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Geometrical effects on the airfoil flow separation and transition

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Abstract

We present results from direct numerical simulations (DNS) of incompressible flow over two airfoils, NACA-4412 and NACA-0012-64, to investigate the effects of the airfoil geometry on the flow separation and transition patterns at $Re = 10^4$ and 10 degrees incidence. The two chosen airfoils are geometrically similar except for maximum camber (respectively $4\%C$ and 0 with C the chord length), which results in a larger projection area with respect to the incoming flow for the NACA-4412 airfoil, and a larger leeward surface curvature at the leading edge for the NACA-0012-64 airfoil. The governing equations are discretized using an energy conservative fourth-order spatial discretization scheme. An assessment on the two-point correlation indicates that a spanwise domain size of $0.8C$ is sufficiently large for the present simulations. We discuss flow separation at the airfoil leading edge, transition of the separated shear layer to three-dimensional flow and subsequently to turbulence. Numerical results reveal a stronger adverse pressure gradient field in the leading edge region of the NACA-0012-64 airfoil due to the rapidly varying surface curvature. As a result, the flow experiences detachment at $x/C = 0.08$, and the separated shear layer transition via Kelvin-Helmholtz mechanism occurs at $x/C = 0.29$ with fully developed turbulent

flow around $x/C = 0.80$. These flow development phases are delayed to occur at much downstream positions, respectively, observed around $x/C = 0.25, 0.71$ and 1.15 for the NACA-4412 airfoil. The turbulent intensity, measured by the turbulent fluctuations and turbulent Reynolds stresses, are much larger for NACA-0012-64 from the transition onset until the airfoil trailing edge, while turbulence develops significantly downstream of the trailing edge for the NACA-4412 airfoil. For both airfoils, our DNS results indicate that the mean Reynolds stress $\overline{u'u'}/U_0^2$ reaches its maximum value at a distance from the surface approximately equal to the displacement thickness, consistent with the experimental observations (Boutillier & Yarusevych, Phys. Fluids, 2012). A quantitative eigen-system analysis on the instantaneous velocity field shows that although the flow over an airfoil is intrinsically anisotropic, the alignments between the vorticity vector and the eigenvectors of S_{ij} and $S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}$ are quite similar to those of the homogeneous isotropic turbulent flows due to the formation of vortex tubes.

Keywords: Direct numerical simulation, airfoil, curvature, separation, transition

2010 MSC: 76D10, 76F65

1. Introduction

Low Reynolds number flow (low- Re regime defined as $Re < 70,000$ by Lissaman [1] and $\sim 15,000 - 500,000$ by Mueller & DeLaurier [2]) past an isolated airfoil is relevant in the context of a variety of applications, such as micro air vehicles, small
 5 wind turbines and low-speed aircraft. Airfoils operated in such environments may suffer from flow separation without reattachment, hence deteriorating their aerodynamic performance. Considering its immense technological importance, many studies in the fields of flow control, aircraft design and optimization are devoted to this topic. These

works rely on the knowledge of the flow pattern development in the separated region,
 10 especially the transition process. To achieve these, detailed analyses of the unsteady
 flow dynamics are required.

To date, most numerical simulations of separated and transitional flow over solid
 surfaces have been devoted to the flat plate (e.g. [3–5]) due to its geometrical sim-
 plicity compared with the airfoil. The separated flow is typically unstable and the
 15 Kelvin-Helmholtz mechanism leads to shear-layer roll-up and eventually shedding of
 vortices, which are then convected downstream. The transition is initially dominated
 by the linear growth of the disturbances, followed by the rapid nonlinear growth which
 eventually results in the fully transition to turbulence. Although there are similarities
 in the initial transition stages of flow over the flat plate and airfoil (see e.g. [6–8]), the
 20 separation and transition characteristics of flow past an airfoil depict different behavior,
 and is strongly influenced by other factors such as the Reynolds number, incidence and
 airfoil profile. The incoming low- Re laminar flow detaches due to the adverse pressure
 gradient (APG) field near the leading edge and forms a shear layer above the airfoil sur-
 face. The separated shear layer is unstable with the formation of roll-up vortices and
 25 leads to transition initiated by the amplification of natural disturbances [5, 9]. The tur-
 bulent flow remains detached as it develops downstream for airfoils at high incidence,
 while it reattaches to the leeward surface and envelopes a region of laminar separation
 bubble at low incidence.

Although an APG is essential for viscous flow separation past an isolated airfoil,
 30 detailed flow features may be significantly different depending on the airfoil geometry,
 such as the airfoil thickness and the camber. At a fixed incidence, airfoils with larger

thickness and maximum camber have a larger leeward surface curvature and projection area perpendicular to the incoming flow. As a result, the incoming flow is experiencing a more bluff-body like airfoil whose leeward surface is more drastically varying from the leading edge to the trailing edge, and the flow separation and transition patterns are consequently affected. Although the physical importance of the geometrical variations is recognized, there are only a few studies on simplified geometries (e.g. flat plate and its variations). Lamballais *et al.* [10] conducted DNS on $Re = 2 \times 10^3$ flow past a forward-facing step with inflow perturbations. The step is rounded at the leading edge at various radii but the downstream part remains flat. The results reveal that larger leading edge radius (towards quarterly rounded corner) increases the separation angle and delays the reattachment point further downstream. The larger separation bubble formed on the flat step has increased the turbulent kinetic energy. Yang & Voke [11] performed a LES study on the similar configuration of a flat plate with semicircular leading edge at $Re = 3.45 \times 10^3$. The instantaneous reattachment position moves up to a distance of 50% the mean reattachment length, and full breakdown to turbulence occurs around the mean reattachment point.

In addition to the leading edge shape, the variation of the surface curvature also affects the flow transition and its downstream development. Wu & Squires [12] performed LES of a turbulent boundary layer flow over a bump on a flat plate. The bump is a prolonged convex surface connected with the flat plate by two short concave regions fore and aft. The streamwise distribution of the mean pressure gradient varies in magnitude as well as in sign (adverse and favorable) as a result of the concave-convex-concave geometry. The streamwise pressure gradient is strongly adverse close to the

fore of the bump, significantly favorable over the upstream concave surface, decreasingly favorable until the bump summit and strongly adverse downstream the summit. Similar trends of pressure gradient variations were also seen by Baskaran *et al.* [13, 14] in their experimental and analytical work on turbulent flow around curved hill and free wing, the numerical work of Wu & Squires [15] on boundary layer over an infinite swept bump and free wing, and in the experiments of Webster *et al.* [16, 17] on the turbulence characteristics in the boundary layer of flat plate with spanwise straight and swept bump. Although the junctions between the convex bump and the flat plate are smoothed by the concave surfaces, the discontinuous curvature at the concave-convex junction triggers the formation of an internal layer which dictates the flow behavior close to the wall [12, 16, 17].

