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We report high quality broadband spatial reflections of Rayleigh surface acoustic waves (SAWs) through a graded grooved surface. High quality means that no wave is allowed to transmit and the incident wave is nearly all reflected to the input side. The graded grooved surface is structured by drilling one dimensional array of graded grooves with increased depths on a flat surface. We investigate SAW dispersion relations, wave field distribution at several typical SAW wavelengths, and time evolution of a Gaussian pulse through the graded grooved surface. Results show that the input broadband Rayleigh SAWs can be slowed, spatially enhanced and stopped, and finally reflected to the input side. The study suggests that engraving the flat surface can be used as an efficient and economical way to manipulate Rayleigh SAWs, which has potential application in novel SAW devices such as filters, reflectors, sensors, energy harvesters, and diodes. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4905948>]

INTRODUCTION

In recent years, based on novel wave characteristics in completely periodic structures, i.e., phononic crystals, people devote more attention to graded periodic structures to improve the quality of the original periodic system or create new physical phenomena. Wave focusing^{1–5} and wide-band wave absorption^{6–8} phenomena have been obtained by these gradually changed periodic structures. Another interesting phenomenon is wave “rainbow trapping.” Rainbow trapping means that the broadband incident wave will be enhanced⁹ and stopped in different spatial positions due to the slow wave effect and finally reflected¹⁰ to the incident side after a certain “trapping time.” This phenomenon can be used as the designed principle of devices for purposes of wave slowing, energy trapping, wave reflecting and other wave manipulation. It has been demonstrated for electromagnetic (EM) wave propagation in tapered metamaterial waveguide¹¹ and EM surface wave¹² propagation in one-dimensional graded or chirped gratings. It also draws the related researchers’ attentions to explore the phenomenon’s potential application for acoustic wave.^{9,13} However, to the best of our knowledge, little work is done to explore the phenomenon’s application for elastic wave.

For elastic wave propagation in completely periodic structures, many researchers have conducted work on surface acoustic waves (SAWs) in half infinite elastic space with surface structured by arrays of stubs, holes, or resonators.^{14–22} Especially, Rayleigh SAWs or sagittal SAWs modulated by one-dimensional periodic gratings^{18,22–25} have been investigated due to easy fabrication of the structures and inherent applications of the interdigital transducers.^{17,26} However, most of these works are focusing on band gap property, transmission attenuation, slow wave effect,^{18,22} etc. As far as

the reflection problem of SAW, Wu’s group^{27,28} has proposed using phononic crystals as high efficiency compact reflective gratings and high performance resonant cavities based on the corresponding band gap characteristic. And very recently, Graczykowski²⁹ has analyzed the incident Rayleigh SAW reflection and energy loss into the bulk by a single step and groove. Based on this work and inspired by the studies of rainbow trapping for electromagnetic and acoustic waves, this paper investigates the reflection problem of Rayleigh SAW propagation through one dimensional array of graded grooves with linearly and smoothly increased depths engraved on the flat surface.

Earlier, Liu’s group³⁰ showed that engaging periodic grooves (slots) with tunable depths on the top and bottom surfaces of a plate can filter elastic waves tunably in low-frequency and high-frequency regimes to make the structures as low-pass, high-pass, and band-pass filters. In parallel, the designed graded grooves on the flat surface of a half-infinite elastic space in this paper can be directly used to filter Rayleigh SAWs in a broadband frequency range. More importantly, the high quality broadband spatial reflecting behavior of the incident Rayleigh SAWs through the graded grooved surface is emphasized. High quality means that no wave is allowed to transmit and the incident wave is nearly all reflected to the input side. The upper limit of the broadband frequency range is up to $V_R \times (\pi/a)$, where V_R is the Rayleigh wave velocity. With the incident Rayleigh SAWs modulated by the graded grooves, the original linear dispersion curves will bend, resulting in zero group velocities at the Brillouin zone boundary. Therefore, the wave fronts will be spatially enhanced, stopped, and finally reflected. In case of the grooves are gradually and smoothly changed, the spatial manipulation of the surface wave will be broadband and especially, the reflection to the input side will be nearly perfect.

