Multisource Least-squares Reverse Time Migration

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ABSTRACT

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Least-squares migration has been shown to be able to produce high quality migration images, but its computational cost is considered to be too high for practical imaging. In this dissertation, a multisource least-squares reverse time migration algorithm (LSRTM) is proposed to increase by up to 10 times the computational efficiency by utilizing the blended sources processing technique.

There are three main chapters in this dissertation.

In Chapter 2, the multisource LSRTM algorithm is implemented with random time-shift and random source polarity encoding functions. Numerical tests on the 2D HESS VTI data show that the multisource LSRTM algorithm suppresses migration artifacts, balances the amplitudes, improves image resolution, and reduces crosstalk noise associated with the blended shot gathers. For this example, multisource LSRTM is about three times faster than the conventional RTM method. For the 3D example of the SEG/EAGE salt model, with comparable computational cost, multisource LSRTM produces images with more accurate amplitudes, better spatial resolution, and fewer migration artifacts compared to conventional RTM. The empirical results suggest that the multisource LSRTM can produce more accurate reflectivity images than conventional RTM does with similar or less computational cost. The caveat is that LSRTM image is sensitive to large errors in the migration velocity model.

In Chapter 3, the multisource LSRTM algorithm is implemented with frequency-selection encoding strategy and applied to marine streamer data, for which traditional random encoding functions are not applicable. The frequency-selection encoding functions are delta functions in the frequency domain, so that all the encoded shots have unique non-overlapping frequency content. Therefore, the receivers can distinguish the wavefield from each shot according to the frequencies. With the frequency-selection encoding method, the computational efficiency of LSRTM is increased so that its cost is comparable to conventional RTM in the examples of the Marmousi2 model and a field data set from the Gulf of Mexico. With more iterations, the LSRTM image quality is further improved. The numerical results suggest that LSRTM with frequency-selection is an efficient method to produce better reflectivity images than conventional RTM.

In Chapter 4, I present an interferometric method for extracting the diffraction signals that emanate from diffractors, also denoted as seismic guide stars. The signal-to-noise ratio of these interferometric diffractions is enhanced by $\sqrt{N}$, where N is the number of source points coincident with the receiver points. Thus, diffractions from subsalt guide stars can then be rendered visible and so can be used for velocity analysis, migration, and focusing of subsalt reflections. Both synthetic and field data records are used to demonstrate the benefits and limitations of this method.

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LIST OF ABBREVIATIONS

KM	Kirchhoff migration
LSM	least-squares migration
LSRTM	least-squares reverse time migration
RTM	reverse time migration
SNR	signal-to-noise ratio
VTI	vertical transversely isotropic

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Chapter 1

Introduction

The goal of seismic imaging is to find out underground structures based on seismic data observed on the surface or within boreholes, given a migration velocity (Claerbout, 1971). Mathematically, the problem can be thought as finding an reflectivity model to explain the observed data with a forward modeling operator. The forward modeling operator is built based on the wave equation and the given velocity model. However, it is prohibitively expensive to solve the problem directly for real data examples. Therefore, conventional migration applies the adjoint of the forward modeling operator to the observed data to produce an image, where the Hessian of the problem is approximated to be an identity matrix. In reality, the conventional migration image usually contains migration artifacts caused by a limited recording aperture and/or coarse source and receiver sampling. To improve the image quality, Nemeth et al. (1999) proposed to solve the problem by an iterative method and named the method least-squares migration (LSM), and it has been shown to have the following advantages: (1) it can reduce migration artifacts; (2) it can balance the amplitudes of the reflectors; and (3) it can improve the resolution of the migration images.

Least-squares migration was originally implemented with Kirchhoff migration (KM) method (Nemeth et al., 1999; Duquet et al., 2000), but was later developed for phase shift migration algorithms (Kaplan et al., 2010; Huang and Schuster, 2012a). In this dissertation, I propose to implement LSM with reverse time migration (RTM)

(Baysal et al., 1983; Whitmore, 1983; McMechan, 1983), where the Green’s functions are calculated by finite-difference solution to the full wave equation instead of asymptotic Green’s function in Kirchhoff migration (Bleistein, 1987), or one-way solution in one-way wave equation migration (Claerbout, 1985). The advantages include: (1) there is no high-frequency approximation; (2) it has no dip limitations; (3) once the known boundaries, e.g., a salt boundary, are embedded in the migration velocity model, RTM can correctly migrate multiples such as prism waves (Jones et al., 2007; Lu et al., 2008; Malcolm et al., 2009); and (4) phase shifts associated with caustics are taken into account when solving the two-way wave equation.

However, least-squares migration is usually considered to be too expensive for practical use. In this dissertation, I propose a new algorithm to combine the blended sources processing technique with least-squares reverse time migration (LSRTM) to increase its computational efficiency. To adapt this method for data recorded with a marine streamer geometry, frequency-selection encoding can be used instead of random time-shift encoding. In the following chapters, the multisource least-squares reverse time migration algorithm is tested with synthetic and real data examples to illustrate its advantages.

1.1 Chapter 2: multisource least-squares reverse time migration

The least-squares migration method (Lailly, 1984; Cole and Karrenbach, 1992; Schuster, 1993; Nemeth et al., 1999; Duquet et al., 2000) has been shown to sometimes produce migration images with better quality than those computed by conventional migration. When least-squares migration is implemented with the RTM method (Tang and Biondi, 2009; Dai and Schuster, 2010a; Dai et al., 2010; Wong et al., 2011; Dai et al., 2012), it can reduce not only the acquisition footprint but also the artifacts

in the RTM image, while enhancing the image resolution.

In this chapter, I propose an efficient multisource least-squares reverse time migration algorithm for a vertical transversely isotropic (VTI) medium. With blended sources processing, many conventionally acquired shot gathers are phase-encoded and blended together to form supergathers to reduce the computational cost and I/O burden of migration (Romero et al., 2000; Dai and Schuster, 2009; Krebs et al., 2009; Dai and Schuster, 2010b). However, blended sources processing introduces crosstalk noise, which needs to be removed from the final migration images. Our synthetic results demonstrate that LSRTM can mitigate both migration artifacts and crosstalk noise introduced by phase encoding, balance the amplitudes of reflectors, and improve the spatial resolution of the image. Moreover, the efficiency of multisource LSRTM can be significantly higher than conventional RTM depending on the number of shots encoded in one supergather, the number of migration operations at every iteration, and the number of iterations needed for an image of acceptable quality.

1.2 Chapter 3: Least-squares reverse time migration of marine data with frequency-selection encoding

The random encoding functions used by Romero et al. (2000); Krebs et al. (2009); Schuster et al. (2011) and Dai et al. (2012), cannot be applied to a seismic survey with marine streamer geometry (Routh et al., 2011; Huang and Schuster, 2012a), because the calculated synthetic data are also of fixed spread geometry, but the observed data are of marine streamer geometry. As a remedy, Routh et al. (2011) proposed a cross-correlation based misfit functional to mitigate the effect of recording pattern mismatch. Alternatively, Huang and Schuster (2012a) proposed a frequency-

selection encoding strategy for least-squares phase shift migration, which is applicable to marine data.

The frequency-selection encoding strategy can also be applied with least-squares reverse time migration, where the time-domain simulation are performed with a single frequency harmonic source instead of the conventional broadband source. Nihei and Li (2006) proposed to use a time-domain finite-difference method to obtain the single frequency response of a point source in a velocity model. Compared to the conventional frequency domain method, their method has significantly lower arithmetic complexity and storage requirements in the 3D case.

In this chapter, the frequency-selection encoding method is applied with least-squares reverse time migration and tested on the Marmousi2 model to show that LSRTM can produce better images than conventional RTM with comparable cost for marine datasets.

1.3 Chapter 4: Super-virtual Interferometric Diffractions as Guide Stars

A significant problem with the seismic imaging method is that subsalt reflections are severely defocused by the strong velocity contrasts and the irregular geometries of salt-sediment interfaces. Upgoing reflection energy is present in the data, but cannot be easily detected in the shot records as coherent arrivals with hyperbolic-like moveout trajectories. This means that velocity estimation methods such as traveltimes tomography cannot be used and, others, such as migration velocity analysis or full-wave inversion will fail unless an accurate starting velocity model is used. Is there another means for estimating subsalt velocities when the other methods fail?

This paper proposes interferometric extraction of subsalt diffractions, with the possibility that they can also be used as migration operators or for velocity analysis.

The key idea is that, similar to surface waves or refractions, 2D subsalt diffractions are associated with stationary source points all along the source line. Thus, application of interferometry can enhance the signal-to-noise ratio of this diffraction energy by $\sqrt{N}$, where N is the number of source points. This means that undetectable diffractions in the shot records can be enhanced, which can then be used to guide velocity analysis and focusing of subsalt reflections. I refer to such diffractors as guide stars because they, similar to VSP data, can be used as Green's functions to build natural migration operators (Schuster, 2002; Brandsberg-Dahl et al., 2007), or estimate migration velocity (Berkhout et al., 2001; Landa et al., 1987). Similar to guide stars used by astronomers for correcting the optical distortion of the atmosphere, diffraction based migration operators can be used to guide the proper focusing of subsalt reflection energy to their points of origin beneath the salt. Both synthetic and field data records are used to demonstrate the benefits and limitations of this method.