In the above studies of the relatively simple geometry of a flat plate and its variations, the geometrical parameters of the leading edge shape and surface curvature have been separately considered and their effects on the flow separation, transition and development are investigated. For flow past an airfoil, the surface geometry is smooth and flow separation and transition are expected to be significantly different compared with the surrogate case of a flat plate with a bump wherein the surface curvature may be discontinuous. Here we present a comparative study on flow past airfoils using the DNS approach. For airfoils the geometrical factors and pressure distributions affect the flow in a combined way and their effects can not be distinctly distinguished. It is perhaps impossible to properly construct a pressure gradient field or any other physical parameter to completely isolate the effects of any single geometrical factor. In our study we focus on two airfoils: the NACA-4412 and NACA-0012-64 profiles with $Re = 10^4$

and 10 degrees incidence. The two airfoils are chosen to be somewhat geometrically similar with the only exception of the camber. Scaled by the chord length C , the two
80 airfoils are similar in that both have the maximum thickness $12\%C$ located at $40\%C$ chordwise position, but differ in that the maximum camber of NACA-4412 is $4\%C$ at $40\%C$ chordwise position while the NACA-0012-64 airfoil has a zero camber. We do not separately consider the effects of different geometrical factors (e.g., leading edge shape, projection area, curvature, etc.), but attribute them to the shape of the camber
85 and investigate its general effects on the flow. We believe this kind of consideration is more realistic and the conclusions are more general since the effects of multiple factors are comprehensively considered. The present study serves as a fundamental investigation on the massive separation and transition of flow past the airfoil. The results and conclusions provide information that may potentially be referred for low Reynolds
90 number flow in similar configurations.

The present work is organized as follows. The computational details including the problem definition, the numerical methods, and the assessments of resolution and computational domain size are presented in section 2. DNS results are analyzed in section 3 with specific focus on the flow separation and laminar-turbulence transition
95 behavior. Some conclusions are presented in section 4.

2. Numerical setup

2.1. Problem definition

Both airfoils, NACA-4412 and NACA-0012-64, are straight, i.e. without spanwise variation in cross-section. They are rescaled to unit chord length C and extended to

Figure 1: Airfoil profiles and leeward surface curvature distributions. r is the local radius.

include sharp trailing edges located at $(x/C, y/C) = (1.0, 0.0)$. The computational domain is discretized by a C-type mesh. The domain size is characterized by the wake length W (distance from the trailing edge to the outflow boundary), the domain radius R (distance from the airfoil surface to the top and bottom boundaries) and the spanwise size L_z . The comparisons of airfoil profiles and leeward surface curvatures are shown in Figure 1, in which the curvature is defined as the reciprocal of the local radius r . It is seen that the curvature of NACA-0012-64 is larger close to the leading edge due to the smaller local radius, while the curvature of NACA-4412 is larger in the region $x/C = 0.1 - 0.5$ due to the non-zero camber, while the curvatures are almost the same over the remaining portion of the leeward surface. Moreover, the NACA-4412 airfoil has a larger projection area with respect to the incoming flow due to its non-zero camber.

2.2. Solution method

The governing equations for the present simulations are the three-dimensional incompressible Navier-Stokes equations and continuity equation in non-dimensional form:

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_k u_i}{\partial x_k} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_k \partial x_k}, \quad (1)$$

$$\frac{\partial u_k}{\partial x_k} = 0, \quad (2)$$

115 in which $Re = U_0 C / \nu$ is the Reynolds number based on the airfoil chord length C and uniform inflow velocity U_0 ; $(u_1, u_2, u_3) \equiv (u, v, w)$ are the velocity components; p is the pressure; $(x_1, x_2, x_3) \equiv (x, y, z)$ are the Cartesian coordinates representing the streamwise, transverse and spanwise directions.

The governing equations are spatially discretized on a collocated mesh. The second-
120 order and energy conservative fourth-order central difference schemes are optional in the code. The fourth-order scheme is suitable for turbulence simulation to conserve energy [18] and is used in production simulations for statistical analysis in the present work. In order to minimize the aliasing error, the convective term is written and discretized in the skew-symmetric form [18, 19]:

$$Conv. = \frac{1}{2} u_k \frac{\partial u_i}{\partial x_k} + \frac{1}{2} \frac{\partial u_k u_i}{\partial x_k}. \quad (3)$$

125 The governing equations are solved by a semi-implicit fractional step method [20]. The time advancement in one physical time step consists of three sub-steps: the predictor step for the velocity from $\mathbf{u}^{n-1}$ to the intermediate velocity $\mathbf{u}^{n-1/2}$, the solution of pressure p^n , and the corrector step for velocity from $\mathbf{u}^{n-1/2}$ to $\mathbf{u}^n$. The mathematical formulations of the three sub-steps are given below:

$$\frac{u_i^{n-1/2} - u_i^{n-1}}{\Delta t} + \left[\frac{3}{2} (Conv.)^{n-1} - \frac{1}{2} (Conv.)^{n-2} \right] = \frac{1}{Re} \frac{\partial^2 u_i^{n-1/2}}{\partial x_k \partial x_k}, \quad (4)$$

130

$$\frac{\partial^2 p^n}{\partial x_k \partial x_k} = \frac{1}{\Delta t} \frac{\partial u_k^{n-1/2}}{\partial x_k}, \quad (5)$$

$$u_i^n = u_i^{n-1/2} - \Delta t \frac{\partial p^n}{\partial x_i}. \quad (6)$$

The velocity modified-Helmholtz equation in the predictor step and the pressure Poisson equation are solved by parallel multigrid solvers with point- and line-relaxed Gauss-Seidel methods. To achieve near-optimal load balancing, the mesh is divided into blocks of equal size and each of them is assigned to a unique processor. All simulations are performed on an IBM Blue Gene/P supercomputer Shaheen at KAUST using up to 8,192 cores. In our unsteady simulations, the time step size is restricted by the CFL condition. In the present work we set CFL=1.0, which is equivalent to a time step size of around $2.5 \times 10^{-4} C/U_0$ during the whole simulation. To complete the simulation, a duration of about $30C/U_0$ is required for the large time scale eddy development in, for example, the detachment and reattachment zones. Although the simulation duration is lesser than those encountered in URANS or LES of similar flows, it is considered sufficient given the high computation cost of DNS and is well accepted by many researchers (see e.g., [9, 21–23]). In order to save computational resources, we conduct the whole simulation by a sequence of steps as follows. The first step is a 2D simulation, and one snapshot of the instantaneous flow field is then used as the initial conditions of a low-resolution 3D simulation using $4096 \times 256 \times 32$ mesh to initiate the development the three-dimensional structures. The results of this low-resolution 3D simulation are then interpolated in the spanwise direction to start the fully 3D simulation using the standard $4096 \times 256 \times 256$ mesh with the second-order discretization option in our code, and lasts for about $20C/U_0$. Finally the simulation

