An efficient wave extrapolation method for tilted orthorhombic media using effective ellipsoidal models

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SUMMARY

The wavefield extrapolation operator for ellipsoidally anisotropic (EA) media offers significant cost reduction compared to that for the orthorhombic case, especially when the symmetry planes are tilted and/or rotated. However, ellipsoidal anisotropy does not provide accurate focusing for media of orthorhombic anisotropy. Therefore, we develop effective EA models that correctly capture the kinematic behavior of the wavefield for tilted orthorhombic (TOR) media. Specifically, we compute effective source-dependent velocities for the EA model using kinematic high-frequency representation of the TOR wavefield. The effective model allows us to use the cheaper EA wavefield extrapolation operator to obtain approximate wavefield solutions for a TOR model. Despite the fact that the effective EA models are obtained by kinematic matching using high-frequency asymptotic, the resulting wavefield contains most of the critical wavefield components, including the frequency dependency and caustics, if present, with reasonable accuracy. The methodology developed here offers a much better cost versus accuracy tradeoff for wavefield computations in TOR media, particularly for media of low to moderate complexity. We demonstrate applicability of the proposed approach on a layered TOR model.

INTRODUCTION

Wavefield extrapolation refers to the advancement of wavefield in small steps through space or time, using extrapolation operators that satisfy the wave equation. It is a key tool in seismic modeling, imaging, and full waveform inversion algorithms. For example, reverse time migration relies on accurate and efficient forward and backward extrapolation of waves in time.

The computational cost of a wavefield extrapolation algorithm is directly linked to the level of complexity in the description of the medium. Involving anisotropy, attenuation, or poroelasticity, or all of them in characterization of the medium can significantly increase the cost of solving the corresponding wave equation. However, embedding the kinematic and dynamic effects of these physical phenomena into a simpler model, requiring lesser number of parameters for characterization, can help considerably reduce the computational burden. The idea relies on finding simpler effective models that exhibit wave behavior similar to that present in the original model. Ideally, the wavefield obtained for the effective model would then match the phase and amplitude properties of the original model.

Seismologists have long recognized that sedimentary rocks cause anisotropic wave propagation behavior (Stoep, 1966). This anisotropic behavior is linked to thin layers of isotropic and transversely isotropic (TI) rocks of different properties. Due to gravity, the layers are naturally aligned horizontally, giving rise to a TI medium with vertical symmetry

axis (VTI). However, many sedimentary formations including sands, carbonates, and shales contain vertical or steeply dipping sets of fractures, causing a vertical orthorhombic (VOR) medium (Wild and Crampin, 1991; Schoenberg and Helbig, 1997). Therefore, orthorhombic symmetry is considered as the simplest realistic symmetry for many geophysical problems (Bakulin et al., 2000; Tsvankin et al., 2010). Moreover, tectonic forces and migration of salt bodies cause tilt and rotation of sedimentary layers. For a VTI medium, it results in a TI medium with tilted axis of symmetry (TTI). However, for a VOR medium, tilt and rotation result in a tilted orthorhombic (TOR) medium.

Alkhalifah et al. (2013) proposed the use of effective isotropic model for wave propagation in anisotropic media. The source-dependent effective isotropic model was generated by embedding the kinematic effects of the anisotropic medium into the isotropic one using the solution to the anisotropic eikonal equation. The effective isotropic model was then used to compute wavefields by employing the much cheaper isotropic wave extrapolation operator. The resulting wavefield yielded a perfect kinematic match for the fastest arriving wave, however, the later arrivals and the amplitude information suffered from significant inaccuracy (Ibanez-Jacome et al., 2014).

In this abstract, we propose the use of effective ellipsoidally anisotropic (EA) models for wave extrapolation in TOR media. The advantages of this scheme are two folds. First, the cost of solving the EA wave equation is similar to that for the isotropic case. Second, due to the increase in number of parameters needed to represent the effective EA model compared to the effective isotropic case, the dynamic and kinematic contents are much better matched by the effective model. In addition, the computed wavefields do not contain shear-wave numerical artifacts. We demonstrate these features through tests on a three layer TOR model.

