Energy Efficiency - Spectral Efficiency Trade-off: A Multiobjective Optimization Approach

Osama Amin, *Member, IEEE*, Ebrahim Bedeer, *Member, IEEE*, Mohamed H. Ahmed, *Senior Member, IEEE*, and Octavia A. Dobre, *Senior Member, IEEE*

Abstract—In this paper, we consider the resource allocation problem for energy efficiency (EE) - spectral efficiency (SE) trade-off. Unlike traditional research that uses the EE as an objective function and imposes constraints either on the SE or achievable rate, we propound a multiobjective optimization approach that can flexibly switch between the EE and SE functions or change the priority level of each function using a trade-off parameter. Our dynamic approach is more tractable than the conventional approaches and more convenient to realistic communication applications and scenarios. We prove that the multiobjective optimization of the EE and SE is equivalent to a simple problem that maximizes the achievable rate/SE and minimizes the total power consumption. Then we apply the generalized framework of the resource allocation for the EE-SE trade-off to optimally allocate the subcarriers' power for orthogonal frequency division multiplexing (OFDM) with imperfect channel estimation. Finally, we use numerical results to discuss the choice of the trade-off parameter and study the effect of the estimation error, transmission power budget and channel-to-noise ratio on the multiobjective optimization.

Index Terms—Energy efficiency, spectral efficiency, multiobjective optimization, OFDM, imperfect CSI, LMMSE.

I. INTRODUCTION

The dramatic growth of wireless communication services and applications represents the main driving force to expand the existing wireless infrastructure and to deploy new systems. Although the communication networks may be able to support the increasing demand of high data rate and ubiquitous services, the energy consumption increases significantly especially at the base stations. This accounts for most of the energy consumption of cellular networks, which represents an important contribution of the information and communication technology industry to the global CO₂ emission [1], [2]. Therefore, wireless communication systems have to be designed based on green metrics that reduce the energy consumption wisely, along with the associated CO₂ emission [1], [2]. Energy efficiency (EE) is a widely used green communication metric, defined either as the number of successfully delivered bits per unit energy, which we adopt in this paper, or its inverse

[3]. Although EE is the major design metric for environment-friendly future wireless communication systems, it conflicts with other traditional metrics such as spectral efficiency (SE) in bit/sec/Hz [4]. The trade-off between EE and SE states that the available system's resources cannot be optimized to improve both EE and SE simultaneously.

Considering the trade-off between the EE and SE in allocating the available resources is a timely and important problem, attention has recently started to be paid for both single carrier [5], [6] and multicarrier transmission systems [7]. The previously mentioned resource allocation problems that deal with the EE-SE trade-off are suitable to fixed objective scenarios and can not deal with the dynamic changes of the design objectives. These traditional approaches use EE as the objective function and impose constraints on the SE, the transmit power and the fairness for multiple access systems. On the other hand, there are scenarios in wireless communication systems in which we need to change the optimization objective function dynamically according to the surrounding circumstances or application requirements.

A motivating scenario is when renewable energy sources are used in addition to diesel generators in base stations to generate the required electrical energy [8]. In such a scenario, when the base station is powered by clean energy sources (i.e., renewable sources), adopting EE as a design metric does not have a green advantage, as the base station is working now with zero CO₂ emissions. Thus, it is more beneficial to improve other quality of service (QoS) metrics, such as SE. On the other hand, when the surrounding environment varies, diesel generators are used to compensate for the shortage of the renewable sources or even replace them. Therefore, considering the EE as the objective function becomes inevitable when the diesel generator is the only energy source, while a multiobjective function taking into account the EE and SE is used when both sources are employed. In the later case, the priority of each function is chosen according to the contribution of each energy source in powering the base station.

In this paper, inspired by the green communication trend and considering the variations of the objective functions according to the provided service or surrounding environment, we use the multiobjective optimization to solve the resource allocation problem for the EE-SE trade-off. First, we propose a general framework to solve the multiobjective optimization of EE and SE for a general communication system and prove that it is equivalent to the multiobjective optimization problem of minimizing the total power consumption and maximizing the

Osama Amin is with King Abdullah University of Science and Technology, Thuwal, Makkah, Kingdom of Saudi Arabia. He is with the Faculty of Engineering, Assiut University, Assiut, Egypt, E-mail: osama.amin@kaust.edu.sa, amin@aun.edu.eg.

Ebrahim Bedeer is with the School of Engineering, University of British Columbia, Kelowna, BC, V1V 1V7, Canada, E-mail: ebrahim.bedeer-mohamed@ubc.ca.

Mohamed H. Ahmed and Octavia A. Dobre are with Faculty of Engineering and Applied Science, Memorial University of Newfoundland, St. John's, NL, A1C 5S7, Canada, E-mail: {mhahmed, odobre}@mun.ca.

SE or achievable rate. Then, we apply the derived framework to orthogonal frequency division multiplexing (OFDM) with estimated CSI.

The rest of the paper is organized as follows. In Section II, we introduce the EE-SE multiobjective optimization general framework. In Section III, we define the model for the OFDM system with estimated CSI and apply the proposed approach to allocate the power for the OFDM subcarriers considering the EE-SE tradeoff. Then, we present numerical simulation results in Section IV and conclude the paper in Section V.

