Electrical Thermal Network for Direct Contact Membrane Distillation Modeling and Analysis

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Abstract—Membrane distillation is an emerging water distillation technology that offers several advantages compared to conventional water desalination processes. Although progress has been made to model and understand the physics of the process, many studies are based on steady-state assumptions or are computationally not appropriate for real time control. This paper presents the derivation of a novel dynamical model, based on analogy between electrical and thermal systems, for direct contact membrane distillation (DCMD). The proposed model captures the dynamics of temperature distribution and distilled water flux. To demonstrate the adequacy of the proposed model, validation with transient and steady-state experimental data is presented.

Index Terms—Dynamical modeling; electrical analogue; electrical-thermal network; direct contact membrane distillation (DCMD); water distillation.

I. INTRODUCTION

MEMBRANE distillation (MD) is a thermally driven distillation process combined with a membrane separation technique. In this process, hot feed stream is passed along one side of a hydrophobic membrane, which is only permeable for water vapor and retains liquid water, whereas the other side is kept at a lower (cooler) temperature. Due to this temperature difference across the membrane, water evaporates at the feed-membrane interface and the induced partial vapor pressure difference drives only water vapor through the membrane where vapor condenses at the cold permeate side of the membrane. MD has three main configurations each with its strengths and weaknesses. A schematic diagram of direct contact membrane distillation (DCMD) is shown in Fig. 1, where both the hot feed and cold permeate streams are in direct contact with the membrane. Another configuration is the air gap membrane distillation (AGMD), in which the permeate stream is separated from the membrane by a stagnant air gap. The third setup is vacuum membrane distillation (VMD), where vacuum is applied at the permeate side in order to enhance flux transfer. [1]

MD requires low grade heat which can be harvested from solar thermal energy and other renewable or waste heat sources [2]. Also, unlike the well-known reverse osmosis (RO), MD operates at a lower water pressure which in turns reduces the capital and operational costs. All these advantages make MD ideal for remote area desalination plants installations

with minimal infrastructure and less demanding membrane characteristics [3]. However, MD is faced with challenges that are yet to be addressed in order for this technology to be competitive with conventional desalination techniques.

In recent years, renewable energy has been integrated into many water desalination technologies, such as RO and MD. However, the unsteady nature of renewable energy sources poses a challenge which requires special attention when coupled with a desalination process. This effect has to be considered when modeling and designing control strategies for the process. While several model based control techniques have been successively applied on the RO process [4]–[7], MD still lacks dynamical models that can be adopted for control applications. Therefore, to ensure successful and economical operation of solar powered membrane distillation (SPMD), the development of a dynamical model for MD that can be used for control purposes is essential.

MD processes are distributed-parameter systems and, as a result, modeling them can be quit complicated. Previous studies on membrane distillation modeling fall into three main approaches. The first approach is based on empirical models that are dimensionless in space and steady in time, [8], [9]. The second approach accounts for the spacial variations of the process but it's steady in time, e.g. in [10] mass and energy conservation laws were used to develop models for DCMD, AGMD, and VMD. Whereas in [11] the steady-state laminar Navier-Stokes equation was used for momentum balance coupled with energy and mass balance equations to study DCMD. The third approach studies the evolution of the MD process with respect to time, e.g. in [12] a black box model based on neural networks was developed for SPMD. A more sophisticated work considered dynamic partial-differential equations (PDEs) to model AGMD as in [13] whereas in [14] the advection-diffusion equation was used to model DCMD process.

Most of the reported studies are limited by the steady-state assumption or computationally not suitable for real-time control and optimization. The lacking of dynamical models for MD that can be adapted for control applications is the motivation for this study. Unlike black box models, this work aims at developing a dynamical physical model that gives more insight into the process. Using analogy between electrical and thermal systems, a dynamical lumped-capacitance electrical-thermal network (ETN) model is derived for the heat and mass transfer processes in DCMD. The proposed ETN modeling yielded a system of ODEs with algebraic constraints, i.e. a differential algebraic equation (DAE) system. DCMD was chosen in this paper because of its simplicity and the availability of

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a DCMD experimental setup in our laboratory at the Water Desalination and Reuse Center (WDRC) at KAUST. Also, DCMD has the largest number of paper published in refereed journals, according to [1]. Moreover, the proposed model can easily be adopted for other MD configurations.

The proposed model accounts for the temperature gradient along the flow direction in both feed and permeate channels as well as for the thermal boundary layer at the membrane interfaces. In general, ETN models offer simplicity to study complex systems and facilitate simulations, at the same time they are reduced order versions of somewhat the equivalent PDE model. They also preserve the controllability and observability properties which could be lost in the discretization of the PDE due to the introduction of many states that are not observable in practice. Indeed electrical-analogy based methods have been used to describe the dynamical behavior of many industrial and biological systems such as heat exchangers [15], and the human cardiovascular system [16]. Moreover, it was shown that the transient diffusion phenomena and the heat transfer due to non-steady fluid flow can be described by an electrical analogue, see [17] and [18] respectively. Literature above motivated the method presented in this paper.

This paper is organized as follows. In Section II, the underlying concepts of mass transfer are given followed by a presentation of the general idea of the proposed approach to model heat transfer. Then, an electrical analogue circuit to DCMD system is designed in Section III. The final equations for the DCMD system based on the electrical analogy are derived in Section IV. The proposed model is validated against experimental data in Section V. Section VI summarizes the obtained results.

II. DIRECT CONTACT MEMBRANE DISTILLATION

A schematic diagram of the direct contact membrane distillation is shown in Fig. 1, (refer to Table I for parameters'

Fig. 1. Schematic diagram of DCMD module.