Figure 2: (color online) Spanwise two-point correlation coefficient of w' -velocity between spanwise-oriented probes.

continues by switching to the energy conservative fourth-order scheme and lasts for about $10C/U_0$. This latter time span of $10C/U_0$ yields results that are used for further statistical analysis and post-processing. It should be noted that after switching to a higher order method, results for an initial time interval of C/U_0 are discarded before the statistical analysis to stabilize any mismatch between different schemes [24].

A uniform flow $(u, v, w) = (U_0, 0, 0)$ is imposed at the inflow boundary. At the outflow plane the convective condition $\partial \mathbf{u} / \partial t + U_B \partial \mathbf{u} / \partial x = 0$ is used where U_B is the bulk velocity. No-slip boundary condition for velocity is prescribed on the airfoil surface. In three-dimensional simulations periodic boundary condition is employed in the spanwise direction.

2.3. Domain size assessment

The computational domain size should be sufficiently large so that the simulation mimics the physically realistic circumstance of flow past an airfoil in an infinite medium. The wake length W , along with proper outflow boundary condition, should

allow the wake vortices to leave the domain without significant reflection. Sohankar *et al.* [25] investigated the wake length effects on the incompressible flow over a rectangular cylinder at incidence. In their simulations, they used fully developed outflow boundary condition ($\partial \mathbf{u} / \partial x = 0$), and concluded that a wake length of at least 26 characteristic lengths is required to minimize the effects of an artificially imposed boundary condition. However, a series of non-reflecting outflow boundary conditions are used in most DNS studies on flow around airfoils. These outflow boundary conditions mitigate the limit, thus permitting a smaller wake length and consequently fewer grid cells in the wake region. For compressible flow over an airfoil $W = 5C$ is deemed sufficient [22], while a smaller value of $W = 2C$ may also be considered reasonable [9]. For incompressible flow cases, Wissink and Rodi [21] used a wake length of $W = 0.9C$ and convective outflow boundary condition for the simulation of flow in a turbine cascade. Zaki *et al.* [26] used a smaller value of $W = 0.5C$ for flow in compressor cascade that yielded reasonable results. In the present study we employ a wake length $W = 10C$, which is significantly larger than most studies, especially compared with a similar case of a separated flow around an inclined plate at $Re = 2 \times 10^4$ and 20 degrees incidence which utilized $W = 3C$ [27].

The domain radius R should be large enough so that the blockage introduced by the airfoil is small and the effect of top and bottom boundaries on the flow are negligible. Jones *et al.* [22] in their comparative study of compressible flow around an airfoil at $Re = 5 \times 10^4$ and 5 degrees incidence, showed that a domain radius $R = 5.3C$ is adequate to capture the potential flow. Based on their conclusions, we employ a domain radius $R = 6C$ for the simulations in this paper.

For three-dimensional simulations, the spanwise domain size should be sufficiently
 190 large so as to resolve the largest flow structure in this direction. In the experimen-
 tal study of flow over a circular cylinder [28], the wake is observed to experience a
 transition from laminar vortex shedding to in-line arranged streamwise vortices as the
 Reynolds number increases in the low- Re regime ($Re \sim O(10^2)$), and the size of the
 flow structure substantially varies in the spanwise direction. For incompressible flow
 195 around an airfoil, the smallest required spanwise domain size is also related to the
 Reynolds number. In the DNS study of Hoarau *et al.* [29] for flow around a NACA-
 0012 airfoil at $Re = 800$ and 20 degrees incidence, the dominant flow structure in
 spanwise direction is about $0.64C$. Moreover, for flow around a NACA-0015 airfoil
 at $Re = 8.96 \times 10^5$ and 16.6 degrees incidence, a spanwise domain size $L_z = 0.2C$
 200 is proved to produce results that are in excellent agreement with experimental results
 [30]. Kitsios *et al.* [31] concluded that the impact of large-scale structures is reduced
 as Re increases. For the present study at $Re = 10^4$, we conclude that L_z should not be
 smaller than $0.64C$ and conservatively choose $L_z = 0.8C$.

The adequacy of the spanwise domain size is confirmed by *a posteriori* quantifica-
 205 tion of the two-point correlation in the spanwise direction. Figure 2 gives the variation
 of correlation coefficient of w' -velocity against the spanwise separation distance at dif-
 ferent positions on the cross-sectional plane, and quantify the coefficient as [31]:

$$r_{w'w'}(\Delta z) = \frac{\overline{w'(z, t)w'(z + \Delta z, t)}}{w'_{\text{R.M.S.}}(z)w'_{\text{R.M.S.}}(z + \Delta z)}, \quad (7)$$

where Δz is the spanwise separation distance between two probes. Following the ap-
 proach of Kitsios *et al.* [31], the coefficient is obtained by averaging over combinations

Figure 3: Spanwise power spectra of T.K.E. against wavenumber taken at probes right downstream of the airfoil trailing edge. The amplitude of each successive spectrum is stepped down by one order of magnitude for clarity.

of probes with the same Δz . The domain size is considered sufficient if the velocity component in the periodic direction is uncorrelated within a separation distance of half the domain size [32]. It is shown in Figure 2 that for present simulations, the correlation coefficient decays to around zero for both airfoils, which confirms the adequacy of the chosen L_z .