The paper is organized as follows. Band structures are first given for the graded units to analyze the mechanism of

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realizing “rainbow trapping.” Then, wave fields at several selected wavelengths in the frequency domain and a Gaussian pulse evolution with a selected central wavelength in the time domain are presented to show the real propagation of Rayleigh SAW through the graded grooved surface.

MODEL AND CALCULATION METHOD

The schematic of half space with the graded grooved surface is shown in Fig. 1. The grooves of unit width $w = 0.2a$ drilled on the flat surface are linearly increased with depth h from 0 to $0.8a$ by graded interval $0.008a$, where a is the periodicity along the horizontal direction. The labeled horizontal axis indicates the location/position (from $0a$ to $100a$) of the graded grooved surface. Since grooves are infinite perpendicular to the principal plane and wave propagation is limited to the principal plane, the three dimensional coupling elastic wave equations are uncoupled by plain strain or in-plane problem of Rayleigh SAW and anti-plain strain or out-of-plane problem of shear horizontal (SH) wave. In this work, only Rayleigh SAW is considered, because SH wave can be mapped to acoustic wave.¹³ The material is selected as a conventional case: silicon. The mass density, elastic modulus, and Poisson ratio are $\rho = 2329 \text{ kg/m}^3$, $E = 170 \text{ GPa}$, and $\nu = 0.28$, individually.

To understand wave propagation in the graded periodic structure, firstly and basically, the dispersion relations for corresponding complete periodic unit cells should be analyzed. The finite element method (FEM) in conjunction with Bloch theory is utilized to compute the modulated surface wave dispersion relations by solving eigenvalues of the unit cell. The unit cell length along the vertical direction is set very larger than the periodicity to approximate the real surface wave propagation situation. Thus, the high-order dispersion curves of plate modes will be squeezed to the linear Rayleigh SAW dispersion curve corresponding to the free bottom flat surface. The computation will not take very long because of that in-plane strain module is used for the problem. The related detail process and tricks for FEM to compute dispersion relations of the modulated SAWs can be referred to the recent documents.^{19,20}

Frequency response analysis by FEM can be done to directly investigate the spectral property of harmonic wave propagation through the structure. The model to simulate frequency response for the Rayleigh SAWs through the graded

grooved surface is shown in Fig. 1(b). The depth along the vertical direction is set $10a$ and connected a perfect match layer (PML) (not shown in Fig. 1(b)). A point source vibrating vertically is applied in the far field away from the graded grooved surface to generate a stable incident Rayleigh SAW. PML along the vertical direction is also applied to absorb and eliminate reflecting of the bulk wave at the left and right boundaries. The source can be frequency swapped to get wave fields at interested frequencies.

To simulate a wave signal traveling through a structure or system, one can do time domain analysis to see the wave trajectory in the whole life journey. A Gaussian pulse centered at a special Rayleigh wavelength can be placed in the far field away from the graded grooved surface to generate the special Rayleigh SAW at the corresponding frequency. Distinct from the model in the frequency domain, PMLs are removed. Plane wave radiation boundary conditions are applied to the outer boundaries to guarantee no waves are reflected.

All the simulations are performed by COMSOL Multiphysics.³¹ Among all the simulations, the finite mesh sizes in space are set smaller than $1/5$ of the lowest wavelength and time step smaller than $1/5$ of the oscillation period. And recomputation is done by finer mesh sizes to make the numerical results convergent and accurate.

