An FDTD algorithm for simulating light propagation in anisotropic dynamic gain media

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ABSTRACT

Simulating light propagation in anisotropic dynamic gain media such as semiconductors and solid-state lasers using the finite difference time-domain FDTD technique is a tedious process, as many variables need to be evaluated in the same instant of time. The algorithm has to take care of the laser dynamic gain, rate equations, anisotropy and dispersion. In this paper, to the best of our knowledge, we present the first algorithm that solves this problem. The algorithm is based on separating calculations into independent layers and hence solving each problem in a layer of calculations. The anisotropic gain medium is presented and tested using a one-dimensional set-up. The algorithm is then used for the analysis of a two-dimensional problem.

Keywords: Anisotropy, Gain, FDTD, Dispersion, Laser, Rate Equations.

I. INTRODUCTION

The problem of anisotropic gain is common in lasers due to crystal orientation. This case is found in solid-state lasers, due to the difficulty of having perfect pumping alignment [1], and in semiconductor lasers, especially vertical cavity surface-emitting laser (VCSEL) [2]. For edge emitting lasers (EELs), the gain is high and the anisotropy is neglected. To simulate propagation of light in gain material using FDTD, the permittivity $\epsilon(\omega)$ needs to be modeled to include the gain. Gain can be static or dynamic. One of the methods to model dynamic gain is the use of the rate equations. Dynamicity of gain is determined by rate of change of population inversion ΔN . In literature, dynamic gain material are well studied [3]. In case of anisotropic gain medium, there are three problems need to be solved at the same time, i.e: dynamic gain through rate equations to calculate the carrier concentrations, (ii) dispersion and (iii) electric fields coupling due to anisotropy. The solution can be obtained in FDTD by either having more equations to be solved to decouple the variables in time as done in reference [4], or the use of matrices as suggested by Taflové [5]. In this work, the problem of many variables is solved without extra equations and without matrices and thus this is the first algorithm that simulates light propagation in dynamic anisotropic medium. Previously, we introduced an algorithm for dispersive material [6] and then extended it to deal with lasing material [7] and anisotropic material [8]. In this paper, the algorithm is extended further to handle lasing material with anisotropic dynamic gain. The main concept of the algorithm is to separate all problems into layers and to solve each layer independently.

II. DERIVATION

Derivation is started with dynamic gain medium, and then is extended to the anisotropic dynamic gain medium. The quantum polarization equation that links the polarization to carrier concentration and electric field in a semiconductor laser is given by [9]

$$\frac{d^2 P(t)}{dt^2} + \Delta\omega_a \frac{dP(t)}{dt} + \omega_a^2 P(t) = \frac{3\omega_a \epsilon \lambda^3 \gamma_{rad}}{4\pi^2} \Delta N(t) E(t) \quad (1)$$

where ω_a is the lasing frequency, λ is the transition wavelength, γ_{rad} is the radiative decay rate, $\Delta\omega_a$ is the broadened linewidth, ΔN is the population difference and ϵ is the dielectric constant. In frequency domain the polarization is given by

$$P(\omega) = \frac{3\omega_a \epsilon \lambda^3 \gamma_{rad}}{\omega_a^2 - j\omega\Delta\omega_a + \omega^2} \Delta N E(\omega) \quad (2)$$

We can assume the change in carrier population $\Delta N(t)$ to be very slow compared to $P(t)$ and $E(t)$ and therefore can be taken as constant over time. The electric flux density D is given by

$$D(\omega) = \epsilon_0 E(\omega) + P(\omega) \quad (3)$$

Therefore, the flux density can be written as

$$D(\omega) = \epsilon_0 E(\omega) + \frac{3\omega_a \epsilon \lambda^3 \gamma_{rad}}{\omega_a^2 - j\omega\Delta\omega_a + \omega^2} \Delta N E(\omega) \quad (4)$$

This can be generalized into the form

$$D(\omega) = \epsilon_0 E(\omega) + \frac{a}{b - c j \omega + d \omega^2} \Delta N E(\omega) \quad (5)$$

From there, the general algorithm constants are

$$C_1 = \frac{4d-2b\Delta t^2}{c\Delta t+2d}, \quad C_2 = \frac{c\Delta t-2d}{c\Delta t+2d}, \quad C_3 = \frac{a}{c\Delta t+2d} \quad (6)$$