Chapter 2

Multisource Least-squares Reverse Time Migration

Least-squares migration has been shown to be able to improve image quality over conventional migration method, but its computational cost is often too high to be practical. In this chapter, I develop two numerical schemes to implement least-squares migration with the reverse time migration method and the blended source processing technique to increase computation efficiency. By iterative migration of supergathers that consist of a sum of many phase-encoded shot gathers, the image quality is enhanced and the crosstalk noise associated with the encoded shot gathers is reduced. Numerical tests on the 2D HESS VTI data show that the multisource LSRTM algorithm suppresses migration artifacts, balances the amplitudes, improves image resolution, and reduces crosstalk noise associated with the blended shot gathers. For this example, the multisource LSRTM is about three times faster than the conventional RTM method. For the 3D example of the SEG/EAGE salt model, with comparable computational cost, multisource LSRTM produces images with more accurate amplitudes, better spatial resolution, and fewer migration artifacts compared to conventional RTM. The empirical results suggest that the multisource LSRTM can produce more accurate reflectivity images than conventional RTM does with similar or less computational cost. The caveat is that LSRTM image is sensitive to large errors in

the migration velocity model.

2.1 Introduction

Conventional migration (Claerbout, 1971) computes the reflectivity image by applying the adjoint operator just once to the data (Plessix, 2006) rather than its iterative application as required by waveform inversion (Tarantola, 1984). Migration can also be thought as the first iteration of iterative full wave inversion, but the Hessian of the misfit functional is not computed, because it is too large to store and invert. Therefore, a variety of approximations have been made to improve the migration images by approximating the inverse Hessian matrix with a fully populated inverse matrix (Hu et al., 2001; Guitton, 2004; Plessix and Mulder, 2004; Yu et al., 2006; Aoki and Schuster, 2009; Wang and Yang, 2010). The simplest approximation is the reciprocal of the diagonal matrix (Beydoun and Mendes, 1989; Rickett, 2003) that is applied to the image to compensate for uneven illumination. This is computationally inexpensive, but it only compensates for amplitude distortion but does not correct for aliasing artifacts or strong footprint noise.

To indirectly account for the inverse Hessian matrix, an iterative conjugate gradient method can be used to solve either the linear (Lailly, 1984; Nemeth et al., 1999; Duquet et al., 2000) or non-linear (Tarantola, 1984; Mora, 1987) optimization problem. In this way, the effects of the source wavelet, limited recording aperture, geometric spreading, etc, are taken into account to produce images with reduced acquisition footprints, balanced amplitudes and improved resolution. Following the work of Lailly (1984) and Beydoun and Mendes (1989), Nemeth et al. (1999) proposed an iterative linear inversion method they denoted as least-squares migration (LSM). They used an operator which is the adjoint to the Kirchhoff migration operator as the forward modeling operator and tested their algorithm with both synthetic data and

field data. Duquet et al. (2000) tested a Kirchhoff LSM method but they also used a regularization term which penalized the difference in images from those estimated in different offset gathers. Their regularization was effective even with migration velocity errors up to 20%.

The inverse problem can also be formulated in the model space instead of the data space to avoid large I/O demands associated with 3D seismic data. To reduce computational cost, Hu et al. (2001) and Yu et al. (2006) computed the inverse Hessian in the wavenumber domain by assuming a locally layered medium to deconvolve the migration Green’s function. Mulder and Plessix (2004a) derived both linear and quasi-linear approaches for LSM with the two-way wave equation, but their implementation was in the frequency domain. Another means for reducing the cost of computing the Hessian inverse was proposed by Tang (2009) who used phase-encoding technique to efficiently calculate the Hessian for a targeted area and performed the inversion in model space. The images he obtained showed high resolution and balanced amplitudes.

In this chapter, I propose an efficient multisource least-squares reverse time migration algorithm for a VTI medium and test it on synthetic traces generated for the 2D HESS VTI and 3D SEG/EAGE salt models. The acquisition geometries are for land data, and the application to marine data geometries requires a different phase encoding scheme (Huang and Schuster, 2012b). In order to increase computational efficiency, the multisource phase-encoding technique (Romero et al., 2000; Dai and Schuster, 2009; Krebs et al., 2009; Dai and Schuster, 2010b) employs encoding functions with random time shifts and random source polarities. Our synthetic results demonstrate that LSRTM can mitigate both migration artifacts and crosstalk noise introduced by phase encoding, balance the amplitudes of reflectors, and improve the spatial resolution of the image. Moreover, the efficiency of multisource LSRTM can be significantly higher than conventional RTM depending on the number of shots

encoded in one supergather, the number of migration operations at every iteration, and the number of iterations needed for an image of acceptable quality. However, these results are sensitive to large errors in the migration velocity model.

This chapter is organized into four parts. The first is this introduction, which is followed by theory and the description of the numerical scheme for implementing LSRTM. The third part presents the numerical results for both the 2D HESS and the 3D SEG/EAGE salt models. In the end, a short summary is provided.

2.2 Theory

The Helmholtz equation is used to derive the forward modeling operators with and without the Born approximation. Seismic inversion with the Born approximation is denoted as linear inversion while the one without the Born approximation is denoted as quasi-linear inversion.

2.2.1 Modeling

Given a background slowness model $\mathbf{s}_o$, the Green's function for the Helmholtz equation is the solution of

$$[\nabla^2 + \omega^2 \mathbf{s}_o(\mathbf{x})^2] G_o(\mathbf{x}|\mathbf{x}_s) = -\delta(\mathbf{x} - \mathbf{x}_s), \quad (2.1)$$

where $G_o(\mathbf{x}|\mathbf{x}_s)$ is the Green's function associated with the background slowness $\mathbf{s}_o$ and an impulsive point source at $\mathbf{x}_s$, $\mathbf{x}$ is the listening location, and ω is the angular frequency. For a point source at $\mathbf{x}_s$ with spectrum $W(\omega)$ (corresponding to the source term $F(\mathbf{x}, \omega) = -\delta(\mathbf{x} - \mathbf{x}_s)W(\omega)$), the solution is $\mathbf{P}_o(\mathbf{x}|\mathbf{x}_s) = W(\omega)G_o(\mathbf{x}|\mathbf{x}_s)$.

If the background slowness perturbation is represented by the amount of $\delta\mathbf{s}(\mathbf{x})$, the true slowness model is described by $\mathbf{s}(\mathbf{x}) = \mathbf{s}_o + \delta\mathbf{s}(\mathbf{x})$. The full wavefield is

obtained by solving the Helmholtz equation with slowness model $\mathbf{s}(\mathbf{x})$,

$$[\nabla^2 + \omega^2 \mathbf{s}(\mathbf{x})^2] \mathbf{P} = F, \quad (2.2)$$

with the same source term $F = -\delta(\mathbf{x} - \mathbf{x}_s)W(\omega)$ as before. Our goal is to calculate the scattered wavefield $\mathbf{P}_1 = \mathbf{P} - \mathbf{P}_o$ induced by the slowness perturbation $\delta\mathbf{s}(\mathbf{x})$ with a linear modeling operator. Plugging $\mathbf{s}(\mathbf{x}) = \mathbf{s}_o + \delta\mathbf{s}(\mathbf{x})$ into equation (2.2), I get

$$[\nabla^2 + \omega^2 \mathbf{s}_o(\mathbf{x})^2 + 2\omega^2 \mathbf{s}_o(\mathbf{x})\delta\mathbf{s}(\mathbf{x})] \mathbf{P} = F, \quad (2.3)$$

where the high order term $O(\delta\mathbf{s}^2)$ is neglected. According to Green's theorem, moving the 3rd term in equation (2.3) to the right side, multiplying both sides with the Green's function $G_o(\mathbf{x}|\mathbf{x}')$ and integrating over the whole volume with index $\mathbf{x}'$, gives the Lippmann-Schwinger equation,

$$\begin{aligned} \mathbf{P}(\mathbf{x}) &= \int G_o(\mathbf{x}|\mathbf{x}') F(\mathbf{x}') d\mathbf{x}' - 2\omega^2 \int \mathbf{s}_o(\mathbf{x}') \delta\mathbf{s}(\mathbf{x}') \mathbf{P}(\mathbf{x}'|\mathbf{x}_s) G_o(\mathbf{x}|\mathbf{x}') d\mathbf{x}' \\ &= \mathbf{P}_o(\mathbf{x}) + \omega^2 \int \mathbf{m}(\mathbf{x}') \mathbf{P}(\mathbf{x}'|\mathbf{x}_s) G_o(\mathbf{x}|\mathbf{x}') d\mathbf{x}', \end{aligned} \quad (2.4)$$