RESULTS AND DISCUSSION

Fig. 2(a) gives the first-order curves of the eigenvalue results, which are the first-modes of surface wave corresponding to several different unit depths. One can see that the characteristic of the slow wave effect introduced by resonance of the stubs divided by the grooves. At the boundary of the Brillouin zone, where the band becomes flat, the group velocity of the SAW goes to zero. This indicates that the surface wave can be slowed down and stopped by the grooves and wave energy can be trapped as discussed in references.^{18,22} In this paper we will show that the energy trapping for the incident wave is a very ideal situation for completely periodic structure and cannot be reached in the real application especially for the graded structures. When the grooves are deeper, the second-order curves of the eigenvalue results will cross the linear Rayleigh SAW dispersion curve shown in solid line in Fig. 2(b). The left parts of the curves beyond the sound line (the linear solid line) are leaky bulk wave

FIG. 1. (a) The local closer view of the graded grooved surface with unit periodicity a , unit groove width w , and graded groove depth h . (b) The FEM model of the graded grooved surface: the grooves of unit width $w = 0.2a$ drilled on the flat surface are linearly increased with depth h from 0 to $0.8a$ by graded interval $0.008a$. The number of the periodicity extending along the horizontal direction is labeled in the horizontal axis.

FIG. 2. (a) The first-order SAW dispersion curves for the completely periodic structures corresponding to several typical groove depths. (b) The second-order SAW dispersion curves for the completely periodic structures with high unit depths, i.e., $h = 0.7a$, and $h = 0.8a$.

modes not shown in Fig. 2. While the right parts under the sound line are the second-order SAW modes as showed in Fig. 2(b). If the groove depth is much deeper, the periodic structured surface will support high-order mode SAWs, and each gap between every adjoining order modes is surface wave band gap.^{14,18,22} Based on the band gaps between adjoining order modes, the periodic structures can be used as SAW reflectors.^{27,29}

If the complete periodic grooves on the surface are replaced by the graded grooves, two interesting phenomena about surface wave propagation arise. The first is that zero group velocities corresponding to different incident wavelengths happened at the Brillouin zone boundary as shown in Fig. 2(a) are spatially characterized by different groove depths along the graded grooved surface. This means that, when a frequency-mixed wave packet propagation through the graded grooved surface, wave signals at different frequencies are spatially split and stopped at different locations, which is called “rainbow trapping” in the studies of electromagnetic and acoustic manipulation.^{12,13} Another interesting implicit phenomenon by rainbow trapping is that only the first-order dispersion relations are needed to account for band gap width of the graded grooved surface. The second-order even more high-order surface wave modes which mean pass bands for higher unit groove depths like $h = 0.7a$, and $h = 0.8a$ in Fig. 2(b) are dropped. Wave components at low frequencies which can pass through the grooves of shallow depth (like $h = 0.5a$) are located in the band gap of the high-depth (like $h = 0.8a$) units and will be stopped by the latter grooves of high depth. While wave components at high frequencies will be stopped earlier by the former grooves of shallow depth before reaching the later groove of high depth, which support the wave propagation by the second surface modes. So, for the graded grooved surface, the band gap width is the frequency range where the first bands with zero group velocities cover. From Fig. 2(a), one can see that the very broad band gap property of the graded grooved surface and the upper limit of the broadband frequency range is up to $V_R \times (\pi/a)$, where V_R is the Rayleigh wave velocity.

The broad band gap of the graded grooved surface means that no surface wave in this frequency range is allowed to transmit. But the wave is not completely trapped. To display this and explore the slow wave effect, surface wave propagation in the frequency domain under a continuous source for

several incident Rayleigh wavelengths are given in Fig. 3. Fig. 3(a) plots total displacement fields corresponding to wavelengths $\lambda_R = 5.4a$, $\lambda_R = 3.8a$, and $\lambda_R = 2.6a$. One can see clearly that the incident waves at different wavelengths are enhanced⁹ and stopped at different locations with groove depths: $h_{stop} = 0.7a$, $h_{stop} = 0.5a$, and $h_{stop} = 0.3a$, individually, because of slow wave effect and zero group velocity at the Brillouin boundary. When the groove depths are not very higher, Bragg reflection mechanism makes the enhanced wavefront wavelengths at the stop location always two times²⁹ of periodicity depicted by the zoomed view in Fig. 3(b). The total displacements on the surface for the selected incident wavelengths are plotted in Fig. 3(c). The point source with vertical displacement amplitude 1 is applied to the far field away from the graded grooved surface. One can see that as the waves travelling along the surface, stable Rayleigh SAWs are formed, slowed, enhanced, and finally stopped with maximum amplitudes. After the stop location, the wave amplitudes fall down to zero, meaning no wave is transmitted.

