Sphere impact and penetration into wet sand

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We present experimental results for the penetration of a solid sphere when released onto wet sand. We show, by measuring the final penetration depth, that the cohesion induced by the water can result in either a deeper or shallower penetration for a given release height compared to dry granular material. Thus the presence of water can either lubricate or stiffen the granular material. By assuming the shear rate is proportional to the impact velocity and using the depth-averaged stopping force in calculating the shear stress, we derive effective viscosities for the wet granular materials.

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Many recent studies involving impact into granular materials have focused on dry, loose materials in order to replicate fluidlike behavior in response to impact. In the case of a sphere impacting a granular bed, this is readily demonstrated with the observations of ejecta sheets [1] during the first moments after impact, jets from the hydrostatic collapse of the cavity [2–5], and sinking into loose [6] or ultralow density [7] grains. In such experiments, the dynamics is governed by interparticle collisions and friction which are noncohesive.

The motion of the impactor, often a dense sphere, in the bed is used to deduce the form of the drag force exerted on the sphere, since it is normally found [8–18] that a sphere comes to rest at a finite depth below the initial surface level of the bed (note the exceptional results of Ref. [7], showing that the sphere reaches a terminal velocity, resulting in infinite penetration).

The final position of the impactor in the bed is a particularly useful quantity as it can be used to measure the depth-averaged stopping force [12], $F_s = mgH/\delta$, where $H = h + \delta$ is the sum of the free-fall height h and the final depth δ below the initial free surface of the bed.

Various empirical scalings for δ have been reported in the literature; $\delta/\delta_0 = (H/\delta_0)^{1/3}$ was found to be a good approximation for a range of impactor-grain combinations for shallow penetration [8,11], where δ_0 is the penetration depth for $V_0 = 0$; δ was found to depend linearly on the impactor momentum by Ref. [9] with $\delta \approx \delta_0 + \alpha V_0$; a modified scaling was proposed by Ref. [15] with $\delta = C_1 V_0 \sqrt{R_0 \rho_s / g \rho_g} + C_2 (\rho_s / \rho_g)^{0.59} R_0^{3/4}$, where $R_0 = D_0/2$ is the sphere radius. The penetration depth was expressed in the form $\delta/D_0 \propto (\rho_s / \rho_g)^\beta (H/D_0)^\alpha$ by Ref. [14], where D_0 and ρ_s are the diameter and density of the impactor and ρ_g is the density of the grains. The exponents β and α were found to be strongly dependent on the geometry of the experiment [14].

Although ambient humidity may play some role in the formation of microscopic liquid bridges for experiments conducted at atmospheric pressure, the materials in the above-mentioned experiments are generally treated as dry and noncohesive. In contrast, wet granular materials are cohesive due to the surface tension of the liquid which wets the grains, and four basic states of wet granular material have been identified [19]: *pendular*—liquid bridges between the contact points of the grains; *funicular*—both liquid bridges

and liquid-filled pores; *capillary*—almost all the pores filled with liquid; and *slurry*—grains are fully immersed, no capillary action. Generally, these states can be distinguished by the level of liquid saturation. In the present Rapid Communication, we investigate the response of wet granular materials to the impact of a dense sphere in terms of the final penetration depth inside the bed. We systematically vary the liquid saturation S of the granular material and seek to fit our data with power-law scalings.

Methods. The experimental setup consists of a plexiglass cylinder filled with a granular material. A steel sphere with diameter D_0 and density $\rho_s = 7850 \text{ kg m}^{-3}$ is released by an electromagnet directly above the center of the cylinder and falls freely under gravity to impact the bed with speed $V_0 = \sqrt{2gh}$, where h is the release height above the surface of the granular bed. To focus on the effects of the bed saturation S we use a single sphere and container size ($D_0 = 20 \text{ mm}$, $D_{\text{cyl}} = 90 \text{ mm}$, and $H_{\text{cyl}} = 200 \text{ mm}$). The results of Ref. [14] indicate that confinement effects due to our cylinder-to-sphere diameter ratio ($D_{\text{cyl}}/D_0 = 4.5$) will result in penetration that is smaller than for an “unbounded” case (requiring $D_{\text{cyl}}/D_0 > 5$), but this effect is constant throughout all the experiments and thus we can neglect this aspect in our analysis.

