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Pore network modeling of drainage process in patterned porous media: a quasi-static study

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Abstract

This work represents a preliminary investigation on the role of wettability conditions on the flow of a two-phase system in porous media. Since such effects have been lumped implicitly in relative permeability-saturation and capillary pressure-saturation relationships, it is quite challenging to isolate its effects explicitly in real porous media applications. However, within the framework of pore network models, it is easy to highlight the effects of wettability conditions on the transport of two-phase systems. We employ quasi-static investigation in which the system undergo slow movement based on slight increment of the imposed pressure. Several numerical experiments of the drainage process are conducted to displace a wetting fluid with a non-wetting one. In all these experiments the network is assigned different scenarios of various wettability patterns. The aim is to show that the drainage process is very much affected by the imposed pattern of wettability. The wettability conditions are imposed by assigning the value of contact angle to each pore throat according to predefined patterns.

Keywords: Pore-network modeling, Porous media, Multiphase flow, Shale-gas

1 Introduction

Shale gas has become an increasingly important source of natural gas in the United States since the start of this century, and interest has spread to potential shale gas sites in the rest of the world, thereafter[1]. In oil and gas exploitations, any prediction of temporal and spatial distributions of the residing gas is sensitive to the correct macroscopic description of the hosting tight formations. This motivated the research at pore scale to gain understanding of the different involved processes and to quantify their effects at macroscopic scale. Although the continuum hypothesis provides an appropriate framework to investigate transport phenomena

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in porous media [2, 3, 4], pore scale modeling is sometimes required to gain more insight. In this context, pore network models stand as one of such techniques that describe pore scale phenomena and provide details about macroscopic quantities (e.g., [5, 6, 7, 8]). The development of pore network models could trace back to 1956, when Fatt [9] accomplished a pioneering work in which pore networks with disparate representations of the pore space had been employed in the studies of two phase flows. Later, the cross sections of the pore bodies and pore throats have extended to encounter more complex geometrical shapes including squares, triangles, polygons, etc. [10, 11]. In the meantime, researchers have developed more complex network topologies with variable coordination numbers. However, most models were generated stochastically without representing natural permeable media. With the great progress in modern imaging techniques such as micro-focused computed tomography (micro-CT) where more information about the microstructure of real sediments become available, more realistic pore networks were constructed (e.g., [12]). Thus, the past decade has seen a quantitative jump in the pore network model capabilities. In this paper, we apply the pore-network modeling to present a quasi-static analysis of two-phase flow in patterned porous media. The aim is to investigate the role of different wettability scenarios on the drainage of a non-wetting phase. First, the network is generated according to log-normal distribution of pore bodies and pore throats and the network is initially assumed fully saturated with wetting phase. Afterwards, we simulate the fluid displacement mechanisms, in which the non-wetting phase invades the space filled with wetting phase. In these cases, the distribution of contact angle is assumed patterned so that we can study the significance of wettability conditions. Besides, we draft some code in Matlab to visualize the network and development of displacement to make it more understandable. Furthermore, several cases with different distribution of the contact angle are also conducted and we highlight the relationship between the saturation of wetting-phase and the pressure difference of wetting phase and non-wetting phase. Furthermore, we have formulated a quasi-steady simulation in order to calculate the global permeability. The Hagen-Poiseuille's equation is applied here to get the local permeability in each pore-throat. In this part, we first find the relationship between mesh size and global permeability if the network is filled with only single phase fluid. Then, for a two-phase flow, we get the relationship of global permeability for non-wetting phase when it's invading the network and the pressure difference between wetting phase and non-wetting phase. Finally, we show how the relative permeability of both wetting phase and non-wetting phase change with the saturation of non-wetting phase.

2 Network Generation and Description

The network in this paper is represented by a two-dimensional or three-dimensional regular lattice with distributed spacing, which is usually called lattice network. In this network, the pore bodies are located at the lattice nodes, and the pore throats are lined along the lattice coordinates. The number of pore throats connected to a pore body is called coordination number. The coordination number for a two-dimensional network, in this work, is assumed four and for a three-dimensional network is assumed six, except the pore bodies at the edges and corners. Fig. 1 shows a general representation of the network in 2D and 3D.

The network geometry is related to the geometrical shape of pore bodies and also cross sections of pore throats. Here, the pore bodies are assumed to be spherical in shape and the cross section for pore throats are circle. The size distribution of pore bodies is given by a truncated log-normal random distribution, with no spatial correlation. The data adopted in this work is listed in Table 1,

This kind of distribution could easily be treated in Matlab using its own function (`lgnrnd`).