Fig. 2. Schematic diagram of the n^{th} DCMD cell.

description). In this configuration, hot water is passed along a hydrophobic membrane from one side, called the feed, and cold water flows in the counter direction along the other side, which is called the permeate. Water vapor is driven from the feed side across the membrane and into the permeate side by the induced partial vapor pressure difference. Both heat and mass transfer processes occur simultaneously as water evaporates at the feed-membrane interface and condenses at the permeate-membrane interface. As a result, the temperature at the membrane boundary layers differs from the bulk temperature of the feed and permeate streams, this is known as the temperature polarization effect. This effect reduces the mass transfer driving force, and as a result lowers the production rate of the system.

The mass and heat transfer processes are described in more details in the following subsections.

A. Mass transfer in DCMD

The transport phenomena is described by the classic gas permeation and heat transfer theories. The mass flux (J) in DCMD is related to the saturated vapor pressure difference across the membrane ΔP through the membrane mass transfer coefficient B_m as follows [19]

$$J = B_m \Delta P = B_m (P_{mf} - P_{mp}). \quad (1)$$

The mechanism dominating the mass transfer through the porous membranes depends on the pore radius (r) and the mean free path of the vapor molecules (λ). For membranes with pore radius in the range of $0.5\lambda < r < 50\lambda$, the membrane mass transfer coefficient is expressed as a parallel combination of Knudsen diffusion (B_{Kn}) and molecular diffusion (B_D) coefficients [8] given by

$$B_m = \frac{1}{1/B_{Kn} + 1/B_D}, \quad (2)$$

where:

$$B_{Kn} = \frac{4}{3} \frac{\varepsilon r}{\chi \delta} \sqrt{\frac{2m_w}{\pi RT}},$$

$$B_D = \frac{\varepsilon r}{\chi \delta} \frac{PD}{P_a} \frac{m_w}{RT}.$$

TABLE I
NOMENCLATURE

Symbol	Description	Unit
A_{cs}	Cross sectional area	(m ²)
A_m	Differential cell membrane area	(m ²)
B_m	Membrane mass transfer coefficient	(Kg/m ² sPa)
C	Thermal capacitance	(J/°C)
C_{elc}	Electrical capacitance per unit length	Farad/m (F/m)
c_p	Specific heat of water	(J/Kg°C)
D	Diffusivity of water vapor and air mixture	(m ² /s)
D_h	Hydraulic diameter	(m)
dz	Differential length along flow direction	(m)
$H_v[T]$	latent heat of vaporization at temperature T	(J/Kg)
h	Heat transfer coefficient	(W/m ² .°C)
I	Electrical current	Ampere (A)
J	Mass flux of distilled water	(Kg/m ² s)
k	Thermal conductivity	(W/m°C)
L	Thermal inductor	
l	Liter	
M	Mass flow rate (MFR)	(Kg/s)
m_w	Molecular mass of water	(Kg/mol)
P	Vapor pressure	(Pa)
P_a	The air pressure	(Pa)
$P_w^{sat}[T]$	Saturation vapor pressure of pure water at temperature T	(Pa)
Q	Heat transfer rate	Watt (W)
$\bar{R}$	Gas constant	(J/mol K)
R	Thermal resistance	(°C/W)
R_{eq}	Equivalent thermal resistance	(°C/W)
R_{elc}	Electrical resistance per unit length	(Ω/m)
T	Temperature	(°C or K)
V	Voltage	Volt (V)
v	Differential cell volume	(m ³)
x_{NaCl}	Molar fraction of NaCl salt	
Z	Impedance	(Ω)
Subscripts		
b	Bulk	
f	Feed	
g	gas (air)	
in	Inlet	
m	Membrane	
n	Index for cell number	
N	Total number of cells	
out	outlet	
p	Permeate	
term	terminating	
w	Water in liquid phase	
Greek letters		
δx	Differential length	(m)
δ	Membrane thickness	(m)
ε	Membrane porosity	
ξ	Membrane tortuosity	
ρ	Density	(Kg/m ³)

The saturated vapor pressure of pure water ($P_w^{sat}[T]$) as a function of temperature is given by the Antoine equation [20]:

$$P_w^{sat}[T] = \exp\left(23.1964 - \frac{3816.44}{T + 227.02}\right). \quad (3)$$

Dissolved salt in the feed stream reduces the saturated vapor pressure. Therefore, to compensate for this the following relation was proposed in [20]:

$$P_{mf} = (1 - x_{NaCl})(1 - 0.5x_{NaCl} - 10x_{NaCl}^2)P_w^{sat}[T_{mf}], \quad (4)$$

where x_{NaCl} is the mole fraction of NaCl in the feed stream. However, the permeate is pure and the saturated vapor at the membrane-permeate interface is $P_{mp} = P_w^{sat}[T_{mp}]$.

B. Heat Transfer in DCMD

To consider spacial variations on the temperature along the feed and permeate flow directions, the DCMD module is divided into control-volume cells. Then, based on the lumped-capacitance method, a dynamical model for heat transfer is developed using the energy conservation law. Fig. 2 depicts the n^{th} DCMD cell, where the bulk temperatures (T_{bfn} , T_{bpn}) are uniform throughout the cell except at the membrane interfaces due to the temperature polarization effect. Therefore, heat transfer takes place in three stages, by conduction and due to mass transfer. In the first stage, heat is transferred from the hot bulk feed stream to the boundary layer at the feed-membrane interface, the heat transfer rate is expressed as Q_{mf}

$$Q_{mf} = A_m(h_f(T_{bfn} - T_{mf_n}) + J_n c_p T_{bfn}).$$

The rate of change of the bulk feed stream energy in the n^{th} cell can now be expressed as

$$C_{bf} \frac{dT_{bfn}}{dt} = Q_{f_n} - Q_{f_{n+1}} - A_m(h_f(T_{bfn} - T_{mf_n}) + J_n c_p T_{bfn}), \quad (5)$$

where Q_{f_n} and $Q_{f_{n+1}}$ are the heat transfer rate into and out of the n^{th} feed cell respectively.