Regarding the notation, bold face letters refer to vectors (lower case) or matrices (upper case). The superscripts H and T represent the conjugate transpose and transpose operation, respectively, $|\cdot|$ denotes the absolute value, and $\hat{\cdot}$ denotes the estimate value.

II. EE-SE TRADE-OFF USING MULTIOBJECTIVE OPTIMIZATION

Optimizing EE or SE under a given set of constraints is suitable only for a specific communication scenario/application or under static environmental circumstances. In this section, we use a multiobjective optimization problem of EE and SE to deal with the dynamic change of design conditions, where tuning the objective functions becomes indispensable. Let us consider a communication system that is used to deliver information with a spectral efficiency η_{SE} bit/sec/Hz. We assume N resources that have to be allocated that may include power sources, subcarriers, antennas, users, relays, bandwidth, etc, and denoted them as $\mathbf{x} = [x_1, x_2, \dots, x_N]^T$. Among these resources, we assume M signal sources that may be obtained from antennas, subcarriers and/or distributed communication nodes. The total power consumed to deliver the required data consists of the transmitted power $\mathbf{p} = [p_1, p_2, \dots, p_M]^T$, and the power consumption at the power amplifiers, and all circuits used in the communication scenario. The EE is expressed as

$$\eta_{EE}(\mathbf{x}) = \frac{C(\mathbf{x})}{\sum_{m=1}^M f_m(\mathbf{x}) + p_c}, \quad (1)$$

where $f_m(\mathbf{x})$ is the variable power consumption term of the m th power source, p_c is the constant power consumption term due to all circuits in the communication system, and $C(\mathbf{x})$ is the achievable rate, which is related to the SE as $C(\mathbf{x}) = B\eta_{SE}(\mathbf{x})$, where B is the transmission bandwidth. We formulate the multiobjective optimization problem as

$$\min_{\mathbf{x}} \eta_{EE}^{-1}(\mathbf{x}) \quad \text{and} \quad \min_{\mathbf{x}} \eta_{SE}^{-1}(\mathbf{x}). \quad (2)$$

We employ the weighted sum method to deal with the trade-off multiobjective using the trade-off parameter, β , with $0 \leq \beta \leq 1$, which describes the priority of each objective function as follows,

$$\begin{aligned} \mathcal{L}_{EE-SE} &= \beta \theta_{EE} \eta_{EE}^{-1}(\mathbf{x}) + (1 - \beta) \theta_{SE} \eta_{SE}^{-1}(\mathbf{x}) \quad 0 < \beta < 1 \\ &= \beta \frac{\theta_{EE} \left(\sum_{m=1}^M f_m(\mathbf{x}) + p_c \right)}{C(\mathbf{x})} + (1 - \beta) \frac{\theta_{SE} B}{C(\mathbf{x})}, \end{aligned} \quad (3)$$

where θ_{EE} and θ_{SE} are normalization factors used to have the same range for the objective functions.¹ It is well known that $C(\mathbf{x})$ is a logarithmic function which is often a strictly-concave function in $\mathbf{x}$. Thus, the multiobjective function in (3) can provide a complete PF since objective functions are quasi-convex with at least one function being strictly quasi-convex, i.e., $-qC(\mathbf{x})$ [9, Ch. 1, pp. 4-11]. The non-convex non-linear fractional objective function in (3) can be transformed to the following equivalent convex function using the Dinkelback approach [10]

$$\mathcal{L}_{EE-SE,1} = \beta \theta_{EE} \left(\sum_{m=1}^M f_m(\mathbf{x}) + p_c \right) + (1 - \beta) \theta_{SE} B - qC(\mathbf{x}), \quad (4)$$

where q is a constant parameter that represents the minimum value of (3) at a specific β [10]. The optimal resource allocation solution of the objective function in (4) can be computed by following the Dinkelback-based iterative approach in [10] for a given priority β . At $\beta = 0$, we do not need to follow the iterative approach because the problem translates into the SE maximization, and reduces to the well known water filling problem. In other words, to draw the PF curve for this multiobjective function, we need to apply the Dinkelback-based iterative algorithm for each point (priority value) in the PF except at the maximum SE solution. In the following, we prove that the EE-SE PF can be obtained by solving a simplified multiobjective problem of total power consumption and achievable rate or SE.

Consider the objective function in (4) for a given β , the term $(1 - \beta) \theta_{SE} B$ is considered constant. Thus, omitting this term does not change the problem solution and the following alternative objective function is used instead,

$$\mathcal{L}_{EE-SE,2} = \beta \theta_{EE} \left(\sum_{m=1}^M f_m(\mathbf{x}) + p_c \right) - qC(\mathbf{x}). \quad (5)$$

Dividing the objective function $\mathcal{L}_{EE-SE,2}$ by $\beta \theta_{EE}$ gives another equivalent objective function that has the same solution and is expressed as

$$\mathcal{L}_{EE-SE,3} = \left(\sum_{m=1}^M f_m(\mathbf{x}) + p_c \right) - \frac{q}{\beta \theta_{EE}} C(\mathbf{x}). \quad (6)$$