which is an integral equation with unknown $\mathbf{P}(\mathbf{x})$ on both sides and $\mathbf{m}(\mathbf{x}') = -2\mathbf{s}(\mathbf{x}')\delta\mathbf{s}(\mathbf{x}')$ representing the reflectivity model. Here, $\mathbf{P}_o(\mathbf{x}) = \int \mathbf{G}_o(\mathbf{x}|\mathbf{x}') F(\mathbf{x}') d\mathbf{x}'$ is the pressure field associated with the background velocity model. Defining $\mathbf{P}(\mathbf{x}'|\mathbf{x}_s) = W(\omega)G(\mathbf{x}'|\mathbf{x}_s)$ and applying the Born approximation $G(\mathbf{x}'|\mathbf{x}_s) \approx G_o(\mathbf{x}'|\mathbf{x}_s)$ to the right side and assuming that $\delta\mathbf{s}(\mathbf{x})$ is small gives the scattered field under Born approximation

$$\mathbf{P}_1 = \mathbf{P}(\mathbf{x}) - \mathbf{P}_o(\mathbf{x}) \quad (2.5)$$

$$\approx \omega^2 \int W(\omega) \mathbf{m}(\mathbf{x}') G_o(\mathbf{x}'|\mathbf{x}_s) G_o(\mathbf{x}|\mathbf{x}') d\mathbf{x}', \quad (2.6)$$

where formula (2.5) represents a non-linear equation for calculation of the scattered

wavefield and equation (2.6) is a linear equation (Mulder and Plessix, 2004a). With $\mathbf{P}_o(\mathbf{x}') = W(\omega)G_o(\mathbf{x}'|\mathbf{x}_s)$, the linear modeling requires the solutions of

$$[\nabla^2 + \omega^2 \mathbf{s}_o(\mathbf{x})^2] \mathbf{P}_o = F,$$

$$[\nabla^2 + \omega^2 \mathbf{s}_o(\mathbf{x})^2] \mathbf{P}_1 = \omega^2 \mathbf{m}(\mathbf{x}') \mathbf{P}_o(\mathbf{x}'). \quad (2.7)$$

These fields can be computed by two finite-difference simulations: one with the original point source F and background slowness model $\mathbf{s}_o$ to generate $\mathbf{P}_o$; The second finite-difference simulation also uses the background slowness model $\mathbf{s}_o$, but the source term is $\omega^2 \mathbf{m}(\mathbf{x}') \mathbf{P}_o(\mathbf{x}')$, where ω^2 becomes the 2nd-order time derivative in the time domain. The adjoint of the linear operator is the reverse time migration (RTM) operator, so the RTM equation is

$$\mathbf{m}_{mig}(\mathbf{x}) = \sum_{\mathbf{x}_s} \int \omega^2 W^*(\omega) \mathbf{P}_1(\mathbf{x}') G_o^*(\mathbf{x}|\mathbf{x}_s) G_o^*(\mathbf{x}|\mathbf{x}') d\mathbf{x}'. \quad (2.8)$$

In the following section, matrix-vector notation will be used to represent the operators, such that the non-linear modeling operator is defined as $\mathbf{A} : \mathbf{d} = \mathbf{A}(\mathbf{m}) = \mathbf{P} - \mathbf{P}_o$, the linear modeling operator is $\mathbf{L} : \mathbf{d} = \mathbf{L}\mathbf{m}$, and the reverse time migration operator is $\mathbf{L}^T : \mathbf{m} = \mathbf{L}^T \mathbf{d}$. The modeling operator $\mathbf{L}$ is called the Fréchet derivative of $\mathbf{A}$ and $\mathbf{L}^T$ is the adjoint of $\mathbf{L}$.

2.2.2 Numerical Scheme: Quasi-linear Inversion

The goal is to find a slowness perturbation $\mathbf{m}(\mathbf{x})$ to fit the input data $\mathbf{d} = \mathbf{A}(\mathbf{m})$ in terms of minimizing the misfit functional

$$f(\mathbf{m}) = \frac{1}{2} \|\mathbf{A}(\mathbf{m}) - \mathbf{d}\|^2, \quad (2.9)$$

where $\mathbf{A}$ represents a non-linear forward modeling operator and $\mathbf{d}$ is the input data. The iterative steepest descent solution is

$$\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} - \alpha \mathbf{L}^T [\mathbf{A}(\mathbf{m}^{(k)}) - \mathbf{d}], \quad (2.10)$$

where $\mathbf{L}^T$ is the reverse time migration operator. The step length α is calculated with a quadratic line search method. As illustrated in Figure 2.1, two trial step lengths α_1 and α_2 along with the current model are used to approximate a quadratic curve. Then, the minimum point of the quadratic function is found to give the optimal step length. For simplicity, the steepest descent method is employed in equation (2.10), but in the numerical examples the preconditioned conjugate gradient method is used.

Equation (2.10) represents a quasi-linear inversion method that is similar to full waveform inversion. The difference is that in equation (2.10), the migration operator $\mathbf{L}^T$ only depends on the background slowness model $\mathbf{s}_o$ and does not change with iterations, as the background slowness model is assumed to be accurate enough.

2.2.3 Numerical Scheme: Linear Inversion

An alternative to quasi-linear inversion is to invert the given data $\mathbf{d}$ by fitting the data with a linear modeling operator $\mathbf{L}$ applied to the reflectivity model $\mathbf{m}$. In other words, the problem can be posed as solving the overdetermined system of equations,

$$\mathbf{d} = \mathbf{Lm} \quad (2.11)$$

with the iterative solution (Nemeth et al., 1999)

$$\begin{aligned}
\mathbf{g}^{(k)} &= \mathbf{L}^T [\mathbf{L}(\mathbf{m}^{(k)}) - \mathbf{d}] \\
\alpha &= \frac{(\mathbf{g}^{(k)})^T \mathbf{g}^{(k)}}{(\mathbf{L}\mathbf{g}^{(k)})^T \mathbf{L}\mathbf{g}^{(k)}} \\
\mathbf{m}^{(k+1)} &= \mathbf{m}^{(k)} - \alpha \mathbf{g}^{(k)},
\end{aligned} \tag{2.12}$$

where α is the analytical step length and $\mathbf{L}^T$ is the migration operator. The above calculation of the step length is based on the assumption that the forward modeling and migration operators are exactly adjoint. In practice, it is difficult to achieve exact adjointness, so usually the step length is not accurate. In this case, similar numerical linea search method as in the quasi-linear approach can be used to improve the convergence.

2.2.4 Preconditioning

Two types of preconditioners are used to regularize the iterative solution represented by equation (2.12). The first one is a high-pass filter in the space domain. It is constructed according to scale-space theory (Lindeberg, 1994), where the basic idea is to obtain a low-wavenumber component of the image (gradient) by convolving it with a 2D/3D Gaussian function. The Gaussian function is parameterized by a scalar t , which controls the smoothness of the image after convolution. The implementation of high pass filtering requires the low-wavenumber component to be subtracted from the original image. For numerical implementation, the pyramid approximation (Burt and Adelson, 1983) is used for the filter. A small $[0.25 \ 0.5 \ 0.25]$ filter is recursively applied along the x and z directions to approximate the Gaussian function. The number of iterations in applying this small filter controls the smoothness of the image after convolution. Then, the filtered image is subtracted from the original to get the high-wavenumber component of the original image. During the LSM iterations,

a proper passband should be defined in the wavenumber domain that separates the artifacts and the model update. Then a high-pass filter should be applied at the early iterations to remove the low-wavenumber back-scattering artifacts. Note a preconditioner should be symmetric positive definite (SPD) for an iterative conjugate gradient algorithm. However, ensuring this high-pass filter is SPD is not automatic so this preconditioner is not used after about 5 iterations.

The second preconditioner is the illumination compensation, which is an approximation to the inverse of the diagonal Hessian. This preconditioner enjoyed early use by Beydoun and Mendes (1989) and Luo and Schuster (1991). Plessix and Mulder (2004) provide a thorough review of its performance and properties. The illumination compensation is a diagonal matrix with positive elements, so it is always SPD and is safe to use for every iteration in LSM.

2.2.5 Phase encoding

The multisource technique with phase encoding (Romero et al., 2000) is used in all the numerical tests to increase computation efficiency. With a fixed-spread acquisition geometry, single shot gathers are encoded and blended together to form supergathers. For land surveys with a rolling spread of receivers, only these shots that share the same receivers should be encoded together. Then, all the forward modeling and migration operations are applied to the encoded supergathers instead of single shot gathers. The encoding functions used in this chapter are the combination of source-side random time shifts and random source polarities. There are several encoding strategies. The static encoding keeps the encoding function to be the same for all the iterations, while a purely dynamic strategy that changes the phase encoding function at each iteration. The hybrid strategy uses static encoding for a number of iterations to reduce I/O costs, and then dynamically changes the encoding function at every N -th iteration, where N is greater than 5.