At the second stage, heat is transferred through the membrane via three mechanisms: the first mechanism (Q_{m1}) is the latent heat of vaporization (H_v) transported by the mass flux (J_n) through the n^{th} cell, expressed as:

$$Q_{m1} = A_m J_n H_v[T_{mf}] = B_m(P_{mf_n} - P_{mp_n})H_v[T_{mf}],$$

the latent heat of vaporization H_v in (KJ/Kg) is expressed as a function of temperature:

$$H_v[T] = -2.426T + 2503.$$

The second and third mechanisms are heat conduction through the membrane material and air trapped in the membrane pores which are combined in (Q_{m2}) as:

$$Q_{m2} = A_m h_m (T_{mf} - T_{mp}),$$

where the membrane heat transfer coefficient h_m is given as

$$h_m = \frac{k_g \varepsilon + k_m (1 - \varepsilon)}{\delta}.$$

Combining these mechanisms to write the energy balance at the membrane interfaces gives the following equations:

$$Q_{mf} = Q_{mp}, \quad (6)$$

where the heat transfer rate at the permeate-membrane interface (Q_{mp}) is expressed as:

$$Q_{mp} = A_m(h_p(T_{mp_n} - T_{bp_n}) + J_n c_p T_{mp_n}) = Q_{m1} + Q_{m2}. \quad (7)$$

Finally, the third stage of heat transfer where the water-vapor condenses at the permeate-membrane interface and heat

is transferred to the bulk permeate mass. The rate of change of energy for the bulk permeate stream is given by:

$$C_{bp} \frac{dT_{bpn}}{dt} = Q_{pn} - Q_{pn+1} + A_m (h_p (T_{mpn} - T_{bpn}) + J_n c_p T_{mpn}), \quad (8)$$

where Q_{pn} and Q_{pn+1} are also the heat transfer rate into and out of the n^{th} permeate cell respectively.

The two heat transfer coefficients at the membrane interfaces (h_f , h_p) can be calculated from empirical correlations. These correlations depend on the flow characteristic (laminar or turbulent) and vary accordingly. In this study, the following relation is used for both heat transfer coefficients [8]:

$$h = 0.13 Re^{0.64} Pr^{1/3} \frac{k_w}{D_h}, \quad (9)$$

where Re and Pr are the Reynolds and Prandtl numbers respectively.

The analysis done so far has not quantified the coupling terms between neighboring cells i.e. (Q_{fn} , Q_{fn+1} , Q_{pn} , and Q_{pn+1}). Equations (5)-(8) can be represented by an electrical analogue circuit. This representation has another advantage, besides facilitating simulation, it allows to couple neighboring cells by modeling the thermal inertia of the system, as will be discussed in the next two sections.

III. ELECTRICAL ANALOGY OF THE DCMD

The analogy between electrical and thermal elements can be derived from the basic laws of each system. Appendix A details the derivation process and a summary of the analogy is shown in Table IV.

Based on the equations derived for the n^{th} DCMD cell, an electrical analogue is constructed to simulate heat and mass transfer processes. The electrical analogue of the n^{th} cell of the DCMD module is shown in Fig. 3. The thermal capacity of the feed and permeate bulk sides is represented by C_{bf} and C_{bp} respectively. In each of the three stages of heat transfer discussed in Section II-B, the heat transfer rate by conduction is proportional to the temperature difference across the thermal resistances R_f , R_m , and R_p , whereas the heat transfer rate due to mass transfer is modeled by the current sources Q_{mf}^n , Q_{m1}^n , and Q_{mp}^n . This completes the analogy of heat transfer within the same cell, and in order to couple neighboring cells, the series impedances Z_f^n and Z_p^n are introduced. Apart from the series impedances, Table II details the expression of each element in the electrical analogue circuit. Another important part of DCMD electrical analogy is to consider the heat transfer by the feed and permeate inlet mass flow rates. Therefore, the network should be fed and terminated properly to account for the heat transfer rates into and out of the system. This will be discussed later as well as the series impedance.

A. Feeding and Terminating the network

The mass flow rate at the feed inlet (M_{fin}) supplies heat in (Watts) at the rate of:

$$Q_{fin} = M_{fin} c_p T_{fin}. \quad (10)$$

Fig. 3. Electrical analogue of the n^{th} cell of the DCMD module.

Therefore, the input impedance of the network should be $1/(M_{fin} c_p)$ in order for a voltage of T_{fin} to develop at the feed input terminal of the network. On the feed outlet terminal, the rate of heat leaving the system is given by:

$$Q_{fout} = M_{fout} c_p T_{fout}. \quad (11)$$

Therefore, the network should be terminated in resistance of $1/(M_{fout} c_p)$ to develop a voltage of T_{fout} across the terminating resistance.

Similar argument can be made for the permeate side in order to properly feed and terminate the network. The feed and permeate inlet temperatures are manipulated by the voltage sources (T_{fin} and T_{pin}), respectively. Whereas the feed and permeate inlet mass flow rates are proportional to the heat transfer rates at module's inlets.