The quantity $q/(\beta \theta_{EE})$ is a positive constant and is equivalently replaced by $(1 - \alpha)/\alpha$, for $0 < \alpha < \alpha_{EE} < 1$, where $(1 - \alpha_{EE})/\alpha_{EE} = q/\theta_{EE}$; as such, (6) becomes

$$\mathcal{L}_{EE-SE,4} = \left(\sum_{m=1}^M f_m(\mathbf{x}) + p_c \right) - \frac{(1 - \alpha)}{\alpha} C(\mathbf{x}). \quad (7)$$

Multiplying the objective function in (7) by α yields the

¹The normalization factor depends on the system parameters such as the maximum power budget and average CSI.

following equivalent objective function

$$\mathcal{L}_{\text{EE-SE},5} = \alpha \left(\sum_{m=1}^M f_m(\mathbf{x}) + p_c \right) - (1 - \alpha) C(\mathbf{x}) \quad (8)$$

$$\equiv \min_{\mathbf{x}} \left(\sum_{m=1}^M f_m(\mathbf{x}) + p_c \right) \quad \text{and} \quad \max_{\mathbf{x}} C(\mathbf{x}). \quad (9)$$

The objective function written in (8) represents a multiobjective function that aims to minimize the total power consumption and maximize the achievable rate using the trade-off parameter α , as presented in (9). Thus, the PF curve for the EE-SE multiobjective optimization can be drawn equivalently by considering the multiobjective optimization of the total power consumption and the achievable rate or SE for a specific weighted factor range of the second problem, i.e., $0 < \alpha < \alpha_{\text{EE}}$. Based on this proved fact, the optimization solution of the EE-SE problem in (2) can be obtained from,

$$\min_{\mathbf{x}} \frac{\alpha}{\theta_p} \left(\sum_{m=1}^M f_m(\mathbf{x}) + p_c \right) - \frac{(1 - \alpha)}{\theta_c} C(\mathbf{x}), \quad (10)$$

where θ_p and θ_c are the normalization factors¹ for the power and the achievable rate objective functions, respectively. The multiobjective problem in (10) reduces to achievable rate or SE maximization at $\alpha = 0$, and to power minimization at $\alpha = 1$, while the energy efficient solution is achieved at α_{EE} . Therefore, we can obtain the EE-SE PF by changing α from 0 to α_{EE} . It is worth emphasizing that we do not need to use the Dinkelback-based iterative algorithm except at $\alpha = \alpha_{\text{EE}}$, which reduces the complexity of the original problem.

III. EE-SE TRADE-OFF FOR OFDM WITH ESTIMATED CSI

A. System model

OFDM is widely utilized in many wireless systems, including the IEEE 802.11 family of standards, LTE/LTE-A, and others. OFDM is chosen over a single carrier solution because it is well suited for wideband transmission and supports higher data rates with reduced equalizer complexity. Thus, we consider a single-link OFDM wireless communication system with single-antenna equipped nodes. The channel is assumed to change slowly and is modeled as a finite impulse response system with order equal to L , $\mathbf{h} = [h(0), h(1), \dots, h(L)]^T$, where each channel tap is assumed to be complex Gaussian distributed with zero-mean and variance σ_h^2 . The noise at the receive-side is modeled as additive white Gaussian noise (AWGN) with zero-mean and correlation matrix equal to $\sigma_n^2 \mathbf{I}$.

At the transmitter, we assume a serial data sequence that is divided into blocks, processed by a given precoded matrix and loaded to M subcarriers. The transmitted training pilot symbols $\mathbf{x}_p$ are inserted in the transmitted data with a known pattern. The receiver is assumed to be equipped with the LMMSE channel estimator that gives $\hat{\mathbf{h}} = (\sigma_n^2 \mathbf{R}_h^{-1} + \mathbf{X}_p^H \mathbf{X}_p)^{-1} \mathbf{X}_p^H \mathbf{x}_r$ [11], where $\mathbf{x}_r$ is the received signal block and $\mathbf{X}_p$ is an $M \times (L+1)$ column wise circulant matrix with the first column equal to $\mathbf{x}_p$. The estimated CSI is assumed to be sent through a reliable feedback channel to the transmitter. The subchannel estimates are computed as $[\hat{H}(1), \hat{H}(W), \dots, \hat{H}(W^{M-1})]^T = \sqrt{M} \mathbf{F}_L \hat{\mathbf{h}}$ [11], where $\hat{H}(W^{m-1})$ is the m th subcarrier channel estimate, $W =$

$\exp(j2\pi/M)$, $\mathbf{F}_L$ is the first $L+1$ submatrix of $\mathbf{F}$, and $\mathbf{F}$ is the $M \times M$ discrete Fourier transform matrix.