Figure 1: General representing of pore-network (a) 2D, and (b) 3D

Specification	Value	Unit
Min. pore body inscribed radius	0.0408	mm
Max. pore body inscribed radius	0.234	mm
Mean pore body inscribed radius	0.114	mm
Standard deviation	0.169	mm

Table 1: Network parameters

Radius and length of pore throats connecting the pore bodies are determined based on the size of neighboring pore bodies. To make our network more reliable, spacing between the layers of the network in each direction is variable. If we name the spacings between layers i and $i + 1$ in x direction as λ_{xi} , the lattice spacings are defined as,

$$\lambda_{xi} = \max\{R(i, j, k) + R(i + 1, j, k) : j = 1 : n_y, k = 1 : n_z\}, i = 1 : n_x \quad (1)$$

Where R denotes the body radius, n_x, n_y, n_z represent the number of pore bodies in each direction, which is assumed similar in other directions. With this center-to-center distance λ and inscribed pore radius R_i and R_j , we can set the dimensionless inscribed radius of the pore throat ij , $\tilde{r}_{ij}$, as,

$$r_{ij} = Q_i Q_j (Q_i^{1/2} + Q_j^{1/2})^{-n}, \quad n > 0 \quad (2)$$

where

$$Q_i = \frac{\tilde{R}_i \sin(\pi/4)}{(1 - \tilde{R}_i \cos(\pi/4))^n}, \quad Q_j = \frac{\tilde{R}_j \sin(\pi/4)}{(1 - \tilde{R}_j \cos(\pi/4))^n}, \quad \tilde{R}_i = \frac{R_i}{d}, \tilde{R}_j = \frac{R_j}{d},$$

n is a parameter, which can control ratio between the radius of pore bodies and pore throats. In this paper, we select $n = 0.1$ to have a significant overlapping between pore body and throat radius distributions. The whole distribution of the radius of pore bodies and pore throats (radius in x direction, and proportion in y direction, Fig. 2),

After the generation of our network, we can visualize it in Matlab, as shown in Fig. 3. To test whether the network is representative to porous media, one can test the change of the

Figure 2: (a) Distribution for pore bodies, and (b) Distribution for pore throats(non-dimensional)

Figure 3: Visualization of pore-network in Matlab (two-dimensional)

porosity with the network size. That is because the size of pore-bodies and pore-throats follow a random distribution, the porosity changes all the time. However, as the network size gets larger, the porosity will fix near one value as depicted in Fig. 4 below.

3 Rules

In quasi-static analysis, our focus is on the threshold capillary force in pore throats, which is treated as the entry pressure for non-wetting phase to occupy the spaces filled with wetting phase. Following assumptions are made for our simulation:

1. The resistance occurs only in the pore throat. It means that if the pressure of non-wetting phase is larger than the threshold capillary force in pore throat, the throat and corresponding pore body will be occupied.

Figure 4: The relationship between porosity and mesh size

2. The pore throat has no volume. It means that the volume of pore bodies is only considered when calculating saturation.
3. If the pore throats and pore bodies are occupied, they are fully occupied with one phase. It means there are only two states to describe each pore body and throat: they are either occupied by wetting phase or by non-wetting phase.
4. The gravity is ignored. It results from the fact that the capillary length for water under normal conditions is commonly 2.7 mm, and the size of pore-body and pore-throats we used here is obviously much less than that.

Thus, the threshold capillary force in pore throats could be computed by,

$$P_c = \frac{2\sigma^{nw} \cos(\theta)}{r} \quad (3)$$

The meaning of the notations is in Table. 2

Specification	Notation	Value	Unit
Contact angle	θ	see Section 4	degree
Interfacial tension	σ^{nw}	0.0725	$kg \cdot s^{-1}$
Throat radius	r	Section 3	mm

Table 2: Notations to Compute Capillary Force

The saturation of wetting phase is defined as

$$S_w = \frac{V_w}{V_t} \quad (4)$$

Where V_w indicates the volume of pore bodies occupied by wetting phase, and V_t represents the total volume of the pore bodies.