The update equation for P will be

$$P^n = C_1 P^{n-1} + C_2 P^{n-2} + C_3 \Delta N^{n-1} E^{n-1} \quad (7)$$

Once current polarization P^n is calculated by knowledge of previously stored values P^{n-1} , P^{n-2} and E^{n-1} , the current electric field E^n can be calculated having the flux density D^n [6]. This form is already solved as a dispersive medium with addition of rate equation to calculate ΔN as in [7]. Finding the population difference ΔN , requires another layer of calculation. The four-level system is shown in Fig. 1. In this system, atoms have four energy levels, E_0 , E_1 , E_2 and E_3 . Populations of these levels respectively are N_0 , N_1 , N_2 , and N_3 . Atoms in the ground state are pumped optically or electrically. Pumping will populate the energy level E_3 . Lasing happens if the pumping is high enough to cause population inversion between E_2 and E_1 . For the four-level system, the ground state E_0 is assumed to have constant population all the time. Therefore, we have three rate equations for the rest of the energy levels. The rate equations for the three levels are given below

$$\frac{dN_3(t)}{dt} = W_p(t) - \frac{N_3(t)}{\tau_3} \quad (8)$$

$$\frac{dN_2(t)}{dt} = \frac{N_3(t)}{\tau_{32}} + \frac{1}{\hbar\omega_a} E(t) \frac{dP(t)}{dt} - \frac{N_2(t)}{\tau_2} \quad (9)$$

$$\frac{dN_1(t)}{dt} = \frac{N_3(t)}{\tau_{31}} + \frac{1}{\hbar\omega_a} E(t) \frac{dP(t)}{dt} - \frac{N_2(t)}{\tau_{21}} - \frac{N_1(t)}{\tau_1} \quad (10)$$

For these equations, one can start solving for N_3 . Having N_3 , N_2 can be calculated. Having N_2 , N_1 can be calculated. Such a scheme is perfect for the general algorithm. We applied this scheme in the calculation and added a layer of calculation to the FDTD algorithm to take care of the rate equations. The population difference $\Delta N(t) = N_1 - N_2$. The FDTD loop is shown in Fig.2.

Now, we move to the anisotropic material where permittivity is a tensor

$$D(\omega) = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{bmatrix} \quad (11)$$

Figure.1: A four-level laser pumping system.

Figure.2: Schematic diagram of the FDTD loop. Each calculation layer depends on the previous layer which makes the solution easier to handle

The diagonal elements can have the form

$$\varepsilon_{xx}(\omega) = \left[\varepsilon_0 + \frac{a_{xx}}{b_{xx} - c_{xx}j\omega + d_{xx}\omega^2} \Delta N_{xx} \right] \quad (12)$$

While the off diagonal elements have the form

$$\varepsilon_{xy}(\omega) = \left[\frac{a_{xy}}{b_{xy} - c_{xy}j\omega + d_{xy}\omega^2} \Delta N_{xy} \right] \quad (13)$$

For now, let's try to find one of the electric fields say E_x . Assume the elements in the tensor are constants, in discrete time, the equations will be

$$D_x^n = \varepsilon_{xx}E_x^n + \varepsilon_{xy}E_y^n + \varepsilon_{xz}E_z^n \quad (14)$$

The full equation will be

$$D_x^n = \left(\varepsilon_0 + \frac{a_{xx}}{b_{xx} - c_{xx}j\omega + d_{xx}\omega^2} \Delta N_{xx} \right) E_x^n + \varepsilon_0 + \left(\frac{a_{xx}}{b_{xx} - c_{xx}j\omega + d_{xx}\omega^2} \Delta N_{xx} \right) E_y^n + \left(\frac{a_{xx}}{b_{xx} - c_{xx}j\omega + d_{xx}\omega^2} \Delta N_{xx} \right) E_z^n \quad (15)$$

Here comes the difficulty where the algorithm has to deal with anisotropy, dispersion and dynamic gain. Eq.15 can be written as

$$D_x^n = \varepsilon_0 E_x^n + P_x^n + P_y^n + P_z^n \quad (16)$$

Assuming we can find the current polarization values P_x^n , P_y^n and P_z^n , then the value E_x^n can be calculated as

$$E_x^n = \frac{D_x^n - P_x^n - P_y^n - P_z^n}{\varepsilon_0} \quad (17)$$

Then electric fields are averaged to correct the location error as discussed in [10].

III. VERIFICATION

Since the case is just an extension to the previously reported algorithm The FDTD reflection coefficients agree with analytical coefficients as reported in [8]. To check on the dynamic gain, we have verification of an anisotropic gain medium having the following form.