The achievable rate is expressed in terms of the channel estimate across subcarriers similarly to [11], while considering the powers per subcarriers $\mathbf{p}$,

$$C(\mathbf{p}) = \Delta f \sum_{m=1}^M \log_2 \left(1 + \frac{|\hat{H}(W^m)|^2 G p_m}{\sigma_{\Delta H}^2 G p_m + \sigma_n^2} \right), \quad (11)$$

where $\Delta f = B/M$ is the subcarrier bandwidth, p_m is the m th subcarrier power, G is the large scale fading power coefficient, and $\sigma_{\Delta H}^2$ is the estimation error variance, which can be expressed as $\sigma_{\Delta H}^2 = (L+1) \sigma_h^2 \sigma_n^2 / (\sigma_n^2 + \sigma_h^2 G p_p)$ [11], with p_p as the pilots' transmitted power, and σ_n^2 is the AWGN noise variance.

B. Optimal power loading for the OFDM systems with estimated CSI

In this subsection, we consider the multiobjective optimization problem of EE-SE trade-off for the OFDM system with imperfect CSI. Since the achievable rate is a strictly concave function, it is strictly quasi-concave [12] and the proposed framework in Section II provides a complete PF curve. The simplified power loading problem for the EE-SE trade-off is defined as

$$\begin{aligned} \min_{\mathbf{p}} \quad & \frac{\alpha}{\theta_p} \left(\kappa \sum_{m=1}^M p_m + p_c \right) - \frac{(1 - \alpha)}{\theta_c} C(\mathbf{p}) \\ \text{subject to} \quad & \text{C1: } \sum_{m=1}^M p_m \leq P_T, \\ & \text{C2: } C(\mathbf{p}) \geq R_{\min}, \\ & \text{C3: } p_m \geq 0, \quad m = 1, 2, \dots, M, \end{aligned} \quad (12)$$

where κ is a constant which depends on the power amplifier efficiency. The constraints imposed in (12) represent the following: C1 reflects the total power budget of the proposed system (P_T), C2 is related to the minimum rate constraint ($R_{\min}$) that is used for rate sensitive applications, and C3 guarantees positive power. The Lagrangian problem of (12) is written as,

$$\begin{aligned} \mathcal{L}_{\text{EE-SE,con}} = & \frac{\alpha}{\theta_p} \left(\kappa \sum_{m=1}^M p_m + p_c \right) - \frac{(1 - \alpha)}{\theta_c} C(\mathbf{p}) \\ & + \lambda_1 \left(\sum_{m=1}^M p_m - P_T + y_1^2 \right) \\ & + \lambda_2 \left(R_{\min} - \Delta f \sum_{m=1}^M \log_2 \left(1 + \frac{|\hat{H}(W^m)|^2 G p_m}{\sigma_{\Delta H}^2 G p_m + \sigma_n^2} \right) + y_2^2 \right), \end{aligned} \quad (13)$$

where we introduce two slack variables y_1^2 and y_2^2 to transform the inequality constraints to equality constraints. By differentiating $\mathcal{L}_{\text{EE-SE,con}}$ with respect to p_m , $m = 1, 2, \dots, M$, λ_1 , λ_2 , y_1 and y_2 , and equating the results to zero, we obtain the

following equations,

$$\frac{(\sigma_{\Delta H}^2 G p_m + \sigma_n^2)^2}{|\hat{H}(W^m)|^2 G \sigma_n^2} \left(1 + \frac{|\hat{H}(W^m)|^2 G p_m}{\sigma_{\Delta H}^2 G p_m + \sigma_n^2} \right) = \left(\frac{\theta_c (1 - \alpha + \lambda_2) \Delta f}{(\theta_p \alpha \kappa + \lambda_1 \theta_c) \ln 2} \right), \quad (14)$$

$$\sum_{m=1}^M p_m + y_1^2 = P_T, \quad (15)$$

$$R_{\min} = \Delta f \sum_{m=1}^N \log_2 \left(1 + \frac{|\hat{H}(W^m)|^2 G p_m}{\sigma_{\Delta H}^2 G p_m + \sigma_n^2} \right) + y_2^2, \quad (16)$$

$$2\lambda_1 y_1 = 0, \quad \text{and} \quad 2\lambda_2 y_2 = 0. \quad (17)$$

The m th subcarrier power is computed from (14) as

$$p_{\text{EE-SE},m} = \rho_m \left[\sqrt{1 + \frac{2(\mu |\hat{H}(W^m)|^2 - \frac{\sigma_n^2}{G})}{\rho_m (2\sigma_{\Delta H}^2 + |\hat{H}(W^m)|^2)}} - 1 \right]^+, \quad (18)$$

where $\mu = \frac{\theta_p(1-\alpha+\lambda_2)\Delta f}{\theta_c(\alpha\kappa+\lambda_1\theta_p)\ln 2}$ and ρ_m is expressed as

$$\rho_m = \frac{\sigma_n^2 (2\sigma_{\Delta H}^2 + |\hat{H}(W^m)|^2)}{2\sigma_{\Delta H}^2 (\sigma_{\Delta H}^2 + |\hat{H}(W^m)|^2) G}. \quad (19)$$