2.4. Resolution assessment

The resolution requirement for DNS is a somewhat complicated issue in non-periodic 3D simulations. It is generally understood that the resolution is adequate if flow structures up to Kolmogorov scale are resolved. However, with the exception of simple cases such as the isotropic turbulence in a periodic box, this scale is dependent on the flow parameters and is difficult to be accurately determined *a priori*, hence the determination of mesh resolution is based on experience and *a posteriori* estimation. In order to directly resolve the boundary layer structures in the viscous sublayer with-

out any turbulence model, the first layer of cells must have a size in the wall-normal direction not exceeding unity. The mesh dimensions in the streamwise wall-tangent
 225 and spanwise directions are chosen judiciously to avoid a large mesh aspect ratio, to ensure convergence and accuracy, and well resolve the free-stream turbulence. In the present study, the mesh size (streamwise wrapping around the airfoil, wall normal and spanwise directions) is $4096 \times 256 \times 256$ (268 million) for both airfoils. The mesh is clustered around the airfoil, which ensures a maximum $\Delta\eta^+$ about 0.9 for the first
 230 layer mesh on the wall with η the wall-normal direction, and maximum $\Delta\xi^+$ and Δz^+ in wall-tangent streamwise and spanwise directions, respectively, calculated in similar manner, are about 3 and 10. These maximum values occur during the course of the entire simulation and occur close to the leading edge where the incoming flow impinges on the wall and turns its direction. It should be noted that the flow in the vicinity of the
 235 leading edge is laminar. The mesh sizes in wall units rapidly decrease downstream of the leading edge critical point, and are much smaller in the region where the flow field is turbulent. This indicates that our mesh resolution is adequate.

Another measure to confirm that the turbulent behavior is well resolved is the spanwise power spectra of turbulent kinetic energy, defined as T.K.E. = $(\overline{u'u'} + \overline{v'v'} + \overline{w'w'})/2$. This is plotted in Figure 3, and the power spectra plotted show reasonable
 240 agreement with the Kolmogorov “-5/3” scaling.

2.5. Quantitative diagnostics

Low-*Re* flow over a bluff body is dominated by the time-dependent large-scale vortex shedding. As the Reynolds number increases, the shedding of the large-scale
 245 vortex still dominates but stochastic velocity and pressure fluctuations are observed,

Figure 4: Example of the time history of v/U_0 in the separated shear layer above the airfoil.

as exemplified by the time history of velocity in Figure 4. A triple-decomposition is needed to decompose the velocity and pressure field data and to extract the turbulent fluctuations [33]. The flow variables may be decomposed as follows:

$$\phi(\mathbf{x}, t) = \bar{\phi}(\mathbf{x}) + \phi^*(\mathbf{x}, t) + \phi'(\mathbf{x}, t), \quad (8)$$

where $\phi(\mathbf{x}, t)$ is the instantaneous value; $\bar{\phi}(\mathbf{x})$ is the time-averaged mean; $\phi^*(\mathbf{x}, t)$ is the coherent component due to large-scale vortex shedding; and $\phi'(\mathbf{x}, t)$ is the stochastic turbulent fluctuation. Each component on the RHS of Eq.(8) can be computed based on the instantaneous flow field:

$$\bar{\phi}(\mathbf{x}) = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} \phi(\mathbf{x}, t) dt, \quad (9)$$

$$\phi^*(\mathbf{x}, t) = \frac{1}{n} \sum_{i=0}^{n-1} \phi(\mathbf{x}, t + i \cdot T) - \bar{\phi}(\mathbf{x}), \quad (10)$$

$$\phi'(\mathbf{x}, t) = \phi(\mathbf{x}, t) - \phi^*(\mathbf{x}, t) - \bar{\phi}(\mathbf{x}), \quad (11)$$

where $t_1 - t_0$ is the duration of the simulation, and T is the vortex shedding period.

In experiments the value of T is normally determined from a reference pressure or velocity signal at some fixed point (e.g., [34, 35]), while in the present simulations it

Figure 5: (color online) Profiles of the mean streamwise velocity.

is computed and averaged through spectral analysis on the flow field at several probes. Since we mainly focus on the flow around the airfoil and in near-wake, the phase jitter (e.g., [36]) is not of great concern and could be omitted [27, 37]. The first term on the RHS of Eq.(10) is the phase-averaging over a number of instantaneous flow fields of the same phase, thus the turbulent fluctuation is erased. In the rest of this paper we use the overbar $\bar{\phi}$ to denote time-averaging during the whole simulation. The results presented have been averaged in the spanwise direction unless otherwise stated.

3. Results and discussion

3.1. Flow separation

We begin our discussion with a comparison of the mean flow fields for the two airfoils. Figure 5 shows the profiles of the normalized mean streamwise velocity $\bar{u}/U_0$ at several streamwise stations. The separated flow is revealed by the negative $\bar{u}$ at the airfoil surface, and the size of the separation zone can also be identified. Compared with the NACA-4412 airfoil, the flow past the NACA-0012-64 airfoil detaches from the leeward surface at a further upstream position. Since the NACA-4412 airfoil has a larger projection area with respect to the incoming flow, the flow downstream of

Figure 6: (color online) Distributions of the mean streamwise wall-tangent velocity and pressure gradient along the airfoil leeward surface.

the leeward surface summit is similar to that in the wake of a bluff body. The mean
 275 streamwise velocity rapidly recovers from almost zero to the free-stream velocity as y
 increases beyond the separated shear layer position. The projection area is smaller for
 the NACA-0012-64 airfoil, thus the flow downstream of the surface summit is more of
 the boundary layer flow type and the velocity gradually recovers.

In the present study the flow detachment in the leading edge region is different
 280 because of the airfoil geometry. The geometrical effects on the flow are expressed by
 two means: the variation of the leading edge curvature and the distribution of the APG
 and favorable pressure gradient (FPG) fields. Figure 6 shows the distributions of the
 mean streamwise wall-tangent velocity and the mean pressure gradient at the airfoil
 leeward surface. The streamwise wall-tangent velocity is measured at the first layer
 285 mesh on the surface and is denoted as u_s , and its negative value corresponds to the
 reverse flow associated with separation. The pressure gradient is negative for the FPG
 field, and positive for the APG field. It is seen from this figure that for the NACA-0012-
 64 airfoil, the drastically varying surface curvature at the leading edge results in a much
 stronger local APG field than the NACA-4412 airfoil, and the pressure gradient then

290 gradually decreases to around zero for the remaining surface. As the incoming flow stagnates at the leading edge, it is accelerated by the FPG field for a short distance, then decelerated by the APG field and the wall viscous drag and eventually detaches. The leading edge detachment occurs at around $x/C = 0.08$ for the NACA-0012-64 airfoil, while it is delayed to around $x/C = 0.25$ for NACA-4412.

295 Based on the simulation results we may conclude that for the NACA-0012-64 airfoil, the drastically varying surface curvature in the leading edge region triggers early flow detachment. A much stronger APG field, formed quite close to the leading edge, contributes to the early flow detachment.

