathcal{I}_D$ and $\mathcal{I}_U$ represent $\sum_i \text{SIR}_i^{-1}$ for the downlink and uplink, respectively. Then, the LT of the interference in downlink and uplink are characterized via the following lemmas.

Lemma 1: Conditioned on $\|x_o\|$, the aggregate interference power LT in a downlink cellular network is given by

$$\mathcal{L}_{\mathcal{I}_D|\|x_o\|}(s) = \exp\left\{-\pi\lambda\|x_o\|^2\mathcal{H}(\eta, m, s)\right\}. \quad (8)$$

Proof: See Appendix A ■

Lemma 2: In an uplink cellular network, the aggregate interference power LT can be *approximated* by

$$\mathcal{L}_{\mathcal{I}_U}(s) \approx \exp\left\{-\nu(\rho, \lambda)\mathcal{H}(\eta, m, s)\right\}, \quad (9)$$

where $\nu(\rho, \lambda) = \frac{\gamma\left(2, \pi\lambda\left[\frac{P_u}{\rho}\right]^{\frac{1}{\eta}}\right)}{1 - e^{-\pi\lambda\left[\frac{P_u}{\rho}\right]^{\frac{1}{\eta}}}}$, such that $\gamma(a, b) = \int_0^b t^{a-1}e^{-t}dt$ is the lower incomplete gamma function [13].

Proof: See Appendix B ■

The ASEP for the downlink and uplink cases are provided by the following theorem.

Theorem 1: For the depicted system model, the ASEP expression in Nakagami- m fading environment for M -QAM modulated signals in the downlink is expressed as in (10) and (11). For the special case of $\eta = 4$ in Rayleigh fading (i.e., $m=1$), the ASEP in the downlink is simplified to

$$\begin{aligned} \text{ASEP}_{DL} = & w_1 \left(1 - \int_0^\infty \Xi(z) \lambda \sqrt{\frac{\pi}{z}} e^{-z} dz\right) \\ & + w_2 \left(1 - \frac{2}{\sqrt{\pi}} \int_0^\infty \Xi(z) \lambda \pi \frac{\text{erfc}(\sqrt{z})}{\sqrt{z}} e^{-z} dz\right), \end{aligned} \quad (12)$$

such that $\Xi(z) = \frac{1}{2}\sqrt{\frac{\pi}{v}} \exp\left\{\frac{u^2}{4v}\right\} \text{erfc}\left(\frac{u}{2\sqrt{v}}\right)$, $u = \pi\lambda\left[{}_2F_1\left(\frac{-1}{2}, 1; \frac{1}{2}; \frac{-z}{\beta}\right)\right]$ and $v = \frac{zN_o}{P\beta}$, and the ASEP in the uplink simplifies to

$$\text{ASEP}_{UL} = \sum_{c=1}^2 w_c \left(1 - \int_0^\infty \mathcal{L}_{\mathcal{I}_U}\left(\frac{z}{\beta}\right) \frac{c \text{erfc}(z\mathbb{1}_{\{c=2\}}) e^{-z(1+\frac{c}{\beta})}}{\sqrt{\pi z}} dz\right) \quad (13)$$

Proof: For the downlink case, with the aid of the integral $\int_0^\infty e^{-uk} e^{-vk^2} dk = \frac{1}{2}\sqrt{\frac{\pi}{v}} \exp\left(\frac{u^2}{4v}\right) \text{erfc}\left(\frac{u}{2\sqrt{v}}\right)$, we can obtain the unconditional ASEP_{DL} in (12) by inserting (5), (6) and (8) into (4) and averaging over the aforementioned Rayleigh distance distribution. Similarly, for the uplink, we directly plug (5), (6), and (9) into (4) to obtain ASEP_{UL} . ■

Note that the Gaussian approximation does not reduce the number of integrals required to calculate the ASEP when compared to the EiD approach in [7], [9]. Nevertheless, it reduces the number of hypergeometric functions inside the

Fig. 1. ASEP versus the received Signal-to-Noise ratio (SNR) for 4-QAM and 16-QAM modulated signals in the a) downlink and b) uplink.

exponential function to only one hypergeometric function for any constellation size (c.f. (10) and (11)) for M -QAM modulation, which significantly reduces the computational complexity. Note that, in some cases (e.g., phase-shift-keying (PSK)) the EiD-based ASEP expressions already have only one hypergeometric function. Though the Gaussian approximation has no effect on the number of hypergeometric functions in this case, it still represents a unified framework and provides a consistent methodology for studying more complex setups.

V. NUMERICAL RESULTS

In this section, we verify the proposed analysis against the exact EiD approach in the depicted downlink and uplink scenarios. For the downlink, we vary the BS transmit powers (P) while keeping N_o constant to vary the SNR, while for the uplink we change ρ against N_o . The network parameters are selected as follows, the path-loss exponent $\eta = 4$, the noise power $N_o = -90$ dBm, the UEs intensity $\lambda_u = 10$ UEs/km², the BSs intensity $\lambda = 3$ BSs/km², and the maximum transmit power $P_u = 1$ W. Note that the effective intensity of the interfering UEs is that of the serving BSs in the resource block of interest, i.e. λ . The desired symbols are modulated