From (17), the values of $\lambda_1, \lambda_2, y_1, y_2$ give four possible cases, Case 1: $\lambda_1 = 0$ and $\lambda_2 = 0$, Case 2: $y_1 = 0$ and $\lambda_2 = 0$, Case 3: $\lambda_1 = 0$ and $y_2 = 0$, and Case 4: $y_1 = 0$ and $y_2 = 0$. The cases with $\lambda_i = 0, i = 1, 2$ refer to the unconstrained problem, where the solution may satisfy the problem constraint(s), $\sum_{m=1}^M p_m \leq P_T$ and/or $C(\mathbf{x}) \geq R_{\min}$ in (12) or not. On the other hand, the cases with $y_i = 0$ and the corresponding $\lambda_i \neq 0, i = 1, 2$ describe the scenarios when the inequality constraints are valid with equality, that is $\sum_{m=1}^M p_m = P_T$ and/or $C(\mathbf{x}) = R_{\min}$. Based on these cases, we introduce Algorithm I as described below:

- **Steps 1 to 2:** for a desired α , we first consider the unconstrained problem (Case 1 above) and find the corresponding solution of the subcarriers power from (18). The obtained solution is feasible only if it meets the problem constraints.
- **Steps 3 to 4:** if the power constraint is violated and the rate constraint is satisfied, then the subcarriers power solution is found based on Case 2. This alternative solution becomes feasible only if it satisfies the rate constraint.
- **Steps 5 to 6:** if the power constraint is satisfied and the rate constraint is violated, then we use Case 3 in finding the solution and investigate the feasibility of the new solution by checking the power constraint.
- **Steps 7 to 8:** when the solution based on Case 1 violates both constraints, a feasible solution does not exist because we need more power than the available budget to meet the rate requirement.

The values of λ_1 in step 4 and λ_2 in step 6 are computed from the total power equality constraint $(\sum_{m=1}^M p_m = P_T)$

Algorithm I

- 1: **INPUT** α .
- 2: Set λ_1 and λ_2 to zero in (17) and find $p_{\text{EE-SE},m}$ from (18).
- 3: **if** the power constraint is violated and the rate constraint is satisfied **then**
- 4: Set λ_2 to zero in (17) and find non-negative λ_1 such that equality power constraint is satisfied from (18). If the rate constraint is violated, then set $p_{\text{EE-SE},m} = 0$.
- 5: **else if** the power constraint is satisfied and the rate constraint is violated **then**
- 6: Set λ_1 to zero in (17) and find non-negative λ_2 such that equality rate constraint is satisfied from (18). If the power constraint is violated, then set $p_{\text{EE-SE},m} = 0$.
- 7: **else if** the power constraint is violated and the rate constraint is violated **then**
- 8: Set $p_{\text{EE-SE},m} = 0$.
- 9: **end if**
- 10: **OUTPUT** $p_{\text{EE-SE},m}$.

and the equality rate constraint ($C(\mathbf{x}) = R_{\min}$), respectively, using the bisection method [13].

The computational complexity of Algorithm I is analyzed based on the fact that the bisection method is used to find λ_i , with a starting interval of $[\lambda_{i,L}, \lambda_{i,U}]$, $i = 1, 2$; as such, step 2 is of complexity $\mathcal{O}(N)$. Step 4 is of complexity $\mathcal{O}(N \log_2(\frac{\Delta \lambda_1}{\epsilon_1}))$ [14], where $\Delta \lambda_1$ is the range between the lower and upper bounds of λ_1 , expressed as $\Delta \lambda_1 = \lambda_{1,U} - \lambda_{1,L}$ and ϵ_1 is the solution accuracy. Similarly, step 6 is of complexity $\mathcal{O}(N \log_2(\frac{\Delta \lambda_2}{\epsilon_2}))$, where $\Delta \lambda_2$ is the range between the lower and upper bounds of λ_2 , written as $\Delta \lambda_2 = \lambda_{2,U} - \lambda_{2,L}$ and ϵ_2 is the solution accuracy. Thus, the complexity of Algorithm I is calculated as $\mathcal{O}(N) + \max(\mathcal{O}(N \log_2(\frac{\Delta \lambda_1}{\epsilon_1})), \mathcal{O}(N \log_2(\frac{\Delta \lambda_2}{\epsilon_2}))) = \mathcal{O}(N \log_2(\frac{\Delta \lambda}{\epsilon}))$ where $\frac{\Delta \lambda}{\epsilon} = \max(\frac{\Delta \lambda_1}{\epsilon_1}, \frac{\Delta \lambda_2}{\epsilon_2})$.

The EE solution is found at $\alpha = \alpha_{\text{EE}}$ and computed by applying the Dinkelbach method [10]; for this case, we develop Algorithm II, which iteratively uses Algorithm I to compute the EE power loading solution $\mathbf{p}_{\text{EE}}$ and α_{EE} that satisfy $\mathcal{L}_{\text{EE-SE,con}}(\mathbf{p}, \alpha_{\text{EE}}) = 0$. Algorithm II starts with an initial value of α_{EE} equal to α_{initial} and employs an error tolerance of δ .