2.3 Numerical results

The least-squares reverse time migration algorithm is tested with synthetic data generated for the 2D HESS VTI and 3D SEG/EAGE salt models. In all the numerical examples, the synthetic data are generated according to a first-order acoustic VTI wave equation (Duveneck et al., 2008). A fixed acquisition geometry for both the sources and receivers are assumed, with the understanding that a different phase encoding strategy (Huang and Schuster, 2012b) is needed for a marine acquisition geometry.

2.3.1 HESS VTI model

The multisource LSRTM algorithm is tested for the HESS VTI model decimated to 1800×750 grid points with a grid interval of 10 m; and 1800 geophones are deployed on the surface with one geophone at each grid point and 1800 point sources are evenly distributed on the surface with a source interval of 10 m. The acquisition geometry is fixed spread, as each geophone is activated for each source. A Ricker wavelet with a 12.5 Hz peak frequency is used as the source wavelet. Figure 2.2 shows (a) the P-velocity model, (b) the delta-distribution model and (c) the epsilon-distribution model. The background velocity model in Figure 2.3(a) is generated by recursively convolving the original P-slowness model with the filter $[0.25 \ 0.5 \ 0.25]$ along both the x and z directions. The corresponding relative slowness perturbation model is shown in Figure 2.3(b), which can be thought of as the true reflectivity distribution.

Figure 2.4(a) shows the RTM image for conventional seismic data (each shot gather separately migrated) after high-pass filtering with the same filter which was used as the preconditioner in LSRTM: the image is convolved with a small filter $[0.25 \ 0.5 \ 0.25]$ along both x and z directions recursively for 32 times, then subtracted from the original image to give the high-pass filtered image. In this image, most of the reflectors

are well delineated except the one below the salt body. However, the problems are: (1) some migration artifacts are still present around the salt body; (2) the amplitudes of reflectors differ significantly from the shallow part to the deep part and (3) the images of the shallow reflectors, especially the water bottom, are of low resolution.

The multisource LSRTM algorithm with linear inversion is then applied to ameliorate these problems. In this example, random source time shifts and random source polarities are used as the encoding functions and are changed at every iteration to suppress crosstalk. All 1800 shot gather are encoded and stacked together to form just one supergather, which is then migrated with just one migration operation. Figure 2.4(b) shows the multisource LSRTM images after 30 iterations of linear inversion. Compared to the conventional RTM image in Figure 2.4(a), the LSRTM image in Figure 2.4(b) shows fewer migration artifacts, more balanced amplitudes, and higher resolution for shallow reflectors. The problem with the multisource LSRTM images is that there are more high-frequency artifacts due to crosstalk noise associated with encoded supergatherers.

2.3.2 Signal-to-Noise Analysis

If a higher signal-to-noise ratio (SNR) is demanded, more iterations should be used. Alternatively, the input data should consist of a number of sub-supergathers rather than one comprehensive supergather that contains all the shot records. Any sub-supergather should contain a distinct blend of encoded shot gathers. The benefit is an enhancement of the SNR but the cost is an extra RTM simulation for each sub-supergather. Schuster et al. (2011) and Dai et al. (2011) have shown that the SNR of the stacked migration images after phase encoding is proportional to the square root of the number of supergatherers (N) used for the migration,

$$\text{SNR} \propto \sqrt{NI}, \quad (2.13)$$

where I indicates the number of stacks by an iterative stacking method. The SNR of the LSRTM image is expected to approximately behave in a similar manner.

To verify the prediction from equation (2.13), the LSRTM algorithm is applied to four and eight supergathers to produce images in Figure 2.4(c) and 2.4(d), respectively, after 30 iterations. All the images in Figure 2.4 are filtered with the same high-pass filter. By examining the gradual change from Figure 2.4(b) to (d), it is clear that increasing the number of supergathers greatly improves the quality of the LSRTM images and the numerical tests largely agree with the prediction from equation (2.13).

To measure how the SNR of the multisource LSRTM image is changing with the number of iterations, the total number of shots is reduced to 300 so that I can afford to carry out LSRTM with conventional single source data. Then, all 300 shots are encoded together in one supergather for multisource LSRTM. Numerically, I use the formula

$$\text{SNR} = \frac{||\mathbf{m}_{ref}||}{||\mathbf{m}^{(k)} - \mathbf{m}_{ref}||}, \quad (2.14)$$

to calculate the SNR at each iteration, where the conventional sources LSRTM image at the same iteration is used as the reference signal. The assumption is that the convergence rates for multisource and conventional sources LSRTM are the same. Figure 2.5 shows that the convergence curves for the two experiments are similar, which suggests the validity of the above assumption. Figure 2.6 shows (a) the LSRTM image at the 1st iteration for conventional sources data after high-pass filtering, (b) the 1st iteration LSRTM image for one 300-shot supergather after high-pass filtering, and (c) the difference between the above images before filtering. Since the crosstalk noise in the shallow part of the image is masked by back-scattering migration artifacts, only the part of the image below 6 km is used for SNR calculation.

Figure 2.7 shows the measured SNR as a function of the number of iterations. The measurements are normalized by the 1st iteration result and compared to the

prediction from equation 2.13. In Figure 2.7 the measurements in general are slightly below the prediction. As explained by Dai et al. (2011), when the LSRTM image is viewed as a weighted summation of the gradients from each iteration, early iterations generally receive large weights because of the large data residual associated with them. In term of reducing random crosstalk noise, the weighted summation is less effective than direct summation.

2.3.3 Estimate of Computational Cost

For a fixed-spread acquisition survey with S shots in total, the computational cost of conventional RTM is approximately $S\alpha$, where α is the cost of one migration operation. On the other hand, for multisource LSRTM with N sub-supergathers and I iterations, the computational cost is about $2NI\alpha$, assuming that each iteration of LSRTM takes twice the computation cost of one RTM operation. The speedup of phase encoding LSRTM relative to conventional RTM can be estimated as

$$\text{speedup} \approx \frac{S}{2NI}. \quad (2.15)$$

Therefore, Figures 2.4(b), (c), and (d) show a speedup of 30, 7.5, and 3.75, respectively.

The above estimation is done with the assumption that for LSRTM the computational cost of each iteration is twice that for conventional RTM. so the speedup is the optimal result which maybe difficult to achieve for practical applications. There are two reasons: (1) complicated line search methods can be used with more computation to enhance the robustness of LSRTM; (2) in real applications, some optimization techniques, that can be applied to conventional sources RTM, may not be available for multisource data.

2.3.4 Quasi-linear inversion

To compare the approaches of linear inversion and quasi-linear inversion, LSRTM is applied to one supergather with quasi-linear inversion to produce the migration image shown in the Figure 2.8. This image is of comparable quality and approximately the same computation cost as Figure 2.4(b). In Figure 2.9, it can be seen that the quasi-linear inversion converges faster and it can reduce the data residual to a lower level compared to linear inversion. The reason is that the linear modeling operator is similar to diffraction stack modeling, where only diffractions and reflections are generated if the background velocity is smooth enough to only create direct waves (Mulder and Plessix, 2004a). In contrast, the non-linear modeling operator can predict all the arrivals in the input dataset. In fact, the input dataset is generated by the same modeling subroutine. Since the background velocity is not 100% accurate, the quasi-linear inversion cannot converge to zero data residual. In terms of reducing crosstalk noise, the linear inversion is expected to be more effective. For quasi-linear inversion, when the data residual approaches the lower limit after certain iterations, the step length calculated by the quadratic line search will be very small. In that case, further iterations receive very small weights and barely contribute to reducing crosstalk noise.

2.3.5 Sensitivity of LSRTM to Errors in Migration Velocity

To study the sensitivity of multisource LSRTM to the background velocity model, another smoothed velocity model is generated by applying a triangle smoothing filter to the original slowness model with a window size of 800 m along both X and Z directions. Figure 2.10 shows (a) the smoothed velocity model and (b) the corresponding relative slowness perturbation. This model contains a larger slowness perturbation compared to Figure 2.3 and is used to migrate the same dataset with the conventional RTM method and the result is shown in Figure 2.11(a). The image quality is con-

siderably degraded compared to Figure 2.4(a), which suggests that the conventional RTM method is sensitive to the accuracy of the migration velocity. The multisource LSRTM algorithm is applied with the new smooth migration velocity to one, four and eight supergather(s) to produce the images shown in Figures 2.11(b), (c), and (d) with the linear inversion scheme and 30 iterations. All the images are filtered with the same high-pass filter. Compared to Figures 2.4(b), (c), and (d), the new images are of lower quality, but the image obtained with eight supergathers (Figure 2.11(d)) shows suppressed migration artifacts, balanced amplitudes and improved resolution at shallow depths, but little or no crosstalk noise. The above results suggest that LSRTM is sensitive to the accuracy of the migration velocity, but in this example is more robust than the conventional RTM method.