THEORY

The TOR wave equation

Let $P(x, y, z, t)$ be the seismic wavefield at a location (x, y, z) at time t . Then, the acoustic wave equation for a VOR medium can be written as (Alkhalifah, 2003):

$$\begin{aligned} \frac{\partial^6 P}{\partial t^6} = & A \frac{\partial^6 P}{\partial x^2 \partial t^4} + B \frac{\partial^6 P}{\partial y^2 \partial t^4} + C \frac{\partial^6 P}{\partial z^2 \partial t^4} + D \frac{\partial^6 P}{\partial x^2 \partial y^2 \partial t^2} \\ & + E \frac{\partial^6 P}{\partial x^2 \partial z^2 \partial t^2} + F \frac{\partial^6 P}{\partial y^2 \partial z^2 \partial t^2} + G \frac{\partial^6 P}{\partial x^2 \partial y^2 \partial z^2}, \end{aligned} \quad (1)$$

where the following definitions have been used for simplification:

$$\begin{aligned} A = & v_1^2(1 + 2\eta_1), \quad B = v_2^2(1 + 2\eta_2), \quad C = v_0^2, \\ D = & v_1^2(1 + 2\eta_1) \left((1 + 2\eta_1) \gamma^2 v_1^2 - (1 + 2\eta_2) v_2^2 \right), \\ E = & -2\eta_1 v_1^2 v_0^2, \quad F = -2\eta_2 v_2^2 v_0^2, \\ G = & -v_0^2 v_1^2 \left((1 + 2\eta_1)^2 v_1^2 \gamma^2 - 2(1 + 2\eta_1) v_1 v_2 \gamma + (1 - 4\eta_1 \eta_2) v_2^2 \right). \end{aligned} \quad (2)$$

Efficient wave extrapolation in tilted orthorhombic media

Here, v_0 is the P-wave vertical velocity, v_1 and v_2 are the P-wave NMO velocities for horizontal reflectors in the $[x, z]$ and $[y, z]$ planes, respectively, η_1 and η_2 are the anellipticity anisotropic parameters in the $[x, z]$ and $[y, z]$ planes, respectively, and the δ parameter is defined in the $[x, y]$ plane (with respect to the x axis). Figure 1(a) shows schematic plot of a VOR model caused by parallel vertical cracks in a medium composed of horizontal layers. Tectonic forces and migration

Figure 1: Schematic plots showing a VOR model (a) caused by parallel vertical cracks in a background VTI medium, and a TOR model (b) due to the tilt and rotation represented using the dip angle θ , and the azimuthal angle ϕ .

of salt bodies may rotate the rocks and tilt the plane containing vertical cracks, giving rise to a TOR model. Figure 1(b) shows the tilt and rotation caused in the original VOR model. In Figure 1(b), θ denotes the layering dip angle measured with respect to the vertical, and ϕ is the azimuthal angle between the original xy -plane and the rotated one.

By computing spatial wavenumbers in the rotated coordinate system, we derive the TOR wave equation under the acoustic assumption to be:

$$\begin{aligned} \frac{\partial^6 P}{\partial t^6} = & A \frac{\partial^4}{\partial t^4}(\alpha) + B \frac{\partial^4}{\partial t^4}(\beta) + C \frac{\partial^4}{\partial t^4}(\gamma) + D \frac{\partial^2}{\partial t^2}(\alpha\beta) \\ & + E \frac{\partial^2}{\partial t^2}(\alpha\gamma) + F \frac{\partial^2}{\partial t^2}(\beta\gamma) + G(\alpha\beta\gamma), \end{aligned} \quad (3)$$