The bed is prepared by either pouring beads into the cylinder and scraping the surface level (in the case of dry beads) or by scooping the material loosely into the cylinder and then scraping level (for the wet beads). For both dry and wet material, tapping the cylinder on the side for approximately 1 min yielded closer packings with higher bulk densities. The beads used were glass beads with a mean diameter $d_b = 520 \mu\text{m}$ ($d_{10} = 441 \mu\text{m}$, $d_{90} = 564 \mu\text{m}$). The grain density is $\rho_g = 2500 \text{ kg m}^{-3}$. For dry material we characterize the bed with the initial voidage ϵ , whereby the initial packing fraction is $\phi = 1 - \epsilon$. The angle of repose for the dry beads is $\theta_r = 22^\circ$ with an internal friction coefficient of $\mu = \tan(\theta_r) = 0.4$.

Wet granular materials were prepared by continuously stirring a specific mass of dry glass beads with a specific mass of water using a mixer with a 45° pitched propeller blade in a standard laboratory Nalgene tray 20 cm wide by 20 cm deep for at least 5 min to ensure even distribution of the liquid. The mixer was manually moved throughout the sample and additional manual stirring was performed if needed to fully incorporate all material. A time

TABLE I. Moisture content $M\%$, bulk density ρ_b (g/cm³), dry packing fraction ϕ_{dry} , voidage ϵ , and saturation S of the granular beds used in this study. In all cases, the liquid was pure water.

Bed state	$M\%$	ρ_{bulk}	ϕ_{dry} (%)	ϵ (%)	S (%)
Dry	0	1.48	59	41	0
Dry	0	1.57	63	37	0
Pendular	2.5	1.21	47.2	47.6	5.9
Pendular	2.5	1.41	54.8	36.4	8.8
Pendular	5	1.25	47.6	41	13.1
Pendular	5	1.45	54.8	34	17.1
Pendular	10	1.32	47.6	35	27.2
Funicular	10	1.57	56.4	23	40.2
Funicular	15	1.42	48.4	28	43.4
Funicular	15	1.66	56.8	15.2	62.6
Funicular	20	1.54	49.2	17.4	63.8
Funicular	20	1.72	55.2	7.8	81.5
Funicular	25	1.70	51.6	7.6	84.7
Capillary	25	1.81	59.4	3.3	93.1

of 5 min was found to be sufficient to achieve a uniform consistency based upon visual inspection and repeatability of results. The moisture content of the wet granular material is given by $M\% = \frac{M_{\text{liq}}}{M_{\text{beads}}}$, where M_{liq} and M_{beads} are the masses of the water and beads, respectively [20,21]. The saturation S is then defined as the ratio of the volume of water in the bed to the total volume of the voidage, given by $S = \frac{V_{\text{liq}}}{V_{\text{liq}} + V_{\text{air}}}$, where V_{liq} and V_{air} are the liquid and air-void volumes in the bed, respectively. Table I shows the liquid moisture contents, voidage, and corresponding saturations used in this study where water is the liquid added to the beads.

The final position of the sphere below the initial level of the grains δ was measured by carefully inserting a millimetric scale into the bed until it touched the top of the sphere. However, if the sphere came to rest above the initial level of the grain, it was measured directly from high-resolution digital images taken of the final position. In accordance with previous studies, we analyze our data for the penetration depth in terms of the total distance covered by the sphere, $H = h + \delta$.

Results. Figure 1(a) shows the final penetration depth versus impact velocity onto dry grains prepared with two packing fractions, $\phi = 0.59$ and $\phi = 0.63$. We immediately note deeper penetrations for the lower packing fraction at all impact velocities and a clear linear dependence on the impact velocity for both. When fitted to the equation $\delta = \delta_0 + \alpha_1 V_0$, we find values of $\alpha_1 = 1.31$ and 0.85 for $\phi = 0.59$ and 0.63 , respectively. Figure 1(b) shows the penetration depth versus saturation for two select release heights, corresponding to $V_0 = 3.19$ and 4.93 m/s, respectively, for both the poured and tapped preparation methods. Here, it is clear that the penetration can either be deeper or shallower than the dry case, depending on the saturation, but, in general, the deepest penetration is observed for the lower saturations. We also note here that the cavity formed behind the impacting sphere does not close for low liquid saturations, which is not the case for dry or highly saturated materials, and thus there appears to be a critical saturation leading to the maximum volume change in the target bed. A similar observation of a critical packing