$$ASEP_{DL} = w_1 \left(1 - \frac{\Gamma(m_o + \frac{1}{2})}{\Gamma(m_o)} \frac{2}{\pi} \int_0^\infty \int_0^\infty 2\pi\lambda x \exp \left\{ -\pi\lambda x^2 \left[{}_2F_1 \left(\frac{-2}{\eta}, m; 1 - \frac{2}{\eta}; \frac{-m_o\Omega z}{m\Omega_o\beta} \right) \right] - z \left(1 + \frac{m_o C}{\Omega_o\beta} \right) \right\} \frac{1}{\sqrt{z}} {}_1F_1 \left(1 - m_o; \frac{3}{2}, z \right) dz dx \right) \\ + w_2 \left(1 - \frac{4m_o}{\pi} \int_0^\infty \int_0^\infty 2\pi\lambda x \exp \left\{ -\pi\lambda x^2 \left[{}_2F_1 \left(\frac{-2}{\eta}, m; 1 - \frac{2}{\eta}; \frac{-m_o\Omega z}{m\Omega_o\beta} \right) \right] - z \frac{m_o C}{\Omega_o\beta} \right\} \int_0^{\frac{\pi}{4}} {}_1F_1 \left(m_o + 1; 2, \frac{-z}{\sin^2 \vartheta} \right) \frac{d\vartheta}{\sin^2 \vartheta} d\vartheta dz dx \right). \quad (10)$$

$$ASEP_{UL} = w_1 \left(1 - \frac{\Gamma(m_o + \frac{1}{2})}{\Gamma(m_o)} \frac{2}{\pi} \int_0^\infty \frac{1}{\sqrt{z}} e^{-z(1 + \frac{m_o C}{\Omega_o\beta})} \exp \left\{ -\nu(\rho, \lambda) \mathcal{H} \left(\eta, m, \frac{m_o z}{\Omega_o\beta} \right) \right\} {}_1F_1 \left(1 - m_o; \frac{3}{2}, z \right) dz \right) \\ + w_2 \left(1 - \frac{4m_o}{\pi} \int_0^\infty e^{-z \frac{m_o C}{\Omega_o\beta}} \exp \left\{ -\nu(\rho, \lambda) \mathcal{H} \left(\eta, m, \frac{m_o z}{\Omega_o\beta} \right) \right\} \int_0^{\frac{\pi}{4}} {}_1F_1 \left(m_o + 1; 2, \frac{-z}{\sin^2 \vartheta} \right) \frac{d\vartheta}{\sin^2 \vartheta} d\vartheta dz \right). \quad (11)$$

using square quadrature amplitude modulation (QAM) scheme, with a constellation size $M \in \{4, 16\}$. Moreover, we consider $m_o = \Omega_o$ and $m = \Omega$.

Figs. 1(a) and 1(b) show the ASEP versus the received SNR for various Nakagami- m fading environments in downlink and uplink, respectively. Indeed, the proposed Gaussian approximation yields very accurate error performance for the system model under study when compared to the exact EiD analysis.

VI. CONCLUSION

This paper presents a simple framework for the average symbol error performance in cellular networks. The proposed approach is critically beneficial for studying network performance in realistic interference environments. Although the number of integrals in the ASEP expressions is not reduced when compared to the EiD approach, this approach alleviates the baseband analysis and aligns it with the stochastic geometry literature. The analytic and computational complexity is substantially reduced while maintaining the accuracy and generality of the EiD framework. The presented model is applied to downlink and uplink scenarios of single-tier cellular networks.

APPENDIX A

PROOF OF LEMMA 1

Conditioned on $\|x_o\|$, the LT of X is given as

$$\mathcal{L}_{\mathcal{I}_D | \|x_o\|}(s) \stackrel{(a)}{=} \exp \left\{ \frac{-2\pi\lambda}{\eta} \|x_o\|^2 \int_0^1 \left[1 - \left(\frac{m}{m+sy} \right)^m \right] \frac{1}{y^{\frac{2}{\eta}+1}} dy \right\} \\ \stackrel{(b)}{=} \exp \left\{ -\pi\lambda \|x_o\|^2 \left[{}_2F_1 \left(\frac{-2}{\eta}, m; 1 - \frac{2}{\eta}; \frac{-s}{m} \right) - 1 \right] \right\} \quad (14)$$

where (a) follows from the gamma distribution of g with parameter m , and (b) is obtained from a simple change of variables $y = x^{-\eta} \|x_o\|^\eta$ and solving the integral using [16].

APPENDIX B

PROOF OF LEMMA 2

$$\mathcal{L}_{\mathcal{I}_U}(s) \stackrel{(c)}{\approx} \exp \left\{ -2\pi\lambda \int_t^\infty \mathbb{E}_{P,g} \left[1 - e^{-s \frac{P_x - \eta g}{\rho}} \right] x dx \right\} \\ \stackrel{(d)}{=} \exp \left\{ -\nu(\rho, \lambda) \int_0^1 \left[1 - \left(\frac{m}{m+sy} \right)^m \right] \frac{1}{y^{\frac{2}{\eta}+1}} dy \right\} \quad (15)$$

where the approximation of (c) follows from the PPP assumption of the active UEs, (d) follows by the independence of P_{u_i} and g_i , incorporating the LT of the gamma random variable g_i , and the change of variables $y = x^{-\eta} \frac{P}{\rho}$. Solving the integral as in Appendix A and substituting $\mathbb{E} \left[P^{\frac{2}{\eta}} \right] = \frac{1}{\pi\lambda} \frac{\rho^{\frac{2}{\eta}} \gamma \left(2, \pi\lambda \left(\frac{P_u}{\rho} \right)^{\frac{2}{\eta}} \right)}{1 - e^{-\pi\lambda \left(\frac{P_u}{\rho} \right)^{\frac{2}{\eta}}}}$ (cf. [14]) completes the proof.

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