2.3.6 3D SEG/EAGE salt model

The quasi-linear multisource LSRTM algorithm is tested on data generated from the 3D SEG/EAGE salt model. This model contains 676 grid points along the X and Y directions, and 201 grid points along the Z direction, with a 20 m grid interval. There are 400 shots evenly distributed on the surface with a 640 m interval along the X and Y directions. A 5 Hz peak frequency Ricker wavelet is used as the source wavelet. Figure 2.12 shows (a) the vertical slice along $x=6.8$ km and (b) the horizontal slice at the depth of 0.8 km. Since there is no anisotropic parameters available for this model, the anisotropic parameters are set to be zero. Figure 2.13 shows the same slices of the smoothed velocity model as in Figure 2.12. This smooth velocity model is obtained by 3D boxcar smoothing the original model with a window size 200 m along X , Y and Z directions.

The synthetic dataset is migrated with the conventional RTM method and the same slices of the image are shown in Figure 2.14 after high-pass filtering. These images will be treated as the benchmark for comparison. Then the 400 common

shots gathers are encoded and stacked together to form 16 supergathers with 25 shots each. The multisource LSRTM method is used to migrate the supergathers. Random time shift and random polarity encoding functions are dynamically used with iterative LSRTM. Figure 2.15 shows the result after 10 iterations for the same slices as in Figure 2.12. Since the input dataset contains 16 supergathers, according to equation 2.13, the SNR of the LSRTM image is expected to be high. When compared to the conventional sources RTM image in Figure 2.14, the multisource LSRTM image in Figure 2.15 is almost free of crosstalk noise. In a vertical slice, the conventional RTM image shows strong amplitudes for the reflectors above the salt body, but in the LSRTM image, the same reflectors are of similar amplitudes along X direction. In the horizontal slice, the arc around $x=2\text{km}$ and $y=2\text{km}$ is barely visible in the conventional RTM image (Figure 2.14(b)), but it is well illuminated in the multisource LSRTM image (Figure 2.15(b)).

The LSRTM image is expected to exhibit higher resolution compared to conventional RTM. By careful examination, the reflector on top of the salt body is more clearly delineated in the LSRTM image due to the resolution improvement. In terms of computational cost, the LSRTM method for Figure 2.15 enjoys a speedup $\frac{400}{2*16*10} = 1.25$. Compared to conventional RTM, multisource LSRTM can produce migration images of better quality with similar computational cost.

Another important advantage of the least-squares migration is the removal of migration artifacts. However, the above dataset is not aliased because the peak frequency of the wavelet is as low as 5 Hz. Therefore, no apparent migration artifacts are observed in the conventional RTM image (Figure 2.14). In order to illustrate the advantage of least-squares migration, the above dataset is decimated to 100 shots with a 1280 m interval along both the X and Y directions, and migrated with conventional RTM. The corresponding images are shown in Figure 2.16. The same 100 shots are then encoded and stacked to form 10 supergathers. These supergathers are migrated

with the multisource LSRTM method to generate images in Figure 2.17. In the horizontal slice, the conventional RTM image contains a few streaks along the X direction, but they are not present in the LSRTM image.

2.4 Discussion and Conclusion

To improve the quality of RTM images and increase the computational efficiency, a least-squares reverse time migration algorithm combined with a blended source phase-encoding technique is proposed. The numerical examples show that both the quasi-linear and linear approaches of multisource LSRTM method can improve the image quality compared to the conventional RTM image.

For the 2D examples, the quality of the LSRTM images depends the accuracy of the migration velocity model and the number of sub-supergathers in the input file. The best image for the 2D HESS VTI model is obtained by migrating eight supergathers with 30 iterations, which shows balanced amplitudes, is almost free of migration artifacts and crosstalk noise, and demonstrates a speedup of 3.75 compared to conventional RTM. The computational speedup depends on the number shots encoded in one supergather, the number of encoded sub-supergathers, and the number of iterations needed for an image of acceptable quality. The numerical tests on the 2D HESS VTI model show a range of computational speedups from 3.75 to 30. For some data sets, it might be feasible to quickly compute a background migration image with a small number of iterations and use many more iterations to delineate a smaller target zone (Dong et al., 2009).

The 3D examples illustrate the advantages of least-squares migration: balancing the reflector amplitudes, improving the spatial resolution, and reducing migration artifacts. The empirical examples show that the multisource LSRTM can produce images of better quality with similar computation cost.

Similar to conventional reverse time migration, the proposed LSRTM is sensitive to the error in the migration velocity model. The numerical tests show that when the velocity model is not accurate enough, the convergence rate of the linearized inversion is slowed down and crosstalk reduction is lessened. The phase encoding scheme presented here is effective only for a fixed-spread geometry, and requires modification for a marine acquisition geometry, as demonstrated in another chapter. Future work will explore the use of preconditioning and regularization methods to accelerate the reduction of crosstalk noise to test the method on field data, and incorporate the updating of the migration velocity with iterations.

Figure 2.1: Schematic plot of the quadratic line search method.

Figure 2.2: The HESS VTI model: (a) the P-velocity model, (b) the delta-parameter, and (c) the epsilon-parameter models.

Figure 2.3: The smoothed HESS P-velocity model (a) and (b) the corresponding slowness perturbation distribution relative to the original slowness model.

Figure 2.4: The migration images obtained with (a) conventional RTM, (b) LSRTM with one supergather and 30 iterations, (c) LSRTM with four supergathers and 30 iterations, and (d) LSRTM with eight supergather and 30 iterations.

Figure 2.5: The convergence curves for (a) LSRTM with 300 shots conventional source data; and (b) LSRTM with a supergather of 300 shots. The data residuals are normalized by the initial value.

Figure 2.6: The first iteration results of (a) conventional RTM, (b) LSRTM with one supergather. The images in Figures (a) and (b) have been high-pass filtered. Figure (c) shows the difference between (a) and (b) before filtering.

Figure 2.7: The measured SNR (solid line with squares) as a function of the number of iterations compared to the prediction $SNR \approx \sqrt{I}$ (dashed line).

Figure 2.8: The LSRTM image obtained with the quasi-linear inversion scheme using one supergather (30 iterations).

Figure 2.9: The convergence curves for quasi-linear and linear inversions. The Red line with stars indicates the convergence for the quasi-linear approach and the blue line (squares) for the linear approach.

Figure 2.10: A smoothed P-velocity model generated by triangle smoothing (a) and (b) the corresponding slowness perturbation relative to the original slowness model.

Figure 2.11: The images obtained with the smooth velocity model in Figure 6 with (a) the conventional RTM, (b) LSRTM with one supergather and 30 iterations, (c) LSRTM with four supergathers and 30 iterations, and (d) LSRTM with eight supergather and 30 iterations.

Figure 2.12: The 3D SEG/EAGE salt model for (a) a vertical slice along $x=6.8$ km and (b) a horizontal slice at 0.8 km depth.

Figure 2.13: The smoothed 3D SEG/EAGE salt model for (a) a vertical slice along $x=6.8$ km and (b) a horizontal slice at 0.8 km depth.

Figure 2.14: The conventional RTM images for 400 evenly distributed shots : (a) a vertical slice along $x=6.8$ km and (b) a horizontal slice at 0.8 km depth.

Figure 2.15: The multisource LSRTM images for 16 supergathers with 25 shots each after 10 quasi-linear iterations: (a) a vertical slice along $x=6.8$ km and (b) a horizontal slice at 0.8 km depth.

Figure 2.16: The conventional RTM images for 100 evenly distributed shots : (a) a vertical slice along $x=6.8$ km and (b) a horizontal slice at 0.8 km depth. Note the streaks along X direction in the horizontal slices.

Figure 2.17: The multisource LSRTM images for 10 supergathers with 10 shots each after 10 quasi-linear iterations: (a) a vertical slice along $x=6.8$ km and (b) a horizontal slice at 0.8 km depth. Note the streaks in the conventional RTM image is removed.

Chapter 3

Least-squares Reverse Time

Migration of Marine Data with

Frequency-selection Encoding

The phase-encoding technique can sometimes increase the efficiency of the least-squares reverse time migration (LSRTM) by more than one order of magnitude. However, traditional random encoding functions require all the encoded shots to share the same receiver locations, thus limiting the usage to seismic survey with a fixed spread geometry. In this chapter, I implement a frequency-selection encoding strategy that utilizes orthogonal encoding functions to handle seismic data with a marine streamer geometry. The encoding functions are delta functions in the frequency domain, so that all the encoded shots have unique non-overlapping frequency content. Therefore, the receivers can distinguish the wavefield from each shot according to the specified frequencies. With the frequency-selection encoding method, the computational efficiency of LSRTM is increased so that its cost is comparable to conventional RTM in the example of Marmousi2 model. With more iterations, the LSRTM image quality is further improved. The numerical results suggest that LSRTM with frequency-selection is an efficient method to produce better images than

conventional RTM.