where

$$\begin{aligned} \alpha = & \cos^2 \phi \cos^2 \theta \frac{\partial^2 P}{\partial x^2} + \sin^2 \phi \cos^2 \theta \frac{\partial^2 P}{\partial y^2} + \sin^2 \theta \frac{\partial^2 P}{\partial z^2} \\ & + \sin 2\phi \cos^2 \theta \frac{\partial^2 P}{\partial x \partial y} + \cos \phi \sin 2\theta \frac{\partial^2 P}{\partial x \partial z} \\ & + \sin \phi \sin 2\theta \frac{\partial^2 P}{\partial y \partial z}, \\ \beta = & \sin^2 \phi \frac{\partial^2 P}{\partial x^2} + \cos^2 \phi \frac{\partial^2 P}{\partial y^2} - \sin 2\phi \frac{\partial^2 P}{\partial x \partial y}, \\ \gamma = & \cos^2 \phi \sin^2 \theta \frac{\partial^2 P}{\partial x^2} + \sin^2 \phi \sin^2 \theta \frac{\partial^2 P}{\partial y^2} + \cos^2 \theta \frac{\partial^2 P}{\partial z^2} \\ & + \sin 2\phi \sin^2 \theta \frac{\partial^2 P}{\partial x \partial y} - \cos \phi \sin 2\theta \frac{\partial^2 P}{\partial x \partial z} \\ & - \sin \phi \sin 2\theta \frac{\partial^2 P}{\partial y \partial z}. \end{aligned} \quad (4)$$

The coefficients A, B, C, D, E, F , and G are as defined by Equation 2.

Effective ellipsoidally anisotropic model

Setting $\eta_1 = \eta_2 = \theta = \phi = 0$ and $\gamma = v_2/v_1$ in the TOR wave equation 3, we get the acoustic wave equation for an EA medium:

$$\frac{\partial^2 P}{\partial t^2} = v_1^2 \frac{\partial^2 P}{\partial x^2} + v_2^2 \frac{\partial^2 P}{\partial y^2} + v_0^2 \frac{\partial^2 P}{\partial z^2}. \quad (5)$$

The extrapolation operator for EA media is much simpler and requires at least five times less computational cost than that for the TOR case. However, it doesn't provide accurate focusing for TOR media. Therefore, embedding the kinematic and dynamic effects of the TOR medium into an EA model can allow us to use the much cheaper EA extrapolation operator, without compromising the accuracy of the computed wavefield. In this abstract, we focus on matching the kinematic behavior of wave propagation in TOR media. The matching is obtained by an additional step of solving the TOR eikonal equation. The cost of this additional step is insignificant compared to computing the TOR wavefield solution (Alkhalifah et al., 2013).

In order to obtain the effective velocities appearing in the EA wave equation, first we write the TOR eikonal equation (Waheed et al., 2014):

$$\begin{aligned} & A(\widehat{\partial_x \tau})^2 + B(\widehat{\partial_y \tau})^2 + C(\widehat{\partial_z \tau})^2 + D(\widehat{\partial_x \tau})^2(\widehat{\partial_y \tau})^2 \\ & + E(\widehat{\partial_x \tau})^2(\widehat{\partial_z \tau})^2 + F(\widehat{\partial_y \tau})^2(\widehat{\partial_z \tau})^2 + G(\widehat{\partial_x \tau})^2(\widehat{\partial_y \tau})^2(\widehat{\partial_z \tau})^2 = 1, \end{aligned} \quad (6)$$

where $\widehat{\partial_x \tau}$, $\widehat{\partial_y \tau}$, and $\widehat{\partial_z \tau}$ denote traveltime derivatives in the rotated coordinate frame $[\widehat{x}, \widehat{y}, \widehat{z}]$. After some algebraic manipulations, we rewrite Equation 6 in terms of the traveltime derivatives in the unrotated coordinate frame as:

$$\begin{aligned} & \frac{(A \cos^2 \phi \cos^2 \theta + B \sin^2 \phi + C \cos^2 \phi \sin^2 \theta)}{c(\tau)} (\partial_x \tau)^2 \\ & + \frac{(A \sin^2 \phi \cos^2 \theta + B \cos^2 \phi + C \sin^2 \phi \sin^2 \theta)}{c(\tau)} (\partial_y \tau)^2 \\ & + \frac{(A \sin^2 \theta + C \cos^2 \theta)}{c(\tau)} (\partial_z \tau)^2 = 1, \end{aligned} \quad (7)$$