3.2. Reverse flow due to separation

300 The flow separation and the formation of the separation zone as well as the recirculation vortex may be quantitatively analyzed from the perspective of reverse flow on the airfoil surface. Figure 7 gives the distributions of several statistical quantities about the streamwise wall-tangent velocity measured at the first cell on the airfoil leeward surface. The wall-tangent velocity is defined positive as the fluid moves from upstream to downstream direction, and negative in the reverse direction. Inspired by the work of Simpson [38] for characterizing the steady free-stream separating turbulent boundary layers, here we define two quantities γ and η : the fraction of time and mass flux that fluid moves from upstream to downstream direction.

$$\gamma = \frac{\int_{u_s \geq 0} dt}{Total\ time}, \quad (12)$$

$$\eta = \frac{\int_{u_s \geq 0} |u_s| dt}{\int |u_s| dt}. \quad (13)$$

Figure 7: (color online) Statistics of the streamwise wall-tangent velocity u_s measured at the first cell on the airfoil leeward surface: (a) fraction of time that flow moves downstream; (b) fraction of mass flux that flow moves downstream; (c) time-averaged velocity; (d) root-mean-square value of velocity; (e) maximum velocity; (f) minimum velocity.

310 The distributions of γ are presented in Figure 7(a). The flow is fully attached to the surface upstream of the leading edge detachment point and is then completely detached just downstream of it. The region of $0 < \gamma < 1$ is alternately occupied by the separated flow and the recirculation vortex. It is seen that the recirculation vortex is only
 315 formed in the region $x/C > 0.53$ for the NACA-4412 airfoil, while multiple vortices are formed and move within a wider region $x/C > 0.19$ for the NACA-0012-64 airfoil. The distributions of η , shown in Figure 7(b), provide the information of reverse flow from the perspective of flow velocity. The η plots are similar to but comparably smaller

in magnitude than the corresponding γ plots, which reflects the relatively smaller velocity magnitude for fluid moving from upstream to downstream compared with the reverse flow, i.e., the region is occupied mainly by the separated flow and the recirculation vortex is weaker. This is more remarkable for the NACA-0012-64 airfoil due to the notable negative u_s in the region $0.6 < x/C < 1.0$, as shown in Figure 7(c). The extreme velocity distributions in Figure 7(e)-7(f) show that for the NACA-4412 airfoil, the maximum velocity is almost zero for the majority of the leeward surface, reflecting the steady and dominant reverse flow in this region. The flow over the NACA-0012-64 airfoil is more oscillatory with larger positive and negative extreme values.

3.3. Laminar-turbulent transition

3.3.1. Initial triggering from 2D to 3D flow

As stated in section 2.2, the initial triggering from 2D to 3D flow is conducted through a precursor low-resolution 3D simulation, whose initial solution is a snapshot of the 2D simulation result. For numerical simulation of flow past an airfoil, the initial triggering is achieved by prescribing a certain type of perturbation to the flow field, such as an unsteady forcing term (e.g. [22]) or a random field (e.g. [29]). The critical point of imposing such perturbation is that it should not be preferred at any frequency or wavelength, otherwise it takes a long time for the flow field to ‘forget’ the initial perturbation. Instead of the artificially constructed perturbation field, Wissink & Rodi [39] relied on the random numerical round-off error to trigger the Kelvin-Helmholtz instability in the separated shear layer over a flat plate. Similar strategy is also employed by Shan *et al.* [9] for flow past a NACA-0012 airfoil. They concluded that the three-dimensional numerical residual fluctuation appearing in the near wake region acts as

the perturbation. In the present study we also rely the numerical noises to trigger the Kelvin-Helmholtz instability and the 3D fluid flow. The numerical noises include the round-off error, the discretization truncation error and the residual, etc. Since this is conducted in a precursor low-resolution simulation, it does not take a long time for
345 the initial 2D flow to develop to statistically steady-state 3D flow. The results is then interpolated in the spanwise direction to start the production simulation.

Classical theory attributes the Kelvin-Helmholtz instability to the vortex shedding, and the waviness leads to transition to turbulence is resulted from the secondary instability mechanism. Recent researches [40, 41] observed that the role of the unstable
350 modes is only to trigger the vorticity rollup but not directly leads to flow transition. In the present study, the formation of the 3D flow field is carried out in the low-resolution precursor simulation, and we are not focusing the initial stages of the transition from 2D to 3D flow, thus the process is not discussed in the following sections.

It is noted here that the numerical noises are only used for the initial triggering
355 from 2D to 3D flow, while we do not think they can contribute a lot to the sustenance of the turbulent flow, although they do exist during the whole simulation. Jones *et al.* [22] carried out DNS and linear stability analysis on flow past a NACA-0012 airfoil, and the results preclude the amplification of round-off error as a route to transition. They proposed a mechanism for the amplification of the perturbation: the perturbation
360 is amplified in the braid region and is propagated upstream along with the reverse flow, and then convected into the braid region of the next developing vortex, where it is further amplified. The upstream propagating of the perturbation is also observed by Spalart and Strelets [5] and Shan *et al.* [9]. Jones *et al.* [22] also concluded that the

Figure 8: (color online) Instantaneous spanwise vorticity field plotted at $\omega_z = \pm 30$.

temporal growth rate of the instability increases with the airfoil incidence due to the
 365 larger magnitude of the reverse flow velocity. Since we are not focusing on the initial
 amplification of the instability in this paper, the related mechanisms are not discussed
 further.

Starting with a 3D flow field, the production simulation is free of any type of arti-
 ficially constructed perturbation, and the inflow is laminar and steady-state as described
 370 in section 2.2. It is shown in previous researches (e.g. [9, 22, 29, 31, 42]) and our
 simulations that the turbulent flow is self-sustained thanks to the amplification of the
 perturbation in the braid region. The results shown hereafter are all obtained by the
 high-resolution production simulation. The results provide information on the whole
 transition process from the incoming laminar flow to the turbulent flow.

375 *3.3.2. General description of the transition process*

The transition of the flow from laminar to turbulent is attributed to the amplification of the disturbance during the flow past the airfoil, where the flow becomes unstable as it separates from the airfoil surface. In the real-world the transition may occur due to myriad sources of noise stemming from the experimental apparatus, the environment etc. In simulations the transition occurs naturally as a result of the roundoff error which plays as the stochastic disturbance. The physical picture of transition is as follows: the flow separated from the airfoil leads to a quasi-2D separated shear layer that is uniform in the spanwise direction; small disturbances grow and amplify as developing downstream; the linear growth phase ends with nonlinearity becoming important; large-scale three-dimensional structures develop; nonlinear interactions in the form of triadic interactions lead to a cascade of large-scale structures to small-scale structures and eventually the flow field is stochastic and turbulent. These physical processes are well exemplified by Figure 8 wherein we show isosurfaces of the instantaneous spanwise vorticity field for both airfoils. In both cases, the separated flow is a vortex (shear) layer almost spanwisely uniform. Since the 2D vortex shedding is absolutely unstable to 3D perturbation [22], corrugations appear on the vortex layer further downstream, amplifying to large-scale 3D structures and smaller scales even further downstream. It is noted that this process is more clear over the NACA-0012-64 airfoil but also seen in the near-wake of the NACA-4412 airfoil.