3.1 Introduction

The least-squares migration method (Lailly, 1984; Cole and Karrenbach, 1992; Schuster, 1993; Nemeth et al., 1999; Duquet et al., 2000) has been shown to sometimes produce migration images with better quality than those computed by conventional migration. Its original implementation was with Kirchhoff migration (Nemeth et al., 1999; Duquet et al., 2000), but was later developed for phase shift migration algorithms (Kaplan et al., 2010; Huang and Schuster, 2012a). When least-squares migration is implemented with the reverse time migration method (Tang and Biondi, 2009; Dai and Schuster, 2010a; Dai et al., 2010; Wong et al., 2011; Dai et al., 2012), it can reduce not only the acquisition footprint but also the artifacts in the RTM image, while enhancing the image resolution. In addition, Romero et al. (2000); Krebs et al. (2009); Tang and Biondi (2009); Schuster et al. (2011); Dai et al. (2011, 2012) employed a phase-encoding multisource approach to increase the computational efficiency by more than an order-of-magnitude compared to conventional LSRTM.

For iterative phase-encoded multisource migration, many shot gathers are encoded with random encoding functions and blended together to form a supergather. One supergather can be modeled and migrated with one finite-difference solution to the wave equation for multiple sources and so provides a high computational efficiency compared to standard LSM. With increasing iteration number, the crosstalk between different shots will be increasingly suppressed. Consequently, the computational cost of LSRTM is reduced to a level comparable to conventional reverse time migration or even lower, depending on the acquisition geometry.

However, the random encoding functions used by Romero et al. (2000); Krebs et al. (2009); Schuster et al. (2011) and Dai et al. (2012), cannot be applied to a seismic

survey with a marine streamer geometry (Routh et al., 2011; Huang and Schuster, 2012a) because, although the calculated synthetic data are of fixed spread geometry, the observed data are recorded with a marine streamer geometry. As a remedy, Routh et al. (2011) proposed a cross-correlation based misfit functional to mitigate the effect of a recording pattern mismatch. Alternatively, Huang and Schuster (2012a) proposed a frequency-selection encoding strategy for least-squares phase shift migration, which is applicable to marine data.

The frequency-selection encoding strategy can also be used with least-squares reverse time migration, where the time-domain simulation are performed with a single frequency harmonic source instead of the conventional broadband source. Nihei and Li (2006) proposed to use a time-domain finite-difference method to obtain the single frequency seismic response of a velocity model. Compared to the conventional frequency domain method, their method has significantly lower arithmetic complexity and storage requirements in the 3D case.

In this chapter, the frequency-selection encoding method is applied with least-squares reverse time migration and tested on the Marmousi2 model to show that LSRTM can produce better images than conventional RTM with comparable cost for marine datasets.

3.2 Theory

A time-domain seismic dataset $d(t, \mathbf{g}, \mathbf{s})$, where $\mathbf{s}$ and $\mathbf{g}$ are source and receiver vectors, can be digitized into a 3D array $\mathbf{d}_{it,ig,is}$ ($it = 1, 2, \dots, n_t; ig = 1, 2, \dots, n_g; is = 1, 2, \dots, n_s$), assuming there are n_t time samples, n_g receivers for a shot, and n_s shots in total. Given time sampling dt , the time domain array can be transformed to the frequency domain as $\tilde{\mathbf{d}}_{i\omega,ig,is}$ ($i\omega = 1, 2, \dots, n_\omega$) and the angular frequency sampling is $d\omega = \frac{2\pi}{n_t * dt}$. In the frequency domain, only these samples that fall into the frequency band of

the seismic data are kept, so for a dataset with peak frequency f (frequency band $0 \sim 2.5f$), n_ω can be calculated as $n_\omega = \frac{2\pi*2.5f}{d\omega}$.

With the frequency-selection encoding, the encoding function is defined as

$$N_s(i\omega, i\omega_s) = \begin{cases} 1 & \text{when } i\omega = i\omega_s \\ 0 & \text{otherwise,} \end{cases} \quad (3.1)$$

where $i\omega_s$ is a function of shot index is , and it represents the selected frequency for the shot. Similar to conventional blended source technique, all the shots are encoded with the encoding function and blended together to form a supergather

$$\tilde{\mathbf{d}}_{i\omega_s, ig} = \sum_{is=1}^{n_s} N_s(i\omega, i\omega_s) \tilde{\mathbf{d}}_{i\omega, ig, is}. \quad (3.2)$$

Now the supergather $\tilde{\mathbf{d}}_{i\omega_s, ig}$ becomes a 2D array, and each frequency component corresponds to a different shot. Note that a supergather can only accommodate up to n_ω shots. It is obvious that the frequency-selection encoding method is applicable to seismic data with a marine streamer acquisition geometry, because at each receiver position, the data components from different shots can be distinguished from one another according to their frequency contents.

In least-squares reverse time migration, a reflectivity model vector $\mathbf{m}$ is sought to best fit the observed data with a Born modeling operator $\mathbf{L}$ by minimizing the misfit functional

$$f(\mathbf{m}) = \frac{1}{2} \|\mathbf{L}\mathbf{m} - \tilde{\mathbf{d}}\|^2 + \frac{\gamma}{2} \|\mathbf{m}\|^2, \quad (3.3)$$

where $\tilde{\mathbf{d}}$ is a vector representing a supergather $\tilde{\mathbf{d}}_{i\omega_s, ig}$ and γ is damping coefficient.

The following numerical scheme can be implemented with Born modeling and the

reverse time migration method:

$$\begin{aligned}
\mathbf{g}^{(k)} &= \mathbf{L}^T [\mathbf{L}(\mathbf{m}^{(k)}) - \tilde{\mathbf{d}}] + \gamma \mathbf{m}^{(k)}, \\
\alpha &= \frac{(\mathbf{g}^{(k)})^T \mathbf{g}^{(k)}}{(\mathbf{L}\mathbf{g}^{(k)})^T \mathbf{L}\mathbf{g}^{(k)} + \gamma \|\mathbf{g}^{(k)}\|^2}, \\
\mathbf{m}^{(k+1)} &= \mathbf{m}^{(k)} - \alpha \mathbf{g}^{(k)}.
\end{aligned} \tag{3.4}$$

At each iteration, a new supergather with new encoding functions should be used to sample a different frequency for each shot. Therefore, if N_ω frequencies are needed to avoid wrap-around effects, the LSM procedure should be iterated at least N_ω iterations to ensure that all the frequencies are visited by a shot. In contrast, the iterative stacking method can be applied to those N_ω supergatherers to produce an image with less computational cost than conventional RTM, because usually N_ω can be much smaller than n_ω due to data redundancy in the frequency domain (Mulder and Plessix, 2004b).

In the following section, I demonstrate the numerical implementation of modeling and migration of a supergather with a time-domain finite-difference method.

3.2.1 Single Frequency Response Modeling for a Shot

Following Nihei and Li (2006) and Sirgue et al. (2008), the single frequency response of a velocity model $v(\mathbf{x})$ for a shot at $\mathbf{s}$ can be modelled with the time-domain finite-difference method by solving the equation

$$(\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2})P(t, \mathbf{x}, \mathbf{s}) = -Re[W(\omega_s)e^{i\omega_s t}]\delta(\mathbf{x} - \mathbf{s}), \tag{3.5}$$

where the source wavelet is a harmonic wave with its amplitude and phase specified by the source $W(\omega_s)$. Assuming that the trace length T is long enough to include all the dominant arrivals, the recorded wavefield at the receiver location $P(t, \mathbf{g}, \mathbf{s})$ reaches

steady state after propagation time T . Therefore, the single frequency response can be extracted with the following formula

$$\tilde{P}(\omega_s, \mathbf{g}, \mathbf{s}) = \frac{1}{T} \int_T^{2T} P(t, \mathbf{g}, \mathbf{s}) e^{-i\omega_s t} dt. \quad (3.6)$$

Note that the simulation time is increased from T to $2T$. For a single shot, the above integration can be computed over a single period ($\frac{2\pi}{\omega_s}$) instead of T . Repeating the above solution for different frequencies and digitizing the records give the frequency domain data $\tilde{\mathbf{d}}_{i\omega, ig, is}$.

3.2.2 Forward Modeling of a Supergather

All the shots in a supergather can propagate at the same time in the time domain,

$$(\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2}) P(t, \mathbf{x}) = - \sum_s \text{Re}[W(\omega_s) e^{i\omega_s t}] \delta(\mathbf{x} - \mathbf{s}). \quad (3.7)$$

At the receiver locations, the observed time domain data $P(t, \mathbf{g})$ need to be transformed into the frequency domain and each shot selects the component according to its frequency encoding

$$\tilde{P}(\omega_s, \mathbf{g}) = \frac{1}{T} \int_T^{2T} P(t, \mathbf{g}) e^{-i\omega_s t} dt. \quad (3.8)$$

In this case, the above integration should be carried out from T to $2T$ where $T = \frac{2\pi}{d\omega}$ (Nihei and Li, 2006). Digitizing $\tilde{P}(\omega_s, \mathbf{g})$ yields a supergather containing the blended full wavefields.