where

$$\begin{aligned} c(\tau) = & 1 - D(\cos \phi \cos \theta (\partial_x \tau) + \sin \phi \cos \theta (\partial_y \tau) + \sin \theta (\partial_z \tau))^2 \\ & \times (-\sin \phi (\partial_x \tau) + \cos \phi (\partial_y \tau))^2 \\ & - E(\cos \phi \cos \theta (\partial_x \tau) + \sin \phi \cos \theta (\partial_y \tau) + \sin \theta (\partial_z \tau))^2 \\ & \times (-\cos \phi \sin \theta (\partial_x \tau) - \sin \phi \sin \theta (\partial_y \tau) + \cos \theta (\partial_z \tau))^2 \\ & - F(-\sin \phi (\partial_x \tau) + \cos \phi (\partial_y \tau))^2 \\ & \times (-\cos \phi \sin \theta (\partial_x \tau) - \sin \phi \sin \theta (\partial_y \tau) + \cos \theta (\partial_z \tau))^2 \\ & - G(\cos \phi \cos \theta (\partial_x \tau) + \sin \phi \cos \theta (\partial_y \tau) + \sin \theta (\partial_z \tau))^2 \\ & \times (-\sin \phi (\partial_x \tau) + \cos \phi (\partial_y \tau))^2 \\ & \times (-\cos \phi \sin \theta (\partial_x \tau) - \sin \phi \sin \theta (\partial_y \tau) + \cos \theta (\partial_z \tau))^2 \\ & - A(\sin 2\phi \cos^2 \theta (\partial_x \tau) (\partial_y \tau) + \cos \phi \sin 2\theta (\partial_x \tau) (\partial_z \tau) \\ & + \sin \phi \sin 2\theta (\partial_y \tau) (\partial_z \tau)) + B(\sin 2\phi (\partial_x \tau) (\partial_y \tau)) \\ & - C(\sin 2\phi \sin^2 \theta (\partial_x \tau) (\partial_y \tau) - \cos \phi \sin 2\theta (\partial_x \tau) (\partial_z \tau) \\ & - \sin \phi \sin 2\theta (\partial_y \tau) (\partial_z \tau)). \end{aligned} \quad (8)$$

Efficient wave extrapolation in tilted orthorhombic media

The eikonal equation, under the acoustic assumption, for EA medium is given as:

$$v_1^2 (\partial_x \tau)^2 + v_2^2 (\partial_y \tau)^2 + v_0^2 (\partial_z \tau)^2 = 1. \quad (9)$$

By comparing Equations 7 and 9, we can define an effective EA model that captures the kinematic effects due to the TOR medium. The effective velocities for EA medium are given as:

$$\begin{aligned} v_{1,eff} &= \sqrt{\frac{(A \cos^2 \phi \cos^2 \theta + B \sin^2 \phi + C \cos^2 \phi \sin^2 \theta)}{c(\tau)}}, \\ v_{2,eff} &= \sqrt{\frac{(A \sin^2 \phi \cos^2 \theta + B \cos^2 \phi + C \sin^2 \phi \sin^2 \theta)}{c(\tau)}}, \\ v_{0,eff} &= \sqrt{\frac{(A \sin^2 \theta + C \cos^2 \theta)}{c(\tau)}}, \end{aligned} \quad (10)$$

where $v_{0,eff}$ is the effective P-wave vertical velocity, $v_{1,eff}$, and $v_{2,eff}$ are the effective P-wave NMO velocities in the $[x, z]$ and $[y, z]$ planes, respectively. The coefficients A, B , and C are as defined by Equation 2.

Once we obtain a solution to the TOR eikonal equation 6, we can compute the right hand side function $c(\tau)$ using Equation 8. Then, we compute the effective velocities using Equation 10, allowing us to use the much cheaper EA wave equation with effective velocities:

$$\frac{\partial^2 P}{\partial t^2} = v_{1,eff}^2 \frac{\partial^2 P}{\partial x^2} + v_{2,eff}^2 \frac{\partial^2 P}{\partial y^2} + v_{0,eff}^2 \frac{\partial^2 P}{\partial z^2}. \quad (11)$$

The effective velocities are a function of the source position and will vary with the location of the source.