B. The Series Impedance

In order to simulate the temperature gradient along the membrane in both the feed and permeate sides, adjacent cells are coupled together via the series impedances (Z_f^n and Z_p^n). Careful analysis should be done to design them in order to obtain the correct temperature drop from one cell to the next. As stated in [15], this impedance cannot be determined by direct analogy. However, it is clear that the value of this impedance should be a function of mass flow rates on both feed and permeate sides and the energy lost/received to/from the other side of DCMD module, i.e. the thermal resistance at the membrane interfaces and through the membrane. From the analysis of the constant jacket temperature heat exchanger analogue in [15], and some intuition, the feed side network can be simplified as shown in Fig. 4, where an equivalent

TABLE II
ELEMENTS OF THE ELECTRICAL THERMAL NETWORK FOR DCMD CELL

Element	Expression	Unite
R_f	$\frac{1}{A_m h_f}$	$^{\circ}\text{C}/\text{W}$
R_m	$\frac{1}{A_m h_m}$	$^{\circ}\text{C}/\text{W}$
R_p	$\frac{1}{A_m h_p}$	$^{\circ}\text{C}/\text{W}$
Q_{mf}^n	$A_m J_n c_p T_{bf_n}$	W
Q_{m1}^n	$A_m J_n H_v [T_{mf}]$	W
Q_{mp}^n	$A_m J_n c_p T_{mp_n}$	W
C_{bf}	$\rho_w c_p v_{bf}$	$\text{J}/^{\circ}\text{C}$
C_{bp}	$\rho_w c_p v_{bp}$	$\text{J}/^{\circ}\text{C}$

Fig. 4. Simplified Electrical analogue of the feed side.

shunt thermal resistance R_{eq} is introduced. Both the series impedance Z_f^n and the shunt resistance R_{eq} are unknown and to be identified empirically. It is apparent that the resistance R_{fz}^n is inversely proportional to R_{eq} and the square of the mass flow rate M_{fn} . The series resistance R_{fz}^n is to take the form reported in [15] as follows:

$$R_{fz}^n = \frac{1}{M_{fn}^2 c_p^2 R_{eq}}. \quad (12)$$

In order to achieve the correct response from the network, several values of the equivalent shunt thermal resistance R_{eq} were tested and verified against experimental data. Based on that, the following parametrization was found to give the best result:

$$Z_f^n = R_{fz}^n + j\omega L_f^n \quad (13)$$

where:

$$R_{fz}^n = \frac{1}{M_{fn}^2 c_p^2 (R_f + R_m + R_p)}$$

$$L_f^n = \frac{R_{fz}^2 C_{bf}}{4}.$$

The same procedure was used to obtain the parametrization for the permeate side series impedance (Z_p^n) as:

$$Z_p^n = R_{pz}^n + j\omega L_p^n \quad (14)$$

where:

$$R_{pz}^n = \frac{1}{M_{pn}^2 c_p^2 (R_f + 0.5R_m + R_p)}$$

$$L_p^n = \frac{R_{pz}^2 C_{bp}}{4}$$

Since heat is stored in the feed and permeate streams and transferred from one cell to the next by their movement, heat transfer along the flow direction is highly influenced by the flow inertia, which resists sudden changes to the flow momentum. Hence, it is important to consider the conservation of momentum in the MD modeling. Therefore, the inductive impedances (L_{fz}^n , L_{pz}^n) serve as the thermal inertia of the

Fig. 5. Completed electrical analogue of the DCMD module.

system to resist any sudden changes in the flow momentum and converts potential energy stored in the thermal capacitor to kinetic energy transferred by the stream mass flow rate and vice versa. This oscillatory behavior is damped by the heat transfer resistance. The energy balance equations are completed by taking into consideration the thermal inductor.

The proposed ETN model is now completed with all elements of the network analyzed and parameterized. In this model the states are the temperatures in each cell and the heat transfer rates into and out of the cell, the manipulated variables are the inlet feed and permeate water temperatures and flow rates, the controlled variables are the water mass fluxes in each cell which when averaged together represent the overall water mass flux of the DCMD module. In the next section, the equations for the DCMD electrical analogy will be driven based on the analysis that has been carried out in order to describe the mass and heat transfer processes.

IV. THE DAE MODEL OF DCMD

With the knowledge gained in the previous Sections II and III, the electrical laws are applied on the completed analogue circuit for DCMD, see Fig.5, to drive the dynamical model.

The coupling between neighboring cells can now be quantified by the current (in thermal analogy, current is the heat transfer rate) through the inductors. At the feed side, the rate of change of the heat transfer rate from the $n-1$ cell to the n^{th} cell is proportional to the temperature difference between them. Taking into consideration the series impedance Z_f^n , this is expressed as:

$$\frac{d Q_{fn}}{dt} = \frac{1}{L_f^n} T_{bf_{n-1}} - \frac{R_{fz}^n}{L_f^n} Q_{fn} - \frac{1}{L_f^n} T_{bf_n}. \quad (15)$$

Using Kirchoff's current law at the n^{th} feed node, it follows that the rate of change for the bulk feed temperature (T_{bf_n}) is

$$\frac{d T_{bf_n}}{dt} = \frac{1}{C_{bf}} Q_{fn} - \frac{1}{C_{bf}} \left(\frac{1}{R_f} + J_n A_m c_p \right) T_{bf_n} - \frac{1}{C_{bf}} Q_{f_{n+1}} + \frac{1}{C_{bf} R_f} T_{mf_n}. \quad (16)$$

Notice that (16) is equivalent to (5), but now (15) describes the dynamics of the heat transfer rates into and out of the n^{th} feed cell (Q_{f_n} and $Q_{f_{n+1}}$ respectively).