FIG. 1. (Color online) Final penetration depth vs (a) impact velocity for the dry grains with $\phi = 0.59$ and 0.63 . The solid lines in (a) indicate fits to $\delta = \delta_0 + \alpha_1 V_0$, where $\alpha = 1.31$ and 0.85 , respectively. (b) Final penetration depth vs saturation for two release heights. (c) Final penetration depth vs total fall height ($H = h + \delta$) for the wet grains poured into the cylinder. (d) Final penetration depth vs total fall height for the wet grains in a tapped cylinder. The solid black lines in (c) and (d) indicate fits to $\delta = AH^{\alpha_2}$ for the dry grains where $A \approx 1.13$ and $\alpha_2 = 0.37$ and 0.32 , respectively, and the dashed and dotted lines in (c) indicate slopes of 0.6 and 0.5 , while in (d) both indicate slopes of 0.45 . Note the region to the left of the red/gray lines in (c) and (d) is not permitted.

state was reported previously for dry materials [22], while the dynamic change in packing fraction was assessed by Ref. [23].

Figures 1(c) and 1(d) show the full data for penetration depth for the wet grains, plotted against the total fall height, $H = h + \delta$, where the material was poured loosely into the cylinder [Fig. 1(c)] and where the cylinder was tapped after filling [Fig. 1(d)]. Here, we have also plotted the corresponding data for the dry grains (solid black lines), which, in agreement with previous studies [8,11–14], is well approximated by a power law of the form $\delta = AH^{\alpha_2}$, where $\alpha_2 = 0.37$ and 0.32 for $\phi = 0.59$ and 0.63 , respectively. The prefactor is $A = 1.13 \pm 0.05$. Note that these power-law exponents for the dry case are very close to the predicted scaling $\delta \sim H^{1/3}$ that is found for shallow penetrations (e.g., Ref. [13]), where it is also found that the penetration scales with other system parameters as $\delta \sim \mu^{-1}(\rho_s/\rho_g)^{1/2}D_b^{2/3}$.

In Fig. 1(c), we see that for all the wet materials tested, $\delta_{0,\text{wet}} < \delta_{0,\text{dry}}$, but for the highest drop heights ($H \gtrsim 100$), $\delta_{\text{wet}} > \delta_{\text{dry}}$ in all cases. Generally, the penetration depth scales as $\delta \sim H^\alpha$, where $\alpha \in (0.5, 0.6)$ as shown by the dotted and dashed lines. In Fig. 1(d) for the higher packings, obtained by tapping the cylinder prior to impact, we again find that $\delta_{0,\text{wet}} < \delta_{0,\text{dry}}$ for all cases but, in contrast to Fig. 1(b), δ_{wet} only exceeds δ_{dry} for the lowest liquid saturations ($S = 8.8\%$ and 17.1%) and with the greatest drop heights, $H > 100$. For the tapped preparations, the penetration depth generally scales as $\delta \sim H^{0.45}$, as indicated by the parallel dashed and dotted lines.

FIG. 2. (Color online) (a) Penetration depth vs total drop height, both scaled by the minimum penetration δ_0 . (b) Minimum penetration δ_0 vs saturation S . The solid triangles in (a) indicate data for the highest saturation, $S = 93.1\%$, where there is a marked increase in δ_0 , shown in (b).

In accordance with previous works [13], we can normalize both the penetration depth and the total drop height by the minimum penetration depth, $\delta_0 = \delta(V_0 = 0)$. Under this simple scaling, we find surprisingly good collapse for both the poured and tapped wet grains, as shown by Fig. 2(a), where the data for each set, i.e., poured and tapped, can be described approximately as $\delta/\delta_0 = (H/\delta_0)^{0.55}$ and $\delta/\delta_0 = (H/\delta_0)^{0.45}$, respectively. The exception is the data for the highest saturation, $S = 93.1\%$, shown by the solid red triangles, which clearly do not collapse. Inspecting the data for δ_0 , shown in Fig. 2(b), we observe a marked increase for this particular saturation, which explains the separation of this data seen in Fig. 2(a).