Algorithm II

- 1: **INPUT** $\alpha_{\text{initial}}, \delta$.
- 2: Set $\alpha_{\text{EE}} = \alpha_{\text{initial}}$.
- 3: **while** $\mathcal{L}_{\text{EE-SE}} < -\delta$ **do**
- 4: Find $P_{\text{EE},m}(\alpha_{\text{EE}})$ from **Algorithm I**.
- 5: Calculate $\alpha_{\text{EE}} = \frac{\theta_p C(\mathbf{p})}{\theta_c (\kappa \sum_{m=1}^M p_{\text{EE},m} + p_c) + \theta_p C(\mathbf{p})}$ and update $\mathcal{L}_{\text{EE-SE}}$.
- 6: **end while**
- 7: **OUTPUT** $p_{\text{EE},m}$ and α_{EE} .

From the complexity discussion of Algorithm I, the complexity of step 4 in Algorithm II is $\mathcal{O}(N \log_2(\frac{\Delta \lambda}{\epsilon}))$; hence, the complexity of Algorithm II is of the order

$\mathcal{O}(N_{\alpha_{EE}} N \log_2(\frac{\Delta\lambda}{\epsilon}))$, where $N_{\alpha_{EE}}$ is the number of required iterations to update α_{EE} .

It is worth noting that the trade-off parameter α is adjusted to obtain a required priority of the objective functions. To optimize the SE objective function, we impose $\alpha = 0$, while to optimize EE, we use α_{EE} that can be found from $\mathcal{L}_{EE-SE,con} = 0$, which is similar to finding q in solving the EE problem. On the other hand, for a specific priority to EE or SE, α can be chosen according to the average performance of EE-SE trade-off that is discussed in the next section.

IV. SIMULATION RESULTS

In this section, we provide a numerical evaluation of the proposed multiobjective optimization problem for an OFDM system with imperfect channel estimation. The simulation parameters are considered for base stations listed in [15].

Simulation Example 1: In the first simulation example, we study the EE and SE behavior versus α without imposing the transmission rate constraint, which is implemented by setting $\lambda_2 = 0$ in Algorithm I, and assuming different ratios of the maximum useful transmission power budget ratio to the total power consumption at the base station (P_T/θ_p) in Fig. 1. We assume a good channel quality with the channel-to-noise ratio (CNR) equal to 20 dB and accurate channel estimate with the MMSE of 0.001.

First, we investigate the choice of the trade-off parameter α to achieve a given priority level for either EE or SE assuming a sufficient power budget with $P_T/\theta_p = 0.11$ in Fig. 1. For $\alpha = 0$, the solution reduces to the maximum SE or achievable rate, and for $\alpha = 1$, the scenario reduces to the minimum transmitted power. On the other hand, maximum average EE is achieved at $\alpha = \alpha_{EE} = 0.5$. As α increases from zero to α_{EE} , the EE increases and SE decreases, while both EE and SE decrease when α changes from α_{EE} to 1; thus, the former represents the region of interest. The average results depicted in Fig. 1 are used as a guide to choose α in order to achieve a predetermined average EE or SE performance. For example, if it is required to design the system for a green perspective while satisfying the average SE to 4.5 bit/sec/Hz, then we choose $\alpha = 0.35$, while if the design targets to maximize the SE while keeping minimum average EE to 4 kb/J, then we choose $\alpha = 0.1$.

Additionally, we aim to study the effect of the useful transmission power ratio, i.e., P_T/θ_p . For a large transmitted power budget ($P_T/\theta_p = 0.11$), we observe the expected trade-off between EE and SE versus α . On the other hand, the unavailability of sufficient transmission power budget limits the SE from improving as α decreases and limits the EE curve from its expected trend, i.e., having a maximum at α_{EE} and reducing as α decreases. Accordingly, and as can be noted from Fig. 1, communication systems with low transmission power efficiency, i.e., very small P_T/θ_p values, have a nearly constant EE and SE performance versus $\alpha = 0 \rightarrow \alpha_{EE}$, which makes the EE solution equivalent to the SE solution.

Simulation Example 2: To study the effect of circuitry power consumption, we assume the scenario discussed in the previous simulation example with $P_T/\theta_p = 0.11$, and then let p_c equal

Fig. 1. EE and SE versus the priority parameter α for different P_T/θ_p values.

Fig. 2. EE and SE versus the priority parameter α for different circuits power consumptions.

100 W and 20 W in addition to its original value of 300 W, to investigate its effect on the EE-SE trade-off. From the results presented in Fig. 2, it can be seen that EE improves as p_c decreases because the total power consumption cost to transmit data decreases. For $\alpha = 0$ the optimization problem reduces to SE maximization, and we observe equal maximum SE regardless of the value of p_c because SE does not depend on the circuitry power consumption, while for $\alpha > 0$, the objective function becomes a linear combination between EE and SE and thus depends on p_c . Therefore, both EE and SE change with p_c for $\alpha > 0$, where EE improves as p_c decreases, while SE improves as p_c increases because both functions are contradicting. The value of α_{EE} increases as p_c decreases, specifically maximum EE is found at $\alpha_{EE} = 0.5, 0.6$ and 0.7 for $p_c = 20$ W, 100 W and 300 W. This shows that communication systems with large p_c are willing to spend more transmission power to guarantee the successfully delivery of the data and improve EE, while systems with small p_c tend to use less transmission power to improve EE.