Similarly for the permeate side, the rate of change of the heat transfer rate (Q_{p_n}) is

$$\frac{d Q_{p_n}}{dt} = \frac{1}{L_p^n} T_{b_{p_{n-1}}} - \frac{R_{pz}^n}{L_p^n} Q_{p_n} - \frac{1}{L_p^n} T_{b_{p_n}}, \quad (17)$$

and the dynamics of the bulk permeate temperature ($T_{b_{p_n}}$) is

$$\begin{aligned} \frac{d T_{b_{p_n}}}{dt} = & \frac{1}{C_{bp}} Q_{p_n} - \frac{1}{C_{bp} R_p} T_{b_{p_n}} - \frac{1}{C_{bp}} Q_{p_{n+1}} \\ & + \frac{1}{C_{bp}} \left(\frac{1}{R_p} + J_n A_m c_p \right) T_{m_{p_n}}. \end{aligned} \quad (18)$$

The coupling between the feed and the permeate dynamics in the n^{th} cell is established through the algebraic constraints (6) and (7), which are written in residue form as

$$0 = \left(\frac{1}{R_f} + J_n A_m c_p \right) T_{b_{f_n}} - \frac{1}{R_f} T_{m_{f_n}} - \left(\frac{1}{R_p} + J_n A_m c_p \right) T_{m_{p_n}} + \frac{1}{R_p} T_{b_{p_n}}, \quad (19)$$

$$0 = \left(\frac{1}{R_m} + \frac{1}{R_p} + J_n A_m c_p \right) T_{m_{p_n}} - \frac{1}{R_p} T_{b_{p_n}} - J_n A_m H_v [T_{m_{f_n}}] - \frac{1}{R_m} T_{m_{f_n}}. \quad (20)$$

The outlet temperatures at the terminal cells of the feed and permeate analogue are also given by the algebraic equations, which are respectively:

$$0 = T_{f_{out}} - T_{p_{in}} - R_{f_{term}} Q_{f_{n+1}}, \quad (21)$$

$$0 = T_{p_{out}} - T_{f_{in}} + R_{p_{term}} Q_{p_1}. \quad (22)$$

The heat and mass transfer equations (15)-(22) represent a nonlinear differential-algebraic system. When considering N number of interconnected cells, the resultant equations can be expressed as a nonlinear descriptor system of the form

$$E \dot{X}(t) = F(X(t), u(t))X + B(u(t)), \quad (23)$$

where $X \in \mathbb{R}^{6N+4}$ represents the differential and algebraic states, $\dot{X}$ refers to the time derivative of the state vector, E is singular, $\text{rank}[E] < N$, and is called the mass matrix, $F(X(t), u(t)) \in \mathbb{R}^{6N+4 \times 6N+4}$ is nonlinear in the states and input, $B(u(t)) \in \mathbb{R}^{6N+4}$ represents the input channels into the system.

This block matrix representation of (23) is further detailed in Appendix B. Also, this representation is computationally efficient to solve the nonlinear DAE system and allows to vary the number of total cells (N). More details about the MATLAB implementation and the model performance and validation results are discussed in the next section.

V. MODEL VALIDATION

A. MATLAB implementation

This work was implemented with MATLAB [21] environment. The script can easily be adjusted to model different DCMD modules and experimental setups, i.e. the membrane

characteristics and the module dimensions, which is very important for process scale up studies and simulations. Also, The feed and permeate mass flow rates and inlet temperatures can follow any desired time varying profiles. The desired level of accuracy can be achieved by varying the number of total cells (N).

Two experimental data sets were used to validate the ETN proposed model. The first one was reported in [11], for a flat-sheet DCMD module with the following effective dimensions: Length of 0.4 m, width of 0.15 m, and feed/permeate channel thickness of 0.001 m. The same module configurations and the membrane characteristics were used in the simulation. This data set consisted of three steady state criteria which are discussed in the next three subsections. The second data set was provided by the Water Desalination and Reuse Center at KAUST to validate the dynamic response of the proposed model for a feed inlet temperature ramp up, which is further detailed in Subsection V-E. In this paper, a total of 10 cells were used to simulate the DCMD experimental setups and the model was solved using MATLAB `ode15s` solver which gave accurate and fast results.

B. Effect of linear velocity on distilled water flux

The distilled water flux is a function of the partial vapor pressure difference across the membrane, which is expressed as a function of temperature. The linear velocity inside the feed and permeate channels highly influence the heat transfer from the bulk stream to the membrane boundary layer, where higher velocity reduces the thickness of the thermal boundary layer and the temperature polarization effect is reduced. Therefore, it is important to study the relation between the feed/permeate stream velocity on the distilled water flux.

This effect was investigated for two feed inlet temperatures, 60°C and 40°C and permeate inlet temperature of 20°C for both cases. The feed and permeate stream velocities were increased from 0.17 m/s to 0.55 m/s and the distilled water flux was recorded. Fig. 6 presents the results obtained from the simulated electrical analogue compared to the experimental data reported in [11]. As it was expected, the flux increased with higher velocities. Also, higher values of flux are achieved with higher feed inlet temperatures. This is due to the exponential increase in the mass transfer driving force. It is clear that the modeling results agree with the experimental values of flux under the two different feed inlet temperatures with less than 10% difference between them.