Previous studies have reported that δ_0 scales primarily with the impactor density as $\delta_0 \sim \rho_s^k$, where the exponent $k = 3/4$ was found by Ref. [12], $k = 1$ by Ref. [6], and $k = 0.59$ by Ref. [15]. In all these works, the initial bed state was kept constant. Here, where ρ_s is constant, we find that δ_0 is now also dependent on the liquid saturation. Excluding the data point for the nearly saturated material ($S = 93.1\%$), we find that $\delta_0/\delta_{\text{dry}}$ scales roughly as $S^{-1/2}$. For the nearly saturated material, the higher value of δ_0 than for the lower saturations may result from the fact that *drained shear* occurs, whereby the loading rate is low enough that the water in the voids can drain out, allowing for dilation of the material [24]. Conversely, for higher impact velocities, *undrained shear* may occur, whereby the pore water pressure increases and thus suppresses the tendency for the material to dilate, reducing the penetration depth.

Yield stress model. Granular materials have previously been modeled as a Bingham fluid [9], with a yield stress τ_0 and an effective viscosity μ_{eff} , whereby the relationship between shear stress and shear rate is given by $\tau = \tau_0 + \mu_{\text{eff}}\dot{\gamma}$. This was first assessed by Ref. [9] for dry materials based on the observation of penetration for $V_0 = 0$. For nonzero impact speeds, the stress can be estimated from the stopping force as $\tau \sim F_s/A = 2D_0\rho_s gH/(3\delta)$ and the shear rate is assumed to

FIG. 3. (Color online) Shear stress τ vs shear rate $\dot{\gamma}$ for a range of saturations for both (a) poured and (b) tapped preparations.

FIG. 4. (Color online) (a) Yield stress τ_0 vs saturation and (b) effective viscosity μ_{eff} vs saturation.

scale as $\gamma \sim V_0(\rho_s/\rho_b)^{1/2}/D_0$. Plots of the stress versus shear are presented in Figs. 3(a) and 3(b) for various saturations, all exhibiting linear trends. However, for the tapped cylinders, the intercept for zero impact velocity can be negative, which is unphysical, as also noted by Ref. [8]. Thus we can use the experimental data of Fig. 2(b) to calculate the yield stress from the minimum penetration as $\tau_0 = mg/A_0$, where $A_0 = \pi D_0^2/4$ for $\delta_0 \geq D_0/2$ and $A_0 = \pi(\delta_0 D_0 - \delta_0^2)$ for $\delta_0 < D_0/2$. Figure 4(a) shows the yield stress plotted against the saturation,

showing maximum stresses of a similar order (4200 and 4900 N/m²) at saturations $S = 64\%$ and 82% , respectively, for the two preparations methods. Also, from the data of τ vs γ , such as that in Fig. 3, we can then calculate the effective viscosities μ_{eff} shown in Fig. 4(b). Here, for the poured preparations, we find that μ_{eff} drops sharply from the dry value and then exhibits a gradual increase with saturation from $\mu_{\text{eff}} \approx 18$ Pa s at $S = 5.98\%$ up to $\mu_{\text{eff}} \approx 26$ Pa s at $S = 84.7\%$. For the tapped cylinder, we again find an initial reduction in μ_{eff} from the dry value, followed by a marked increase as the saturation increases from $S = 17\%$ to 63% . This data also indicates that the preparation method has a profound influence on the response to impact. Previously, it was shown that tapping a granular bed increases both the packing fraction and the coordination number (number of contacts per particle) [25], thus the restructuring of the contacts and voids appears to have a strong influence on both the apparent yield stress and effective viscosity of the wet granular material.

Concluding remarks. We have measured the penetration depth for a range of impact velocities for a single sphere size onto wet granular materials for a broad range of liquid saturations. It was found that the minimum penetration depth (for zero impact velocity) for the wet materials was always smaller than that for the dry case, depending on the target preparation method (i.e., pouring or tapping). We found that the data collapsed with the power-law scaling $\delta/\delta_0 = (H/\delta_0)^\alpha$ with $\alpha = 0.55$ and 0.45 , respectively, for the poured and tapped preparations, thus indicating that the preparation method has a strong influence on the penetration depth. Using the penetration depth data and assuming the Bingham fluid model, we derived the apparent yield stress and corresponding effective viscosities for the range of liquid saturations. The data herein constitute the first reports of a broader ongoing investigation into the dynamical response of wet granular material following impact. The influence of impactor density, grain size, liquid viscosity, and surface tension will be addressed in a future publication.