Simulation Example 3: In this example, we study the effect

Fig. 3. EE and SE versus the priority parameter α for different minimum SE constraints corresponding to minimum rate constraints.

of rate constraint on the EE-SE multiobjective problem. For this purpose, we consider the scenario with sufficient power budget, i.e., $P_T/\theta_p = 0.11$, and plot EE and SE versus α for different rate constraints in Fig. 3, where $\eta_{SE,min}$ is the SE corresponding to the minimum rate, expressed as $\eta_{SE,min} = R_{min}/B$. Unlike the unconstrained rate case from which the SE curve decreases as α increases and reaches zero at $\alpha = 1$, the constrained SE curves do not go below the floor of the predetermined constraints, which is either 2.3 or 4.6 b/s/Hz. The EE curves are affected by the rate constraint starting from the rate breaking point, defined as the point in which the SE starts to have a constant flooring behavior while α increases. Particularly, if the rate breaking point is located before the maximum EE point, then both the EE peak and the decreasing part of the curve are affected, which is the case of $\eta_{SE,min} = 4.6$ b/s/Hz; otherwise, only the decreasing part of the EE is affected and achieves larger EE when compared with the unconstrained case, which is the case of $\eta_{SE,min} = 2.3$ b/s/Hz. If the rate breaking point lies outside our region of interest, then the rate constraint does not affect apparently the EE and SE average curves in this region.

Simulation Example 4: To study the effect of the channel estimation error on the trade-off between EE and SE, we plot the PF curve of EE and SE for $P_T/\theta_p = 0.11$ assuming unconstrained rate problem and different MMSE values in Fig. 4. One can notice that the having a good channel estimator is important to achieve a good EE-SE curve. Specifically, channel estimators with maximum MMSE of 0.001 are needed to achieve near optimal EE-SE relation. SE shows more sensitivity to estimation errors than EE, where SE loses around 30% from its performance as the MMSE decreases from 0.001 to 0.01, while EE loses only 5% from its performance.

V. CONCLUSION

In this paper, we introduced the general framework of the multiobjective optimization to deal with the resource allocation problem for any communication system while considering the EE-SE trade-off. We proved the necessary conditions that the weighted sum method provides the optimal EE-SE trade-off

Fig. 4. EE versus SE for different MMSE values and at $P_T/\theta_p = 0.11$.

relationship. The multiobjective optimization of the EE-SE is equivalent to the multiobjective problem that minimizes the total power consumption and maximizes the SE or achievable rate. We applied the proposed framework to the OFDM system with estimated CSI and proposed a power loading algorithm to achieve the optimal EE-SE relationship. Through simulation examples, we explained how to select the trade-off parameter to switch between different practical communication scenarios, with different design requirements.

REFERENCES

- [1] A. Fehske, G. Fettweis, J. Malmudin, and G. Biczok, "The global footprint of mobile communications: The ecological and economic perspective," *IEEE Commun. Mag.*, vol. 49, no. 8, pp. 55–62, Aug. 2011.
- [2] Z. Hasan, H. Boostanimehr, and V. Bhargava, "Green cellular networks: A survey, some research issues and challenges," *IEEE Commun. Surveys Tuts.*, vol. 13, no. 4, pp. 524–540, Fourth Quarter 2011.
- [3] O. Amin, S. Bavarian, and L. Lampe, "Cooperative techniques for energy-efficient wireless communications," in *Green Radio Communication Networks*. Cambridge University Press, 2012, pp. 125–149.
- [4] Y. Chen, S. Zhang, S. Xu, and G. Y. Li, "Fundamental trade-offs on green wireless networks," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 30–37, Jun. 2011.
- [5] X. Hong, Y. Jie, C.-X. Wang, J. Shi, and X. Ge, "Energy-spectral efficiency trade-off in virtual MIMO cellular systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 10, pp. 2128–2140, Oct. 2013.
- [6] S. Huang, H. Chen, J. Cai, and F. Zhao, "Energy efficiency and spectral-efficiency tradeoff in amplify-and-forward relay networks," *IEEE Trans. Veh. Technol.*, vol. 62, no. 9, pp. 4366–4378, Nov. 2013.
- [7] C. Xiong, G. Y. Li, S. Zhang, Y. Chen, and S. Xu, "Energy- and spectral-efficiency tradeoff in downlink OFDMA networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 11, pp. 3874–3886, Nov. 2011.
- [8] Y.-K. Chia, S. Sun, and R. Zhang, "Energy cooperation in cellular networks with renewable powered base stations," in *Proc. IEEE Wireless Communications and Networking Conference*, 2013, pp. 2542–2547.
- [9] J. Branke, K. Deb, K. Miettinen, and R. Slowinski, *Multiobjective Optimization: Interactive and Evolutionary Approaches*. Springer, 2008.
- [10] W. Dinkelbach, "On nonlinear fractional programming," *Management Science*, vol. 13, no. 7, pp. 492–498, Mar. 1967.
- [11] S. Ohno and G. B. Giannakis, "Capacity maximizing MMSE-optimal pilots for wireless OFDM over frequency-selective block Rayleigh-fading channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 9, pp. 2138–2145, Sept. 2004.
- [12] A. Cambini and L. Martein, *Generalized Convexity and Optimization: Theory and Applications*. Springer, 2008, vol. 616.
- [13] S. P. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [14] A. A. Puntambekar, *Analysis and Design of Algorithms*. Technical Publications, 2008.