3.2.3 Born Modeling of a Supergather

In LSRTM, the Born modeling operator is used to fit the observed reflection data with a reflectivity model $m(\mathbf{x})$. Following Dai et al. (2012), the Born modeling of a

supergather can be computed in the time domain as

$$(\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2})d(t, \mathbf{x}) = m(\mathbf{x}) \frac{\partial^2 P(t, \mathbf{x})}{\partial t^2}, \quad (3.9)$$

subsequent to equation 3.7. The frequency domain data $\tilde{d}(\omega_s, \mathbf{g})$ can be extracted from $d(t, \mathbf{g})$ according to

$$\tilde{d}(\omega_s, \mathbf{g}) = \frac{1}{T} \int_T^{2T} d(t, \mathbf{g}) e^{-i\omega_s t} dt. \quad (3.10)$$

Digitizing $\tilde{d}(\omega_s, \mathbf{g})$ yields a supergather $\tilde{\mathbf{d}}_{i\omega_s, ig}$ containing the reflection waves. Equations 3.7, 3.9, and 3.10 represent the numerical computation of the Born modeling operator $\mathbf{L}$ in equation 3.3.

3.2.4 Migration of a Supergather

The migration operator can be formulated as the adjoint of the Born modeling operator. Corresponding to equation 3.9, the migration formula is

$$(\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2})Q(t, \mathbf{x}) = - \sum_{\omega_s} \sum_g \text{Re}[\tilde{d}(\omega_s, \mathbf{g}) e^{i\omega_s t}] \delta(\mathbf{x} - \mathbf{g}), \quad (3.11)$$

$$m(\mathbf{x}) = \int_T^{2T} \frac{\partial^2 P(t, \mathbf{x})}{\partial t^2} \times Q(t, \mathbf{x}) dt, \quad (3.12)$$

where $P(t, \mathbf{x})$ is the source wavefield obtained with equation 3.7, $Q(t, \mathbf{x})$ is the receiver wavefield and equation 3.12 indicates a dot product imaging condition after both source and receiver wavefields reach steady state.

With the above definitions of the Born modeling and migration operators, the LSRTM scheme in Equation 3.4 can be implemented numerically. In the following examples, the preconditioned conjugate gradient method is used as the iterative solver.

3.3 Numerical results

In this section, the proposed frequency-selection LSRTM algorithm is first tested on 2D synthetic data with the modified Marmousi2 model, and then tested on a marine-streamer field dataset from Gulf of Mexico.

3.3.1 The Marmousi2 Model

The proposed method is tested on the data associated with the Marmousi2 model, where the original model is modified to be a size of $8 \text{ km} \times 3.5 \text{ km}$ with a 10 m grid interval (Figure 3.1)(a). The synthetic data are generated with a marine streamer geometry, where 400 shots are excited with a 20 m offset interval at the depth of 10 m. Each shot is recorded with a 2 km long cable with 201 receivers and 10 m interval. The minimum source-receiver offset is zero meters and the maximum offset is 2 km. A Ricker wavelet with 20 Hz peak frequency is used as the source wavelet. The record length is 8 sec in time and a 2-8 constant density finite difference code is used for the RTM.

The source wavelet and its frequency spectrum are plotted in Figure 3.2 and the frequency range is chosen to be 0-50 Hz. According to the Nyquist theorem, the frequency sampling rate should be 0.125 Hz. Therefore, there are 400 samples in the frequency domain (excluding the 0-Hz component), which means 400 shots can be accommodated in one supergather. In this example, all 400 shot gathers can be uniquely encoded in a supergather.

3.3.2 Single Frequency Modeling Test

When a 25-Hz harmonic source is fired at $x = 4 \text{ km}$ and recorded at $x = 6 \text{ km}$, the data trace (Figure 3.3(a)) reaches steady state after 8 secs, assuming 8 secs is long enough to include all the arrivals. According to equation 3.6, the latter

half of the trace (8-16 secs) is used to extract the frequency response. The red line in Figure 3.3(b) shows the frequency spectrum of the latter 8 secs in panel (a), which clearly only contains a 25-Hz frequency component. In contrast, the frequency spectrum of the former 8 secs is far from a spike, and the peak frequency amplitude is weaker compared to the red line. Therefore, it is of significant importance to run a time-domain finite-difference simulation long enough to reach steady state before extracting frequency responses. Figure 3.3(c) shows the zoom view of the spectrums between 20-30 Hz.

When the single frequency modeling is repeated 400 times over all the frequencies spanned by the Ricker wavelet, the resulting reconstruction (black line in Figure 3.4(a)) is almost identical to the red line, which is the recorded trace with a 20-Hz Ricker wavelet as source for time-domain simulation. Also, their spectrums are very similar. Figure 3.4 verifies that the single frequency simulation based on individual frequencies yields the same results as a broadband simulation.

3.3.3 Frequency-selection LSRTM

Due to data redundancy, the frequency domain data can be represented by a coarser sampling than the Nyquist rate. Figure 3.5 illustrates how to estimate the necessary sampling. According to Mulder and Plessix (2004b), the maximum frequency sampling

$$\Delta f_{max} = \frac{1}{T_{max} - T_{min}}. \quad (3.13)$$

In this example, as explained in Figure 3.5, T_{min} is about 1.49 sec and T_{max} is about 2.91 sec calculated with average velocity 2.5 km/s, so that the Δf_{max} is about 0.70 Hz. I choose 0.625 Hz as the frequency sampling for convenience, and thus the range of 0-50 Hz can be represented by 80 frequency samples. The frequency-selection encoding scheme is as follows. Each shot is assigned one out of 400 frequencies, and

then blended to form a supergather. For the next iteration, the frequency assigned to each shot is increased by 0.625 Hz. When the frequency of a shot exceeds 50 Hz, it will be wrapped around to the low frequencies. Therefore, 80 such LSRTM iterations can sample the frequency domain of interest.

A reflectivity model is derived from the true velocity by high-pass filtering and shown in Figure 3.6 as the benchmark. A pseudo-spectral method is used to compute the true data of 400 CSGs and Figure 3.7 depicts a shot gather that is fired at $x = 3 \text{ km}$ after muting the direct waves. Those 400 shot gathers are then transformed into the frequency domain and 80 supergathers are formed with the above frequency-selection encoding strategy.

First, the iterative stacking method is applied to the 80 supergathers, where those 80 supergathers are migrated with the migration velocity in Figure 3.1(b) and stacked together. The image is shown in Figure 3.8(b) and it is almost identical to the conventional shot domain RTM image in Figure 3.8(a).

To reduce the migration artifacts and improve the image resolution, the same 80 supergathers are migrated with the LSRTM algorithm with one supergather for each iteration. Figure 3.9(a) plots the image for the first iteration, which contains strong ringing artifacts. As iterations proceed, the LSRTM image quality gradually improves (see Figure 3.9(b) for 20-iteration result). Figure 3.9(c) shows the image after 80 iterations which is of higher resolution than the conventional RTM image in Figure 3.8(a). The zoom views of the shallow part (Figure 3.11) and the deep part (Figure ??) show similar resolution enhancement. The drawback is that there is high frequency noise present in the LSRTM image.

3.3.4 Computational Cost

To compare the computational cost of frequency-selection LSRTM to conventional shot-domain RTM, there are two factors that must be considered. The first one is the

difference in the size of computation grid. For shot-domain RTM, the computation grid only need to be long enough to cover the shot and all the receivers, but for migration of a supergather, the computation grid has to be of the full model size. In this example, this factor is $\frac{8}{2} = 4$. The second factor is the simulation time difference. As shown in the single frequency modeling test, to obtain a spike frequency spectrum and avoid crosstalk between different shots, the simulation time is doubled for frequency-selection LSRTM. In this example of Marmousi2 model, LSRTM image is obtained after 80 iterations. Assuming each iteration cost twice that of a single RTM operation, the computational cost of Figure 3.9(c) is $\frac{80*2*2*4}{400} = 3.2$ times that of conventional RTM (Figure 3.8(a)). If the LSRTM is computed without phase-encoding technique, its computational cost would be 160 times that of conventional RTM, assuming 80 iterations are still needed.