Figure 5: The relationship between pressure difference of wetting and non-wetting phase and the saturation of wetting phase

4 Quasi-Static Simulations

With a normal distribution of contact angle (mean=45°, deviation = 15°, maximum = 90°, minimum = 0°), and a pressure step set to be 1, we can get the relationship between pressure difference of wetting and non-wetting phase and the saturation of wetting phase as shown in Fig. 5. The non-linearity is reasonable due to the random distribution of the contact angle and radius of pore throats. Next, several cases with various distribution of contact angle (for the wetting phase) have been studied. The wettability patterns considered in this work may be described as follows: we set the whole mesh size to 100×100 , and the notation for each node is $N_{i,j}$,

- **Case 1:** For $i=1:20, j=1:100$: $\theta = 0$; For $i=21:40, j=1:100$: $\theta = 20$; For $i=41:60, j=1:100$: $\theta = 40$; For $i=61:80, j=1:100$: $\theta = 60$; For $i=81:100, j=1:100$: $\theta = 80$;
- **Case 2:** For $i=1:20, j=1:100$: $\theta = 0$; For $i=21:40, j=1:100$: $\theta = 80$; For $i=41:60, j=1:100$: $\theta = 0$; For $i=61:80, j=1:100$: $\theta = 80$; For $i=81:100, j=1:100$: $\theta = 0$;
- **Case 3:** For $j=1:20, i=1:100$: $\theta = 0$; For $j=21:40, i=1:100$: $\theta = 80$; For $j=41:60, i=1:100$: $\theta = 0$; For $j=61:80, i=1:100$: $\theta = 80$; For $j=81:100, i=1:100$: $\theta = 0$;
- **Case 4:** For $i=1:50, j=1:50$: $\theta = 0$; For $i=1:50, j=51:100$: $\theta = 80$; For $i=51:100, j=1:50$: $\theta = 80$; For $i=51:100, j=51:100$: $\theta = 0$.

Under a pressure difference between wetting and non-wetting phase equals 300Pa for case 1-3, and 500Pa for case 4 with the side boundaries considered as no flow, the static occupancy station of each phase could be visualized as Fig. 6:

From these 4 figures, we can easily witness the significant effect of contact angle distribution on the considered two-phase system. This may reveal that, the flow direction of non-wetting phase may be controlled by utilizing different distribution of contact angle. For example, we can develop the following two patterns shown in Fig. 7. From the above two figures, we can get an idea that if we can control the distribution of contact angle in multi-phase flow, we may in effect manage the flow direction for the non-wetting phase, which is very useful in modern industry, for example, petroleum engineering and shale-gas extraction. Furthermore, an interface using

Figure 6: Simulation Results (a) Case 1, (b) Case 2, (c) Case 3 and (d) Case 4

Matlab to demonstrate the occupancy in three dimensions is developed as shown in Fig. 8. On this interface, the user can set the parameters used in the pore-network, including the distribution of radius of pore bodies, mesh size, contact angle and interfacial tension.

5 Quasi-Equilibrium Simulation

In this part, we want to get more information of global permeability in our pore-network, which is widely concerned in petroleum industry. The Hagen-Poiseuille's equation is applied here to get the local resistance in each pore-throat,

$$Q = \frac{dV}{dt} = \frac{\pi r^4}{8\mu} \frac{|\Delta p|}{L} \quad (5)$$

In addition, we apply the Shift-Matrix method, proposed by (Sun et al. [13] and Salama et al. [14], to calculate the pressure and velocity of flow in our network. To focus on the physical rules, we treat the parameters in a dimensionless manner here. Firstly, we find the relationship between mesh size and global permeability if the network is filled with only single phase fluid, Fig. 9.

Figure 7: Some interesting flow of non-wetting phase

Figure 8: An interface to demonstrate the pore-network

Then, for a two-phase flow, with a certain distribution of pore size and contact angle, we get the relationship of global permeability for non-wetting phase when it is invading the network and the pressure difference between wetting phase and non-wetting phase, as shown in Fig. 10.

This curve is easy to understand, because more pores are occupied by non-wetting phase as pressure differences goes up. The relative permeability-saturation relationship is shown in Fig. 11.

Figure 9: Dimensionless global permeability for single phase

Figure 10: Permeability for non-wetting phase

Figure 11: Relative permeability

6 Conclusion

Employing the pore network models in a quasi-static simulation, we can investigate the effect of contact angle patterns to the invading non-wetting fluid in porous media. It is shown that the flow of the non-wetting fluid can be directed in a very interesting way. The invading fluid will move, preferably, along the path of least resistance. One component of such resistance is related to the imposed wettability conditions.

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