Effective tilted ellipsoidally anisotropic model

In a similar manner, we can obtain an effective tilted ellipsoidally anisotropic (TEA) model for wave extrapolation in TOR media. Doing the necessary algebra as outlined above, we obtain the effective TEA velocities as:

$$\begin{aligned} v'_{1,eff} &= v_1 \sqrt{\frac{1+2\eta_1}{c'(\tau)}}, \quad v'_{2,eff} = v_2 \sqrt{\frac{1+2\eta_2}{c'(\tau)}}, \\ v'_{0,eff} &= \frac{v_0}{\sqrt{c'(\tau)}}, \end{aligned} \quad (12)$$

where

$$\begin{aligned} c'(\tau) &= 1 - D (\cos \phi \cos \theta (\partial_x \tau) + \sin \phi \cos \theta (\partial_y \tau) + \sin \theta (\partial_z \tau))^2 \\ &\quad \times (-\sin \phi (\partial_x \tau) + \cos \phi (\partial_y \tau))^2 \\ &\quad - E (\cos \phi \cos \theta (\partial_x \tau) + \sin \phi \cos \theta (\partial_y \tau) + \sin \theta (\partial_z \tau))^2 \\ &\quad \times (-\cos \phi \sin \theta (\partial_x \tau) - \sin \phi \sin \theta (\partial_y \tau) + \cos \theta (\partial_z \tau))^2 \\ &\quad - F (-\sin \phi (\partial_x \tau) + \cos \phi (\partial_y \tau))^2 \\ &\quad \times (-\cos \phi \sin \theta (\partial_x \tau) - \sin \phi \sin \theta (\partial_y \tau) + \cos \theta (\partial_z \tau))^2 \\ &\quad - G (\cos \phi \cos \theta (\partial_x \tau) + \sin \phi \cos \theta (\partial_y \tau) + \sin \theta (\partial_z \tau))^2 \\ &\quad \times (-\sin \phi (\partial_x \tau) + \cos \phi (\partial_y \tau))^2 \\ &\quad \times (-\cos \phi \sin \theta (\partial_x \tau) - \sin \phi \sin \theta (\partial_y \tau) + \cos \theta (\partial_z \tau))^2. \end{aligned} \quad (13)$$

The θ and ϕ models remain the same as for the original TOR medium. Once we compute the velocities using Equation 12, we can compute the approximate wavefield solution for the

	Layer 1	Layer 2	Layer 3
v_0	1.6 km/s	1.9 km/s	2.2 km/s
v_1	1.8 km/s	2.3 km/s	2.5 km/s
v_2	2.0 km/s	2.4 km/s	2.6 km/s
η_1	0.1	0.15	0.2
η_2	0.15	0.2	0.3
θ	20°	30°	40°
ϕ	20°	5°	-10°

Table 1: Medium parameters for a three layer TOR model. Each layer has dimensions 1 km $\times$ 1 km $\times$ 1 km. Layer 1 refers to the top layer, layer 2 is the middle layer, while layer 3 is the bottom layer.

TOR medium using wave equation for TEA medium with effective velocities:

$$\begin{aligned} \frac{\partial^2 P}{\partial t^2} &= \left(v_{1,eff}^2 \cos^2 \phi \cos^2 \theta + v_{2,eff}^2 \sin^2 \phi + v_{0,eff}^2 \cos^2 \phi \sin^2 \theta \right) \frac{\partial^2 P}{\partial x^2} \\ &\quad + \left(v_{1,eff}^2 \sin^2 \phi \cos^2 \theta + v_{2,eff}^2 \cos^2 \phi + v_{0,eff}^2 \sin^2 \phi \sin^2 \theta \right) \frac{\partial^2 P}{\partial y^2} \\ &\quad + \left(v_{1,eff}^2 \sin^2 \theta + v_{0,eff}^2 \cos^2 \theta \right) \frac{\partial^2 P}{\partial z^2} \\ &\quad + \left(v_{1,eff}^2 \sin 2\phi \cos^2 \theta - v_{2,eff}^2 \sin 2\phi + v_{0,eff}^2 \sin 2\phi \sin^2 \theta \right) \frac{\partial^2 P}{\partial x \partial y} \\ &\quad + \left(v_{1,eff}^2 - v_{0,eff}^2 \right) \left(\cos \phi \frac{\partial^2 P}{\partial x \partial z} + \sin \phi \frac{\partial^2 P}{\partial y \partial z} \right) \sin 2\theta. \end{aligned} \quad (14)$$

The TEA wave extrapolation operator is also cheaper to solve compared to the one for TOR medium.