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{3\omega_a \varepsilon_\infty \lambda^3 \gamma_{rad}}{\omega_a^2 - j\omega \Delta\omega_a + \omega^2} \Delta N E(\omega) = \varepsilon_\infty + \frac{\kappa \Delta N}{\omega_a^2 - j\omega \Delta\omega_a + \omega^2} E(\omega) \quad (18)$$

where κ is just a constant summing all the numerator variables and is equal to 1.5×10^{-8} , $\omega_a = 4.72 \times 10^{14} \text{ Hz}$, $\Delta\omega_a = 0.1 \times \omega_a$ and $\varepsilon_\infty = 1.5\varepsilon_0$. The reflection test was performed. The simulation was done with half space filled with material having permittivity given in Eq.18 and half space is air. A plane wave is impinged on the material and reflection coefficient is calculated. To check for the dynamic reaction, ΔN was assumed to be -1×10^{25} , -5×10^{25} , -10×10^{25} and $-20 \times 10^{25} \text{ e/cm}^2$. FDTD reflection coefficients agree with analytical ones as shown in Figure 3. If the magnitude of the applied pulse is increased to higher values then ΔN gets smaller and the gain becomes smaller causing saturation. For very high magnitudes, ΔN will oscillate until it reaches equilibrium. These results, shown in Fig3.A, agree well with theoretical behavior given by L. Coldren [11]

Figure.3: A) The population difference at a point in the dynamic gain medium after applying three pulses with different magnitudes. Small magnitudes have no effect on population difference. High magnitudes reduce population difference and very high magnitudes make it oscillate and saturate. B) the calculated vs analytical reflection coefficients of half space filled with dynamic gain material after changing the population difference from 1×10^{25} to $20 \times 10^{25} \text{ e/cm}^2$

IV. 2D SIMULATION

The goal of this work is to show the ability of this algorithm to simulate propagation in anisotropic gain medium. Signal growth, oscillation and anisotropy are the evidence of this ability. A 2D simulation is performed by having a circular disk at the center of the simulation space. The permittivity tensor is

$$D(\omega) = \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & 0 \\ \varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \begin{bmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{bmatrix} \quad (19)$$

where

$$\varepsilon_1 = \varepsilon_0 - \frac{\kappa_1}{b_1 - c_1 j\omega + d_1 \omega^2} \Delta N_1, \quad \varepsilon_2 = \frac{\kappa_2}{b_2 - c_2 j\omega + d_2 \omega^2} \Delta N_2 \quad (20)$$

The parameters are $\omega_a = 2\pi \times 4.72 \times 10^{14} \text{ Hz}$ or $\lambda = 635 \text{ nm}$, $\kappa_1 = \kappa_2 = 1.5 \times 10^{-8}$, $c_1 = c_2 = 0.2 \times \omega_a$. Pumping rate $W_p(t) = 1 \times 10^{29} \text{ m}^{-3}$, $\tau_1 = 1 \times 10^{-9} \text{ s}$, $\tau_2 = 1.35 \times 10^{-7} \text{ s}$, $\tau_3 = 1 \times 10^{-10} \text{ s}$, $\tau_{32} = 0.99 \times 10^{-10} \text{ s}$, $\tau_{21} = 1.35 \times 10^{-7} \text{ s}$ and $\tau_{31} = 1 \times 10^{-6} \text{ s}$. The simulation space is $6 \times 6 \mu\text{m}^2$ and the disk has a radius of $2 \mu\text{m}$. Discretization steps dx and dy are 20 nm . The tensor is chosen this way so anisotropy can be seen in the TE simulation plane only. The rate equations in the E_x and in the E_y were chosen to be the same but having different

rate equation is not a problem as they are totally independent. The excited pulse is a Gaussian plane wave initiated by E_y and H_z to give propagation in the positive x -axis. The scattered field-total field technique is used to have a perfect plane wave. The Gaussian pulse hits the circular disk and a growing oscillation at the lasing wavelength is seen. This shows the gain action on the E_y plane. Even though the E_x fields are not initiated at all, the E_x fields inside the disk start to have growing oscillation in the direction of the propagation when the plane wave hits the disk in E_y plane. This shows the anisotropy action (see Figure 4).

Figure. 4: A) 2D propagation of a plane wave in E_y plane impinging the anisotropic dynamic gain disk. B) Propagation in the E_x plane. C) E_y pulse measured at a line on the center of the disk at different times. it is growing in time and oscillating around lasing frequency. D) E_x pulse measured at a line on the center of the disk at different times. it is growing in time and oscillating around lasing frequency

V. CONCLUSION

An algorithm is presented to simulate the wave propagation in the anisotropic dynamic gain medium. The algorithm solves the problem of many variables by separating calculations into separate independent calculation layers. This method enables treating dynamic gain and anisotropy. The algorithm was verified in a 1D medium and further studied in a 2D structure. This algorithm can be used as a useful tool to analyze and study propagation in solid-state lasers and VCSELs, semiconductor laser where the gain is low and anisotropy or polarization have an effect on laser performance.

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