C. Effect of linear velocity on feed and permeate outlet temperature

Another criterion to validate the model is to compare the feed and permeate outlet temperatures to the experimental measurements reported in [11]. Table III shows the outlet temperatures obtained from simulating the model with five feed and permeate linear velocities starting from 0.17 m/s to 0.55 m/s compared to the experimental data with these conditions: 1% NaCl concentration, and counter current flow setup. The model results are accurate to less than 3% error.

TABLE III

COMPARISON BETWEEN EXPERIMENTAL [11] AND ETN SIMULATION RESULTS OF FEED AND PERMEATE OUTLET TEMPERATURES. (COUNTER-CURRENT FLOW, FEED INLET TEMPERATURE OF 60°C, PERMEATE INLET TEMPERATURE OF 20°C, AND NaCl CONCENTRATION OF 1%).

velocity (m/s)	Feed outlet temperature °C			Permeate outlet temperature °C		
	Experimental	ETN modeling	% error	Experimental	ETN modeling	% error
0.17	50.1	49.58	1.04	29.1	28.62	1.65
0.28	52.1	52.36	0.11	27.4	28.08	2.48
0.39	53.9	54.06	0.3	26.3	26.67	1.40
0.5	55.1	55.16	0.11	25.8	25.52	1.09
0.55	55.3	55.54	0.43	25.4	25.09	1.22

Fig. 6. Flux as a function of feed linear velocity. Experimental data (Exp.) extracted from [11] compared to modeling results (Mod.) for different linear velocities.

Fig. 7. Temperature distribution along the flow in the feed and permeate sides. Feed linear velocity is 0.5 m/s, feed inlet temperature of 60°C, permeate inlet temperature of 20°C.

D. Temperature distribution along the flow direction

In order to further validate the proposed model, the temperature distribution along the flow direction was investigated and compared to the results obtained by solving the Navier-Stokes equation reported in [11]-Fig.8(b). Similar temperature distribution for both the feed and permeate channels by the proposed model. This shows that, despite the simplicity of the proposed model, accurate results can be obtained.

Fig. 8. Feed outlet temperature for a ramp feed inlet temperature from 30°C to 68°C. Black is experimental measurements, red is simulation results.

E. Dynamic response of the system

In collaboration with the Water Desalination and Reuse Center at KAUST, experimental data was collected from a flat sheet DCMD module to validate the dynamical response of the proposed model. The effective length, width, and channel thickness are 0.1m, 0.05m, and 3mm respectively. The experiment was designed to ramp up the feed inlet temperature from 30°C to 68°C with increment of 0.1°C per two minutes approximately, while maintaining the permeate inlet temperature constant at 20°C. Counter-current flow mode was used in this experiment, the feed flow rate was 90 l/h, and the permeate flow rate was 60 l/h and the feed stream was a sample from the Red Sea water. Throughout the whole experiment, the feed outlet temperature and the distilled water flux were electronically recorded.

Fig. 8 depicts the simulated and experimental feed outlet temperature, where it is clear that the simulation results closely match the experimental data. The simulation results of distilled water flux are shown in Fig. 9, where good agreement between the measured and simulated flux is shown.

VI. CONCLUSION

The work has demonstrated the adequacy of electrical thermal networks to model the behavior of direct contact membrane distillation. Based on the analogy between electrical and thermal systems, an electrical thermal network was designed. The model was then validated for both steady state and dynamic response by two sets of experimental data. In both cases, simulation results showed great agreement with

Fig. 9. Distilled water flux for a ramp feed inlet temperature from 30°C to 68°C. Black is experimental measurements, red is simulation results..

experimental measurements. This model provides an insight to the important physical parameters of MD, and therefore, it is appropriate to be used for fault diagnosis and detection and isolation. On the other hand, the proposed model is simple enough to be used for control design and process real time optimization.

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APPENDIX A

ELECTRICAL ANALOGUES FOR THERMAL ELEMENTS

The analogy can be derived from the basic equations of electrical and thermal systems. Let's start by considering the one dimensional heat conduction through an element of cross sectional area A_{cs} and thermal conductivity k and thickness of δx , the heat transfer rate is given by the Fourier's law as:

$$Q = -kA_{cs} \frac{\partial T}{\partial x}. \quad (24)$$

The difference of heat transfer rates between two parallel surfaces is equal to the heat absorbed to raise the temperature of the control volume, as given by:

$$\frac{\partial Q}{\partial x} = -\rho A_{cs} c_p \frac{\partial T}{\partial t}. \quad (25)$$

Substituting (24) into (25) gives the one dimensional heat diffusion equation as:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial x^2}. \quad (26)$$

In order to complete the analogy, a section of a uniform transmission line is considered as depicted in Fig. 10, where the line resistance and capacitance (R_{elc} , C_{elc} respectively) are

Fig. 10. Schematic diagram of a transmission line.

given per unit length. For some length δx , the total resistance and capacitance are $R_{elc}\delta x$ and $C_{elc}\delta x$ respectively.

Ohm's law gives:

$$V(x + \delta x) - V(x) = -IR_e \cdot \delta x. \quad (27)$$

Applying Kirchoff's current law gives:

$$I(x + \delta x) - I(x) = -C_e \cdot \delta x \frac{\partial V}{\partial t}. \quad (28)$$

Taking the limit as $\delta x \rightarrow 0$ for (27) and (28) respectively gives:

$$I = -\frac{1}{R_e} \frac{\partial V}{\partial x}, \quad (29)$$

$$\frac{\partial I}{\partial x} = -C_e \frac{\partial V}{\partial t}. \quad (30)$$

Combining equations (29) and (30) results in the telegraph equations:

$$\frac{\partial^2 V}{\partial x^2} = R_e C_e \frac{\partial V}{\partial t}. \quad (31)$$

Comparing (29) with (24) and (30) with (25), leads to the analogy between electrical and thermal systems given in Table IV.