395 The main difference between the two airfoils is that the spanwise distortion of the shear layer takes place at a further upstream location for the NACA-0012-64 airfoil compared with its NACA-4412 counterpart, representing earlier amplification of distur-

bances and laminar-turbulent transition. The difference of the transition process could also be analyzed by re-examining the velocity distribution in Figure 5. The Rayleigh
 400 inflection point theorem demonstrates that the inviscid parallel flow is unstable to small disturbances where there is an inflection point on the velocity profile [43]. Due to an earlier leading edge detachment, the $\bar{u}$ -velocity profile for the NACA-0012-64 airfoil has a inflection point as early as $x/C = 0.1$, while the mean flow over the NACA-4412 airfoil depicts an inflection point delayed further downstream to approximately
 405 $x/C = 0.4$. These locations are consistent with observations of shear layer instability and transition regions for both airfoils.

Another measure of the transition process is an examination of the instantaneous skin friction coefficient plotted in Figure 9. The complete scenario from quasi-2D flow to small-scale stochastic 3D flow structure is observed in this figure. The flow is
 410 attached to the airfoil surface near the leading edge, and the contour $C_f = 0$ plotted as a solid curve, indicates the first separation locus. After this, there is spanwise distortion and formation of large-scale 3D structures which breaks up into small-scale structures further downstream. Consistent with the findings in Figure 8, the transition of flow over the NACA-0012-64 airfoil is earlier at every stage in the whole development process.

415 3.3.3. Transition via Kelvin-Helmholtz mechanism

The unsteady development of the separated shear layer over the airfoil is presented in Figures 10 and 11. The instantaneous Q -field, where $Q = 0.5(\|\Omega\|^2 - \|S\|^2)$ with S and Ω the symmetric and antisymmetric components of $\nabla \mathbf{u}$, is plotted in approximately one
 420 shedding cycle at the mid-span of each airfoil with positive values denoting rotation dominating over strain. The shear layer holds steady close to the leading edge as it

first separates from the airfoil surface, then the spanwise distortion gradually appears as visualized by the increasing Q and large-scale vortex sheds away from the shear layer. The clockwise-rotating large-scale vortex moves close to the trailing edge as it develops downstream, during which process it breaks up into smaller vortices. The
425 initial transition of the shear layer flow from quasi-2D to 3D via the Kelvin-Helmholtz mechanism is clearly seen by the formation and shedding of the vortices since the K-H is the primary instability type in the separated shear layer (e.g. [42, 44]).

Because of the different leeward surface curvatures and leading edge shapes, the shedding vortex exhibits different development patterns as marked by symbols in the
430 figures. For the NACA-4412 airfoil, the separated shear layer is more steady and the clockwise-rotating vortex initially forms at around $x/C = 0.60$ and sheds away at $x/C = 0.80$. The two legs (marked by arrows) attaching to the shedding vortex (marked by the plus sign) rotate clockwise with the vortex, and the right one breaks up above the trailing edge as shown in Figure 10(c). The vortex also breaks up in the wake
435 and is hardly observed in Figure 10(f). In the meantime, two subsequent vortices have already shed away from the shear layer and are entrained into the wake. The formation, deforming and breakdown of the shedding vortex occur much earlier for the NACA-0012-64 airfoil. The vortex sheds away from the shear layer at $x/C = 0.3 - 0.4$, and breakup is observed at $x/C = 0.5 - 0.7$ as shown in Figure 11(d)-11(f). The large-scale
440 vortex completely breaks up into smaller ones at around $x/C = 0.8$. The small-scale structures are observed close to the trailing edge and in the near-wake, representing a fully developed turbulent flow.

3.3.4. Spanwise distortion of quasi-2D flow

As already seen in Figures 8 and 9, the initial stage of the transition to the 3D flow is visibly characterized by the corrugated structures and the spanwise distortion of the quasi-2D shear layer. Here we quantify the spanwise distortion by the mean spanwise standard variation of the transverse velocity. The quantity is defined as:

$$\sigma_v = \left[\frac{\int_0^{L_z} [v(x, y, z, t) - \langle v \rangle(x, y, t)]^2 dz}{U_0^2 L_z} \right]^{\frac{1}{2}}, \quad (14)$$

in which the angle bracket denotes spanwise-averaging. This quantity is zero for the quasi-2D flow and increases as the flow develops three dimensionally. It is expected to be large where the flow is governed by the large-scale shedding vortices, i.e. drastically varying velocity in the spanwise direction for the shedding flow. This quantity is computed at several positions above the airfoil surface and the results are shown in Figure 12. For both airfoils, the spanwise distortion is first observed roughly in the shear layer and is much significant than the flow within the separation zone, reflecting the dominance of the Kelvin-Helmholtz instability in the shear layer during the transition process. The distortion rapidly grows as the flow develops downstream as the large-scale structures break up into smaller ones and the spanwise uniformity vanishes. It is seen in the figure that for the NACA-4412 airfoil, the transition from quasi-2D to 3D flow starts at around $x/C = 0.6$ where non-zero σ_v at the shear layer is clearly observed. The distortion above the trailing edge is the most violent at the position of $50\%\delta$ since the separated vortices (see Figure 10) shed downwards into the wake. The distortion of the shear layer for flow past the NACA-0012-64 airfoil occurs further upstream at $x/C = 0.1$, and the distortions at all positions are much significant than

those in the NACA-4412 case, indicating the rapid transition towards 3D flow. The
 465 flow is relatively fully developed at around $x/C = 0.8 - 0.9$ (see Figure 13) and small-
 scale structures dominates the following development process, hence the variation of
 the flow in the spanwise direction is not affected by the large-scale vortex shedding
 which significantly distorts the flow, and σ_v slightly decreases at the trailing edge.