One point should be noticed about the results of Fig. 3. Here, rainbow trapping does not mean the complete standstill of the incident wave. Because the source is continuous and always works, if the wave is in complete standstill as the zero group velocity of the dispersion relation for the unit cell shown in Fig. 2, the wave field amplitude will emerge and go to infinity. Actually, when the incident Rayleigh surface wave goes through the graded grooved surface, wave energy is divided into three parts: transmitted wave energy, reflected wave energy, and leaky wave energy (radiated bulk wave energy). From Fig. 3(a), one can see that very little wave is leaky to the bulk, due to the very smooth changing of groove depths along the surface. One can image that most of the wave energy are reflected when reaching the stop location (at this location, no wave is allowed to transmit).

To see clearly the reflective property of wave propagation through the graded grooved surface, a pulse travels as the time going is done by time domain analysis.

A Gaussian pulse with a central wavelength $\lambda_R = 2.6a$ evaluation along the graded grooved surface as the time going is depicted in Fig. 4. The vertical and horizontal displacements are plotted in Figs. 4(a) and 4(b), individually. The black line is the source signal indicating only vertical displacement is applied to the point and a stable Rayleigh

FIG. 3. (a) Total displacement fields for the incident harmonic Rayleigh SAWs with several typical wave lengths. (b) The closer view of the identical wave front at the stop location. (c) The evaluation of the total displacements of the incident Rayleigh SAWs along the top surface.

SAW forms in the far field. The red line represents the displacements of the start point (the first vertex of the first groove) of the graded grooved surface, while the blue line represents the displacements of one typical point (here, named stop point) at the stop location. One can see that the

maximum amplitudes of the vertical and horizontal displacements at the stop point are basically two times that of the start point due to the wave field enhancement at the stop location. What is worthy of notice is that there are two Gaussian packets for the start point and the two packets are

FIG. 4. (a) The transient vertical displacements and (b) the transient horizontal displacements for the applied source point, the start point of the graded grooved surface, and the stop point at the stop location as a function of time for the incident Gaussian pulse with a central wavelength $\lambda_R = 2.6a$. (c) Surface wave profiles at the special moments of the travel time to illustrate wave incidence, enhancement, and reflection. $T = \lambda_R/v_R$, where v_R is the linear phase velocity of the Rayleigh SAW.

nearly symmetric with respect to the packets (in blue) for the stop point. This clearly shows that the reflection of the incident Rayleigh SAW when travelling along the graded grooved surface. The reflection can also be directly seen by Gaussian surface wave profiles snapshotted at several special time points depicted in Fig. 4(c).

Here, we point out that the amplitude value of the second reflective packet is near that of the first incident packet especially in vertical displacement (the ratio of the vertical displacement amplitudes of the second reflective packet and the first incident one is 87.9%). This means that when a wave going through the graded grooved surface, the reflection and leak are very small due to the smooth changing of the surface property. Consequently, the incident Rayleigh surface wave is near perfectly reflected to the incident side after reaching the stop location. If the incident wave is with a shorter wavelength, the stop location will shift to the start point of the graded grooved surface. In this case, the reflective wave amplitude will be much closer to the incident wave amplitude because of the reduction of the wave travel distance. So, very high quality broadband spatial reflector for Rayleigh SAWs is obtained.

CONCLUSION

We show Rayleigh SAW dispersion relations and wave fields in the frequency and time domain modulated by the graded grooved surface, which can be easily made by engraving depth-gradually changed array of grooves on the flat surface. The phenomena of broad band gap, slow wave effect, spatial wave front enhancement are discussed to reveal the high quality broadband spatial reflection property of the graded grooved surface. The work shows that the structure can be used directly as high performance SAW filters and reflectors. The study also shows that the design of graded grooved surface has potential application on Rayleigh SAW manipulation like broadband direction emission and energy harvesting, etc.

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