APPENDIX B

DAE MODEL OF DCMD

This appendix details the block matrix representation of the descriptor system (23). In order to take advantage of a tridiagonal structure of (23), the state vector X combines both

TABLE IV
ELECTRICAL ANALOGUES OF THERMAL SYSTEM

Electrical			Thermal		
Element	Expression	Unit	Element	Expression	Unit
Voltage	V	V	Temperature	T	°C or K
Current	I	A	Heat transfer rate	Q	Watt (W=J/s)
Resistor	$R_{elc}\delta x$	Ω	Resistor	$\frac{\delta x}{kA_{cs}}$	°C/W
Capacitor	$C_{elc}\delta x$	F	Capacitor	$C = c_p m$	J/°C

differential and algebraic states in the following order:

$$X = \begin{bmatrix} Q_{f1} \\ T_{bf1} \\ \vdots \\ T_{bfN} \\ Q_{fN+1} \\ Q_{p1} \\ T_{bp1} \\ \vdots \\ T_{bpN} \\ Q_{pN+1} \\ T_{f_{out}} \\ T_{p_{out}} \\ T_{mf1} \\ \vdots \\ T_{mfN} \\ T_{mp1} \\ \vdots \\ T_{mpN} \end{bmatrix}. \quad (32)$$

As a result, the matrix E is the singular mass matrix, given as:

$$E = \begin{bmatrix} I_{4N+2 \times 4N+2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_{2N+2 \times 2N+2} \end{bmatrix}, \quad (33)$$

where I is the identity matrix and $\mathbf{0}$ is the zero matrix of appropriate size.

The matrix $F(X(t), u(t))$ represents the nonlinear dynamics of (23) and is composed of several blocks which account for the dynamics of the feed and permeate sides along with the algebraic coupling between them, the symbol Z refers to a block matrix for algebraic variables. $F(X(t), u(t))$ is given as

$$F(X(t), u(t)) = \begin{bmatrix} A_f & \mathbf{0} & Z_{f1} & Z_{f2} & \mathbf{0} \\ \mathbf{0} & A_p & Z_{p1} & \mathbf{0} & Z_{p2} \\ T_{fo} & T_{po} & I & \mathbf{0} & \mathbf{0} \\ Z_1 & Z_2 & \mathbf{0} & Z_3 & Z_4 \\ \mathbf{0} & Z_5 & \mathbf{0} & Z_6 & Z_7 \end{bmatrix} \quad (34)$$

Where:

- $A_f \in \mathbb{R}^{2N+1 \times 2N+1}$: A tridiagonal matrix representing the feed differential dynamics.
- $Z_{f1} \in \mathbb{R}^{2N+1 \times 2}$: Feed last cell
- $Z_{f2} \in \mathbb{R}^{2N+1 \times N}$: Coupling to the membrane-feed interface
- $A_p \in \mathbb{R}^{2N+1 \times 2N+1}$: A tridiagonal matrix representing the permeate differential dynamics.
- $Z_{p1} \in \mathbb{R}^{2N+1 \times 2}$: Permeate first cell
- $Z_{p2} \in \mathbb{R}^{2N+1 \times N}$: Coupling to the membrane-permeate interface
- $T_{fo} \in \mathbb{R}^{2 \times 2N+1}$: Outlet temperature of the feed
- $T_{po} \in \mathbb{R}^{2 \times 2N+1}$: Outlet temperature of the permeate
- $I \in \mathbb{R}^{2 \times 2}$: The identity matrix.
- $Z_1 \in \mathbb{R}^{N \times 2N+1}$: Sparse matrix
- $Z_2 \in \mathbb{R}^{N \times 2N+1}$: Sparse matrix
- $Z_3 \in \mathbb{R}^{N \times N}$: Diagonal matrix
- $Z_4 \in \mathbb{R}^{N \times N}$: Diagonal matrix
- $Z_5 \in \mathbb{R}^{N \times 2N+1}$: Sparse matrix
- $Z_6 \in \mathbb{R}^{N \times N}$: Diagonal matrix
- $Z_7 \in \mathbb{R}^{N \times N}$: Diagonal matrix

The matrix $B(u(t))$ is a nonlinear function of the manipulated variables (M_{fin} , M_{pin} , T_{fin} , T_{pin}). For convenient representation, the matrix is indexed at the left side:

$$B(u(t)) = \begin{matrix} & \begin{matrix} 1 \\ 2 \\ \vdots \\ 4N+2 \\ 6N+4 \end{matrix} & \begin{bmatrix} 8 & a_2 & M_{fin}^4 & 0 \\ & 0 & & 0 \\ & \vdots & & \vdots \\ & 0 & & 0 \\ & 0 & -8 & a_7 M_{pin}^4 \\ & \vdots & & \vdots \\ & 0 & & 0 \end{bmatrix} & \begin{bmatrix} T_{fin} \\ T_{pin} \end{bmatrix} \end{matrix} \quad (35)$$