NUMERICAL TESTS

In this section, we test the accuracy properties of the effective EA model in approximating the wavefield solution for the TOR medium. We consider a three layer TOR model with parameters shown in table 1. Each layer is flat and has dimensions of 1 km $\times$ 1 km $\times$ 1 km. A constant value of $\gamma = 1$ is used in every layer.

Figure 2 shows wavefield snapshots (crossline, inline, and depth slices) at 0.5 second obtained using the expensive TOR wavefield extrapolator for the three layer model using the algorithm by Song and Alkhalifah (2013). A grid spacing of 20 m is used in both directions, while the peak frequency of the source wavelet is 20 Hz. We also overlay the TOR eikonal solution (in red) at 0.5 second. As expected, the first-break of the wavefield correctly matches the eikonal solution.

However, since computing the wavefield solution using a TOR wavefield solver is computationally cost prohibitive, we use a much cheaper EA wave extrapolator employing effective EA velocities ($v_{0,eff}, v_{1,eff}, v_{2,eff}$), computed using Equation 10. Figure 3 shows the wavefield snapshots at 0.5 second obtained by solving the EA wave equation 11 using effective velocities. Despite ignoring several model parameters ($\eta_1, \eta_2, \theta, \phi, \gamma$), the effective EA wavefield solution matches the TOR eikonal solution (in red), as the kinematic effects due to these parameters have been captured by the effective EA model.

Efficient wave extrapolation in tilted orthorhombic media

Figure 2: Wavefield snapshots at 0.5 second using the TOR wavefield extrapolator. Also mapped on top are traveltime contours (in red) for: the inline slice at $y = 1.5$ km (a), the crossline slice at $x = 1.5$ km (b), and the depth slice at $z = 1.5$ km (c).

Figure 3: Wavefield snapshots at 0.5 second using the EA wavefield extrapolator employing the obtained effective EA model. Also mapped on top are traveltime contours (in red): the inline slice at $y = 1.5$ km (a), the crossline slice at $x = 1.5$ km (b), and the depth slice at $z = 1.5$ km (c).

In Figure 4, we plot the difference between the velocities in the original model (v_0, v_1, v_2) with the respective effective velocities ($v_{0,eff}, v_{1,eff}, v_{2,eff}$). The difference is attributable mainly to the anisotropy and the tilt ignored by the EA model. Figure 5 shows traces obtained from the effective model based wave extrapolation (dashed red) to those obtained using the costlier TOR wavefield solver (solid blue). Notice that the effective EA based extrapolator matches the kinematic information pretty well. Also, the amplitude fit has remarkable accuracy, considering that the effective EA model based wave extrapolation is several times cheaper than TOR wave extrapolation. In addition, from Figure 5(c), we see that the later arrivals are also matched with reasonable accuracy.

CONCLUSIONS

Computing effective EA model by fitting kinematic features corresponding to the TOR medium allows us to use the much cheaper EA wave extrapolation operator. Using solution to the TOR eikonal equation for matching ensures the kinematics of first arriving wave are matched perfectly. For low to moderately complex media, kinematics of the later arrivals and amplitude information also match with reasonable accuracy. The amplitude fit is expected to be better for effective TEA model, however the cost is slightly higher compared to the effective EA case. Therefore, the formulations developed here allows us a better cost versus accuracy tradeoff in choosing between using the effective EA or the effective TEA medium for wavefield computations in TOR media. Despite using high

Figure 4: Difference in velocities (in km/s) for the TOR and the effective EA model: (a) $v_0 - v_{0,eff}$, (b) $v_1 - v_{1,eff}$, and (c) $v_2 - v_{2,eff}$.

Figure 5: Traces from TOR wavefield extrapolator (solid blue), compared with the traces obtained using the effective EA model based wave extrapolator (dashed red) at (a) $y = 1.1$ km, $z = 1.5$ km, (b) $x = 1.5$ km, $z = 1.3$ km, and (c) $x = 1.5$ km, $y = 1.5$ km.

frequency asymptotics for matching the kinematics of wavefields, the resulting effective model includes most of the critical wavefield components, including frequency dependency and caustics, if present. In addition to the numerical test shown here, several interesting examples including an effective TEA model test will be presented at the 84th SEG annual meeting.

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