The transition from quasi-2D to 3D flow is naturally accompanied by the emer-
 470 gence and growing of streamwise vorticity $\omega_x = \partial v / \partial z - \partial w / \partial y$, normalized by U_0/C ,
 which can be used as a sensor for the development of the 3D flow. Figure 13 presents
 the instantaneous streamwise vorticity distributions at several spanwise positions. By
 referring Figures 10 and 11, it is noted that the streamwise vorticity is initially formed
 roughly along the separated shear layer for both airfoils. For the NACA-4412 airfoil,
 475 the streamwise vorticity is observed to be remarkable in the region $x/C > 0.8$, while
 the three-dimensionality of the flow upstream of this position is rather weak. The flow
 fully develops to 3D in the near-wake region where ω_x shows strong spanwise non-
 uniformity. The distribution of ω_x for the NACA-0012-64 airfoil confirms the earlier
 transition to 3D flow. Notable 3D structure is observed at $x/C = 0.4$ and downstream,
 480 and the flow in the separation zone is strongly three-dimensional, as shown by the ev-
 ident spanwise variation of ω_x in the region $x/C = 0.4 - 0.8$. Small-scale structures
 are especially clearly observed in (a2) both in the separation zone and in the wake,
 reflecting the relatively fully developed 3D flow for the NACA-0012-64 case.

3.3.5. Turbulence statistics

485 A commonly used indicator for transition onset is the distribution of the mean off-
 diagonal Reynolds stress $-\overline{u'v'}/U_0^2$, shown in Figure 14. The Reynolds stress value

above a threshold of 0.001 is plotted in the figure because this threshold value has been chosen to indicate the location of transition onset in various numerical (e.g., [23, 45]) and experimental (e.g., [46, 47]) investigations. Based on this criterion, the transition first occurs in the separated shear layer at the streamwise position $x/C = 0.71$ and $x/C = 0.29$ for the two airfoils, respectively. After transition, there is a concentration of the Reynolds stress with the maximum value respectively at around $x/C = 1.15$ and $x/C = 0.65$, which indicates a strong momentum transfer in the cross-sectional plane and the large-scale structures rapidly break up into small-scale structures.

The maximum R.M.S. values of velocity fluctuations along the streamwise direction are plotted in Figure 15. The fluctuations are non-zero near the leading edge owing to two-dimensional unsteadiness. As the flow develops downstream, all components of velocity fluctuations grow approximately linearly in time: in the region $x/C < 0.7$ for NACA-4412 and $x/C < 0.3$ for NACA-0012-64 airfoils. The fluctuations then increase rapidly where the shear layer breaks down and reach the peak values downstream of the trailing edge at around $x/C = 1.15$ for NACA-4412 and $x/C = 0.6$ for NACA-0012-64 airfoils, indicating the transition to fully 3D flow and breakdown to turbulence. Although the fluctuations are growing at different rates, the corresponding peak values in the two cases are approximately the same. It is noted that for flow over the two airfoils the transitional flow via Kelvin-Helmholtz mechanism is three-dimensional, and it commences with the fluctuation of streamwise velocity which is the largest component. As a result, before the fluctuations reach their maximum values, the u' -component grows earlier and faster and is always larger than the other two components.

Figure 16 shows the profiles of the mean diagonal turbulent Reynolds stresses at

510 nine stations above the leeward surface. The dotted curve denotes the position of the
 displacement thickness of the boundary layer. Boutillier & Yarusevych [48] experi-
 mentally investigated the separated shear layer transition over a NACA-0018 airfoil at
 $Re = 10^5$ and 0-15 degrees incidence, and concluded that the maximum R.M.S. value
 of the streamwise velocity fluctuation occurs at a distance away from the leeward sur-
 515 face approximately equal to the displacement thickness. Although different airfoils and
 flow parameters are used in the present simulations, similar conclusions could also be
 deduced for both airfoils, i.e., the mean Reynolds stress $\overline{u'u'}/U_0^2$ attains its maximum at
 the position of displacement thickness. It is also noted that the three diagonal Reynolds
 stress components are distinct for the NACA-4412 airfoil, reflecting the anisotropy of
 520 the turbulence in this case. However, the three components are closer to each other in
 the trailing edge region for NACA-0012-64 implying that the turbulent flow in this re-
 gion is relatively fully developed and rapidly becoming isotropic. In addition to these
 diagonal components, the distributions of off-diagonal turbulent Reynolds stresses
 are plotted in Figure 17. The off-diagonal Reynolds stress components vary in a man-
 525 ner similar to the diagonal components in that they are more pronounced close to the
 trailing edge, but their magnitudes are smaller in all streamwise locations than the
 diagonal components. Since there is no spanwise component or disturbance in the uni-
 form incoming flow, the breakdown of shear layer and roll-up of shedding vortices are
 more pronounced in the cross-sectional plane, leading to a larger momentum transfer
 530 $\overline{-u'v'}/U_0^2$ in this plane than in the spanwise direction. It is also noted in Figure 17 that
 similar to $\overline{u'u'}/U_0^2$, the Reynolds stress $\overline{-u'v'}/U_0^2$ reaches its maximum at a distance
 from the leeward surface of approximately the displacement thickness for both airfoils.

As a result of the earlier transition for flow over the NACA-0012-64 airfoil, the turbulent flow is well developed, and all Reynolds stress tensor components are larger than those in the NACA-4412 case and reflects the intense momentum transfer in this case.

3.3.6. Eigen-system analysis

Finally, we perform an interesting statistic on the transitional flow. Figure 18 presents the probability density functions (PDFs) of the cosine of the angles between the vorticity vector $\vec{\omega}$ and the eigenvectors of S_{ij} and $S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}$, where S and Ω are respectively the symmetric and anti-symmetric parts of $\nabla\mathbf{u}$. The notations $\vec{e}_+^*$, $\vec{e}_-^*$ and $\vec{e}_z^*$ represent the eigenvectors of S_{ij} reordered such that $\vec{e}_z^*$ is most aligned with $\vec{\omega}$ and the eigenvalue corresponding to $\vec{e}_+^*$ is the larger one of the remaining two eigenvalues. $\vec{\lambda}_+^*$, $\vec{\lambda}_-^*$ and $\vec{\lambda}_z^*$ denote the eigenvectors of $S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}$ and are reordered similarly. The eigen-system analysis is carried out for the instantaneous flow field in the region above the airfoil leeward surface and in the near-wake. The alignment patterns are generally very similar for both cases. The eigenvectors $\vec{e}_z^*$ and $\vec{\lambda}_z^*$ are aligned with $\vec{\omega}$ with a very high probability, while the other eigenvectors are mostly perpendicular to $\vec{\omega}$ due to the symmetry of the matrix. Horiuti [49] performed such statistics for the homogeneous isotropic turbulence in a box with periodic boundary conditions, and arrived at similar conclusions except that $\vec{e}_+^*$ showed a rather flat distribution as for a random Gaussian field. Although the flow over an airfoil with no spanwise variation is anisotropic in nature in that the developments of initial unsteadiness, breakdown to turbulence, turbulent fluctuations and other dynamic behavior are much more significant in the cross-sectional plane rather than in the spanwise direction. The similarity of present distributions to those of isotropic turbulence reveal high correlation between $\vec{\omega}$

and the eigenvectors as a kinematic consequence of the presence of vortex tubes [50].