- [1] J. O. Marston, E. Q. Li, and S. T. Thoroddsen, *J. Fluid Mech.* **704**, 5 (2012).
- [2] S. T. Thoroddsen and A. Q. Shen, *Phys. Fluids* **13**, 4 (2001).
- [3] D. Lohse, R. Bergmann, R. Mikkelsen, C. Zeilstra *et al.*, *Phys. Rev. Lett.* **93**, 198003 (2004).
- [4] J. R. Royer, E. I. Corwin, A. Flior, M.-L. Cordero, M. L. Rivers, P. J. Eng, and H. M. Jaeger, *Nat. Phys.* **1**, 164 (2005).
- [5] J. O. Marston, J. P. K. Seville, Y.-V. Cheun, A. Ingram, S. P. Decent, and M. J. H. Simmons, *Phys. Fluids* **20**, 023301 (2008).
- [6] D. Lohse, R. Rauhe, R. Bergmann, and D. van der Meer, *Nature (London)* **432**, 689 (2004).
- [7] F. Pacheco-Vazquez, G. A. Caballero-Robledo, J. M. Solano-Altamirano, E. Altshuler, A. J. Batista-Leyva, and J. C. Ruiz-Suarez, *Phys. Rev. Lett.* **106**, 218001 (2011).
- [8] J. S. Uehara, M. A. Ambroso, R. P. Ojha, and D. J. Durian, *Phys. Rev. Lett.* **90**, 194301 (2003); **91**, 149902(E) (2003).
- [9] J. de Bruyn and A. M. Walsh, *Can. J. Phys.* **82**, 439 (2004).
- [10] M. P. Ciamarra, A. H. Lara, A. T. Lee, D. I. Goldman, I. Vishik, and H. L. Swinney, *Phys. Rev. Lett.* **92**, 194301 (2004).
- [11] M. A. Ambroso, C. R. Santore, A. R. Abate, and D. J. Durian, *Phys. Rev. E* **71**, 051305 (2005).
- [12] M. A. Ambroso, R. D. Kamien, and D. J. Durian, *Phys. Rev. E* **72**, 041305 (2005).
- [13] H. Katsuragi and D. J. Durian, *Nat. Phys.* **3**, 420 (2007).
- [14] A. Seguin, Y. Bertho, and P. Gondret, *Phys. Rev. E* **78**, 010301 (2008).
- [15] D. I. Goldman and P. Umbahnhwar, *Phys. Rev. E* **77**, 021308 (2008).
- [16] S. von Kann, S. Joubaud, G. A. Caballero-Robledo, D. Lohse, and D. van der Meer, *Phys. Rev. E* **81**, 041306 (2010).

- [17] L. Kondic, X. Fang, W. Losert, C. S. O'Hern, and R. P. Behringer, [Phys. Rev. E **85**, 011305 \(2012\)](#).
- [18] J. R. Royer, E. I. Corwin, P. J. Eng, and H. M. Jaeger, [Phys. Rev. Lett. **99**, 038003 \(2007\)](#).
- [19] N. Mitarai, and F. Nori, [Adv. Phys. **55**, 1 \(2006\)](#).
- [20] P. Pierrat, and H. S. Caram, [Powder Technol. **91**, 83 \(1997\)](#).
- [21] C. L. Flemmer, [Powder Technol. **66**, 191 \(1991\)](#).
- [22] P. B. Umbanhowar and D. I. Goldman, [Phys. Rev. E **82**, 010301R \(2010\)](#).
- [23] J. R. Royer, B. Conyers, E. I. Corwin, P. J. Eng, and H. M. Jaeger, [Europhys. Lett. **93**, 28008 \(2011\)](#).
- [24] B. M. Das, *Advanced Soil Mechanics* 3rd ed. (Taylor & Francis, New York, 2008).
- [25] S. M. Iveson, J. D. Lister, K. P. Hapgood, and B. J. Ennis, [Powder Technol. **117**, 3 \(2001\)](#).