The tridiagonal matrices A_f and A_p are given as

$$\text{tridiag}(A_f) = \begin{bmatrix} & & -4a_1 M_{fin}^2 & -8a_2 M_{fin}^4 \\ a_5 & -a_3 J_1 - a_4 & & -a_5 \\ 4a_2 M_{f1}^4 & -4 a_1 M_{f1}^2 & & -4a_2 M_{f1}^4 \\ \vdots & \vdots & & \vdots \\ 4a_2 M_{fN-1}^4 & -4a_1 M_{fN-1}^2 & & -4a_2 M_{fN-1}^4 \\ a_5 & -a_3 J_N - a_4 & & -a_5 \\ 8a_2 M_{fN}^4 & -4a_1 M_{fN}^2 & & \end{bmatrix}$$

$$\text{tridiag}(A_p) = \begin{bmatrix} & -4a_6 M_{p1}^2 & -8a_7 M_{p1}^4 & \\ a_9 & -a_8 & -a_9 & \\ 4a_7 M_{p2}^4 & -4a_6 M_{p2}^2 & -4a_7 M_{p2}^4 & \\ \vdots & \vdots & \vdots & \\ 4a_7 M_{pN}^4 & -4a_6 M_{pN}^2 & -4a_7 M_{pN}^4 & \\ a_9 & -a_8 & -a_9 & \\ 8a_7 M_{pin}^4 & -4a_6 M_{pin}^2 & & \end{bmatrix}$$

where $\text{tridiag}(A_f)$ and $\text{tridiag}(A_p)$ refers to the three diagonal vectors of A_f and A_p . Each column is a diagonal vector, starting from the lower, main, and then upper diagonal vector respectively.

The rest of the block elements of the matrix $F(X(t), u(t))$ are detailed below.

$$Z_{f1} = \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ -8a_2 M_{fN}^4 & 0 \end{bmatrix} \quad Z_{f2} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ a_4 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & a_4 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & a_4 \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

$$Z_{p1} = \begin{bmatrix} 0 & 8a_7 M_{p1}^4 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}$$

$$Z_{p2} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ a_{10} J_1 + a_8 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & a_{10} J_2 + a_8 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & a_{10} J_N + a_8 \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

$$T_{fo} = \begin{bmatrix} 0 & \dots & 0 & -\frac{1}{M_{fN} c_p} \\ 0 & \dots & 0 & 0 \end{bmatrix} \quad T_{po} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \frac{1}{M_{p1} c_p} & 0 & \dots & 0 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} 0 & \frac{1}{R_f} + a_{11} J_1 & 0 & \dots & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & \dots & 0 & \frac{1}{R_f} + a_{11} J_N & 0 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} 0 & \frac{1}{R_p} & 0 & \dots & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & \dots & 0 & \frac{1}{R_p} & 0 \end{bmatrix}$$

$$Z_3 = \begin{bmatrix} -\frac{1}{R_f} & 0 & \dots & 0 \\ 0 & -\frac{1}{R_f} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -\frac{1}{R_f} \end{bmatrix}$$

$$Z_4 = \begin{bmatrix} -\frac{1}{R_p} - a_{11} J_1 & 0 & \dots & 0 \\ 0 & -\frac{1}{R_p} - a_{11} J_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -\frac{1}{R_p} - a_{11} J_N \end{bmatrix}$$

$$Z_5 = \begin{bmatrix} 0 & -\frac{1}{R_p} & 0 & \dots & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & \dots & 0 & -\frac{1}{R_p} & 0 \end{bmatrix}$$

The diagonal vectors of Z_6 and Z_7 are $\text{diag}(Z_6)$ and $\text{diag}(Z_7)$ respectively, which are given as

$$\text{diag}(Z_6) = \begin{bmatrix} -\frac{1}{R_m} - A_m H_v [T_{mf1}] J_1 \\ -\frac{1}{R_m} - A_m H_v [T_{mf2}] J_2 \\ \vdots \\ -\frac{1}{R_m} - A_m H_v [T_{mfN}] J_N \end{bmatrix}$$

$$\text{diag}(Z_7) = \begin{bmatrix} \frac{1}{R_m} + \frac{1}{R_p} + a_{11} J_1 \\ \frac{1}{R_m} + \frac{1}{R_p} + a_{11} J_2 \\ \vdots \\ \frac{1}{R_m} + \frac{1}{R_p} + a_{11} J_N \end{bmatrix}$$

The parameters a_1 to a_{11} are

$$a_1 = \frac{c_p^2 R_{feq}}{C_{bf}} \quad a_2 = \frac{c_p^4 R_{feq}^2}{C_{bf}}$$

$$a_3 = \frac{A_m c_p}{C_{bf}} \quad a_4 = \frac{1}{C_{bf} R_f}$$

$$a_5 = \frac{1}{C_{bf}} \quad a_6 = \frac{c_p^2 R_{peq}}{C_{bp}}$$

$$a_7 = \frac{c_p^4 R_{peq}^2}{C_{bp}} \quad a_8 = \frac{1}{C_{bp} R_p}$$

$$a_9 = \frac{1}{C_{bp}} \quad a_{10} = \frac{A_m c_p}{C_{bp}}$$

$$a_{11} = A_m c_p$$

where

$$R_{feq} = R_f + R_m + R_p$$

$$R_{peq} = R_f + 0.5R_m + R_p$$

The mass flow rates (M_{fn} and M_{pn}) coming out of the n^{th} cell are indexed

$$M_{fn} = M_{fn-1} - A_m J_n; \quad n = 2, 3, \dots, N$$

$$M_{pn} = M_{pn+1} + A_m J_n; \quad n = 1, 2, \dots, N-1$$

where as

$$M_{f1} = M_{fin} - A_m J_1$$

$$M_{pN} = M_{pin} + A_m J_N$$

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