- [15] M. Deruyck, E. Tanghe, W. Joseph, W. Vereecken, M. Pickavet, L. Martens, and B. Dhoedt, "Model for power consumption of wireless access networks," *IET Science, Measurement and Technology*, vol. 5, no. 4, pp. 155–161, Aug. 2011.

Osama Amin received B.Sc. degree in Electrical and Electronics Engineering from Aswan University, Aswan, Egypt, in 2000, M.Sc. degree in Electrical Engineering from Assiut University, Assiut, Egypt in 2004 and Ph.D. degree in Electrical and Computer Engineering, University of Waterloo, Canada in 2010. In June 2012, he joined Assiut University as an Assistant Professor in the Electrical and Electronics Engineering department. Currently, he is with King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah, Kingdom of Saudi Arabia. His general research interests lie in communications systems and signal processing for communications with special emphasis on wireless applications. Specific research areas include green communications, cognitive radio, cooperative communications, and channel estimation. He has served as a Technical Program Committee member for *IEEE VTC*, *ISSPIT* and *CROWNCOM* conferences.

Ebrahim Bedeer received the B.Sc. and M.Sc. degrees from Tanta University, Tanta, Egypt, in 2002 and 2008, respectively and the Ph.D. degree from Memorial University, St. Johns, NL, Canada in 2014, all in Electrical Engineering. Currently, he is a post-doctoral fellow at University of British Columbia, Okanagan campus, Kelowna, BC, Canada. His current research interests are in the area of wireless communications, with a focus on dynamic resource allocation, cognitive radio networks, and green communications.

Mohamed Hossam Ahmed received the B.Sc. and M.Sc. degrees in electronics and communications engineering from Ain Shams University, Cairo, Egypt in 1990 and 1994, respectively. He obtained the Ph.D. degree in Electrical Engineering in 2001 from Carleton University, Ottawa, where he worked from 2001 to 2003 as a senior research associate. In 2003, he joined the Faculty of Engineering and Applied Science, Memorial University where he works currently as an Associate Professor. Dr. Ahmed published more than 100 papers in international first-

class journals and conferences. He serves as an Editor for *IEEE Communication Surveys and Tutorials* and for *EURASIP Journal on Wireless Communications and Networking (JWCN)*, and as an Associate Editor for *Wiley International Journal of Communication Systems and Wiley Communication and Mobile Computing (WCMC)*. He served as a Guest Editor of a special issue on Fairness of Radio Resource Allocation, *EURASIP JWCN* in 2009, and as a Guest Editor of a special issue on Radio Resource Management in *Wireless Internet*, *Wiley Wireless and Mobile Computing Journal*, 2003. Dr. Ahmed is a *Senior Member* of the IEEE. He serves as a cochair of the Signal Processing Track in *ISSPIT'14* and served as a cochair of the Transmission Technologies Track in *VTC'10-Fall*, and the multimedia and signal processing symposium in *CCECE'09*. Dr. Ahmed won the Ontario Graduate Scholarship for Science and Technology in 1997, the Ontario Graduate Scholarship in 1998, 1999, and 2000, and the Communication and Information Technology Ontario (CITO) graduate award in 2000. His research interests include radio resource management in wireless networks, multi-hop relaying, cooperative communication, vehicular ad-hoc networks, cognitive radio networks, and wireless sensor networks. Dr. Ahmed's research is sponsored by NSERC, CFI, Bell/Aliant and other governmental and industrial agencies. Dr. Ahmed is a registered Professional Engineer (P.Eng.) in the province of Newfoundland, Canada.

Octavia A. Dobre received the Dipl. Ing. and Ph. D. degrees in ECE from the Polytechnic University of Bucharest (formerly the Polytechnic Institute of Bucharest), Romania, in 1991 and 2000, respectively. In 2000 she was the recipient of a Royal Society scholarship at Westminster University, UK, and in 2001 she held a Fulbright fellowship at Stevens Institute of Technology, USA. Currently, she is an Associate Professor with Memorial University, Canada.

Her research interests include blind signal identification and parameter estimation techniques, cognitive radio systems, transceiver optimization algorithms, dynamic spectrum access, cooperative communications, resource allocation, and coherent optical communications. She published over 150 journal and conference papers in these areas. Dr. Dobre's research has been supported by the Natural Sciences and Engineering Research Council of Canada, Canada Foundation for Innovation, Research and Development Corporation, Defence and Research Development Canada, Communications Research Centre Canada, Altera Corporation, DTA Systems, and ThinkRF among others.

Dr. Dobre is a Senior Editor for the *IEEE Communications Letters*, as well as an Editor for the *IEEE Transactions on Wireless Communications* and *IEEE Communications Surveys and Tutorials*. She has served as an Editor and Guest Editor for several other prestigious journals. She is the General Chair of *IEEE CWIT 2015*, and has served as Technical Program Co-Chair of symposia at several conferences, including *IEEE ICC* and *GLOBECOM*. She is a registered Professional Engineer (P. Eng.) in the province of Newfoundland, Canada.