It is also noted that the range of $\cos\theta\{\vec{\omega} - \vec{e}_z\}$ is approximately $[-1, -0.6]$ and $[0.6, 1]$ and

the ranges of $\cos\theta\{\vec{\omega} - \vec{e}_+\}$ and $\cos\theta\{\vec{\omega} - \vec{e}_-\}$ are $[-0.7, 0.7]$, both quantitatively close to

the values derived by Andreotti [51] for an artificial random flow as $[-1, -1/\sqrt{3}]$ and

560 $[1/\sqrt{3}, 1]$ and $[-1/\sqrt{2}, 1/\sqrt{2}]$.

4. Conclusion

We present direct numerical simulation results of flow past two airfoils, NACA-4412 and NACA-0012-64, at $Re = 10^4$ and 10 degrees incidence to investigate the

effects of the airfoil geometry on the flow separation and transition patterns. The sim-

565 ulations are conducted using an in-house code with energy conservative fourth-order

spatial discretization scheme. Spatial resolutions and adequacy of the spanwise domain

size have been addressed. The two-point correlation shows that a spanwise domain size

of $0.8C$ is adequate for the present Reynolds number. The resolution is proved to be

fine enough by the agreement between the spanwise power spectra of turbulent kinetic

570 energy and Kolmogorov's "-5/3" law.

For both airfoils, the flow detachment occurs close to the leading edge as a result

of the adverse pressure gradient field and the varying of surface curvature. Compared

with the NACA-4412 airfoil, the flow over the NACA-0012-64 airfoil experiences a

much stronger adverse pressure gradient field and more drastically varying leading

575 edge surface curvature, and hence results in flow detachment closer to the leading edge.

The separated shear layer forms at the leading edge is unstable and disturbances are

amplified via the Kelvin-Helmholtz mechanism. As flow develops downstream, the

transition from quasi-2D to 3D flow first takes place in the shear layer as indicated by the corrugated structures and shedding vortices. The unsteadiness then rapidly grows in a nonlinear way and the flow transits to fully three-dimensional. Due to the earlier detachment, the flow past the NACA-0012-64 airfoil experiences the whole laminar-turbulent transition scenario that is much earlier than that of the NACA-4412 airfoil, mainly characterized by the distortion of the quasi-2D flow, the vortex shedding and the breakdown from large-scale to small-scale structures. The onset of laminar-turbulent transition is observed in the separated shear layer at the streamwise position $x/C = 0.71$ for the NACA-4412 airfoil, and the transitional flow develops to fully turbulent state downstream of the trailing edge ($x/C > 1.0$). For the NACA-0012-64 airfoil, the transition onset and fully development of turbulent flow respectively takes place at around $x/C = 0.29$ and $x/C = 0.6 - 0.8$, both of which are evidently much earlier.

Based on the high-fidelity DNS results, quantitative analysis is performed for the high-order turbulence statistics, namely, the turbulent Reynolds stresses. It is observed that the mean Reynolds stress $\overline{u'u'}/U_0^2$ reach a maximum at a distance from the airfoil leeward surface of approximately equal to the displacement thickness (roughly at the shear layer), consistent with the experimental measurements for the NACA-0018 airfoil [48]. Moreover, we find in the present simulations that the same pattern also applies for the off-diagonal Reynolds stress $\overline{u'v'}/U_0^2$.

A further eigen-system analysis reveals that although the transitional flow over the airfoil is intrinsically anisotropic, the alignments between the vorticity vector and the eigenvectors of S_{ij} and $S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}$ are qualitatively in close agreement with the homogeneous isotropic turbulent flow in a periodic box [49]. They are also quantitatively

consistent with the derived values for an artificial random flow [51].

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Figure 10: (color online) Instantaneous contours of the mid-span Q -invariant at equally spaced instants in about one shedding cycle for the NACA-4412 airfoil. Contour values $Q=[10,1000]$ with 20 exponentially distributed levels.

Figure 11: (color online) Instantaneous contours of the mid-span Q -invariant at equally spaced instants in about one shedding cycle for the NACA-0012-64 airfoil. Contour values $Q=[10,1000]$ with 20 exponentially distributed levels.

Figure 12: (color online) Spanwise standard deviation of the transverse velocity v/U_0 . δ is the local displacement thickness.

Figure 13: (color online) Instantaneous streamwise vorticity ω_x at three spanwise positions (from top to bottom): $z/C=0.2$, 0.4 and 0.6 . The left and right columns are respectively the NACA-4412 and NACA-0012-64 airfoils.

Figure 14: (color online) Distributions of the mean off-diagonal Reynolds stress $\overline{-u'v'}/U_0^2=[0.001,0.06]$ with 10 equally distributed levels. The transition onset is indicated by the threshold value 0.001.

Figure 15: (color online) Streamwise distributions of the maximum R.M.S. values of turbulent velocity fluctuations.

Figure 16: (color online) Distributions of the mean diagonal turbulent Reynolds stresses. The dotted green line indicates the displacement thickness.

Figure 17: (color online) Distributions of the mean off-diagonal turbulent Reynolds stresses. The dotted green line indicates the displacement thickness.

Figure 18: (color online) Instantaneous PDFs of the cosine of the angle between the vorticity vector $\vec{\omega}$ and the reordered eigenvectors. $\vec{e}_+$, $\vec{e}_-$ and $\vec{e}_z$ are the reordered eigenvectors of S_{ij} . $\vec{\lambda}_+$, $\vec{\lambda}_-$ and $\vec{\lambda}_z$ are the reordered eigenvectors of $S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}$. See text for details.

Highlights:

1. DNS of separated and transitional flow past airfoils of different geometries.
2. Detailed discussion of the settings of computational parameters.
3. Airfoil with a sharper leading edge leads to earlier separation and transition.
4. Maximum turbulent Reynolds stress occurs at the displacement thickness position.
5. This transitional flow is similar to isotropic flow in eigen-system analysis.