3.3.5 Field Data Test

The proposed methods are tested on a 2D marine dataset. There are 496 shots with a shot interval of 37.5 m. Each shot is recorded by a 6 km long cable with 480 receivers at an interval of 12.5 m. The nearest offset is 198 m. Each trace is multiplied by $\sqrt{i/\omega}$ in the frequency domain and then scaled by $\sqrt{t}$ in the time domain to correct for 3D geometrical spreading (Zhou et al., 1997). Then, the CSGs are filtered with a Wiener filter to transform the original wavelet to a Ricker wavelet with a 25 Hz peak frequency. The original wavelet is estimated by stacking traces with a strong water bottom reflection, and windowing the water-bottom reflection event. Figure 3.13 shows a shot gather with source at $x = 11.3 \text{ km}$ after preprocessing. For convenience, the trace length is set to be 10 secs, so that the frequency sampling is 0.1 Hz. For the frequency band of 0-62.5 Hz, there are 625 frequency channels to accomodate up to 625 shots.

3.3.6 Shot-domain RTM

The dataset is first migrated with conventional shot-domain RTM method after pre-processing with migration velocity is obtained by waveform inversion (Boonyasiriwat et al., 2010), and the image is shown in Figure 3.14(a), which contains strong artifacts near the shallow reflectors, which are caused by head waves and diving waves (Liu et al., 2011). In the bottom right corner of the image, there are low-frequency horizontal stripes.

3.3.7 Frequency-selection LSRTM

According to equation 3.13, the maximum frequency sampling for this field dataset is larger than 1 Hz, assuming average water depth 0.5 km, maximum depth 2.5 km. However, due to the fact that the shot spacing 37.5 m is larger than half of the dominant wavelength (30 m) in the water, denser sampling is needed than what equation 3.13 predicts. Empirical tests suggest a frequency sampling of 0.3 Hz. Therefore, each shot over iterations visits 208 frequency samples in the frequency domain. To form a supergather, all the shots are encoded with frequency-selection functions and blended together. For the next iteration, the frequency of each shot is shifted by 0.3 Hz to form a new supergather. The LSRTM algorithm iterates for 208 iterations to go over all the frequencies and the image is shown in Figure 3.14(b). The low-frequency artifacts in the shallow part and in the right bottom corner are removed. Furthermore, the resolution of the LSRTM image is enhanced compared to shot-domain RTM image, which leads to better delineation of the faults as shown in the zoom views in Figures 3.15 and 3.16. The drawback is that there are still some high-frequency noise present in the LSRTM image.

3.3.8 Computational Cost

In this field data example, LSRTM image is obtained after 208 iterations. Calculated similarly to the Marmousi model example, the computational cost of Figure 3.14(b) is $\frac{208*2*2*18}{496*6} \approx 5$ times that of conventional RTM (Figure 3.14(a)).

3.4 Discussion and Conclusion

In this chapter, I implemented a frequency-selection encoding strategy to speed up the least-squares reverse time migration of marine data. The traditional random phase encoding method is not applicable to marine data due to the mismatch in acquisition geometry between the observed data and the calculated synthetic data. With frequency-selection encoding, all the shots are encoded with encoding functions that are orthogonal to each other in the frequency domain, so the calculated synthetic data can be effectively decoded at the receiver locations for comparison with the observed data. Because of the data redundancy in the frequency domain, the frequency sampling rate can be large, which leads to significant computational savings. Numerical tests on part of the Marmousi2 model show that the frequency-selection encoding can significantly improve the efficiency of the LSRTM and reduce its cost to the level of conventional shot domain RTM. Empirical results suggest that the LSRTM with frequency-selection encoding is an efficient method to produce better images than conventional RTM.

Figure 3.1: The Marmousi2 model: (a) the modified Marmousi2 velocity model and (b) the smooth migration velocity.

Figure 3.2: A 20 Hz Ricker wavelet (a) and its associated frequency spectrum.

Figure 3.3: Harmonic simulation for a 25-Hz source at $x = 4km$ and recorded at $x = 6km$ (a). Panel (b) plots the frequency spectrums of the first 8 secs (black) and second 8 secs (red) and panel (c) shows the zoom view of the part between 20-30 Hz.

Figure 3.4: Comparison of recorded traces with different simulation method. The black line indicates the recorded trace from time-domain simulation with a broadband wavelet; the red line is the stack of 400 harmonic traces (0-8 secs) and blue line (8-16 secs).

Figure 3.5: Ray diagrams for the reflections for the ocean bottom and the deepest reflector. The difference in arrival times of these two phases is used to estimate the necessary frequency sampling rate.

Figure 3.6: The true reflectivity of the Marmousi2 Model.

Figure 3.7: A common shot gather with shot location at 3 km offset.

Figure 3.8: Migration images obtained by (a) the conventional shot-domain RTM method and (b) the iterative stacking method.

Figure 3.9: The frequency-selection LSRTM image after (a) 1 iteration, (b) 20 iterations, and (c) 80 iterations.

Figure 3.10: Zoom view comparison of (a) shot-domain RTM image and (b) frequency-selection LSRTM image for the shallow part.

Figure 3.11: Zoom view comparison of (a) shot-domain RTM image and (b) frequency-selection LSRTM image for the deep part.

Figure 3.12: The migration velocity model for the field data.

Figure 3.13: A common shot gather with shot location at 11.3 km offset after pre-processing.

Figure 3.14: The migration images obtained with (a) the conventional RTM method and (b) the frequency-selection LSRTM method. Red and blue boxes indicate the area for zoom views.

Figure 3.15: Zoom view of the red box for (a) the conventional RTM image and (b) the frequency-selection LSRTM image.

Figure 3.16: Zoom view of the blue box for (a) the conventional RTM image and (b) the frequency-selection LSRTM image.

Chapter 4

Super-virtual Interferometric Diffractions as Guide Stars

A significant problem in seismic imaging is seismically *seeing* below salt structures: large velocity contrasts and the irregular geometry of the salt-sediment interface strongly defocus both the downgoing and upgoing seismic wavefields. This can result in severely defocused migration images so as to seismically render some subsalt reserves invisible. The potential cure is a good estimate of the subsalt and salt velocity distributions, but that is also the problem: severe velocity contrasts prevent the appearance of coherent subsalt reflections in the surface records so that MVA or tomographic methods can become ineffective. I now present an interferometric method for extracting the diffraction signals that emanate from diffractors, also denoted as seismic guide stars. The signal-to-noise ratio of these interferometric diffractions is enhanced by $\sqrt{N}$, where N is the number of source points coincident with the receiver points. Thus, diffractions from subsalt guide stars can then be rendered visible and so can be used for velocity analysis, migration, and focusing of subsalt reflections. Both synthetic and field data records are used to demonstrate the benefits and limitations of this method.

4.1 Introduction

Many of the world's giant oil reservoirs discovered in the 21st century are offshore marine fields, and a significant number of them are below salt. For example, deep drilling in the Gulf of Mexico is exclusively below a large salt horizon that blankets the Gulf of Mexico beneath depths of 5 km or more. Another example is offshore Brazil where large scale imaging, drilling, and extraction of subsalt hydrocarbons are carried out. The main challenges with deep subsalt deposits are that they are difficult to identify with the seismic method, and they are extremely expensive to drill and extract. Thus, improving the accuracy of subsalt imaging with the seismic method is an important goal of many large oil companies.

A significant problem with the seismic imaging method is that subsalt reflections are severely defocused by the strong velocity contrasts and the irregular geometries of salt-sediment interfaces. Upgoing reflection energy is present in the data, but cannot be easily detected in the shot records as coherent arrivals with hyperbolic-like moveout trajectories. This means that velocity estimation methods such as travelttime tomography cannot be used and, others, such as migration velocity analysis or full-wave inversion will fail unless an accurate starting velocity model is used. Is there another means for estimating subsalt velocities when the other methods fail?

This chapter proposes interferometric extraction of subsalt diffractions, with the possibility that they can also be used as migration operators or for velocity analysis. The key idea (see Figure 4.1) is that, similar to surface waves or refractions, 2D subsalt diffractions are associated with stationary source points all along the source line. Thus, application of interferometry can enhance the signal-to-noise ratio of this diffraction energy by $\sqrt{N}$, where N is the number of source points. This means that undetectable diffractions in the shot records can be enhanced, which can then be used to guide velocity analysis and focusing of subsalt reflections. I refer to such diffractors as guide stars because they, similar to VSP data, can be used as Green's functions

to build natural migration operators (Schuster, 2002; Brandsberg-Dahl et al., 2007), or estimate migration velocity (Berkhout et al., 2001; Landa et al., 1987). Similar to guide stars used by astronomers for correcting the optical distortion of the atmosphere, diffraction based migration operators can be used to guide the proper focusing of subsalt reflection energy to their points of origin beneath the salt. Both synthetic and field data records are used to demonstrate the benefits and limitations of this method.

The first part of this chapter presents the interferometric theory for extracting diffraction energy in seismic records. This is followed by synthetic and field data examples that show both the benefits and limitations of this method, and finally the last section presents a summary.

4.2 Theory

I will first present the far-field reciprocity equations of correlation and convolution types, and then show how they can be used to construct super-virtual diffractions. The use of the far-field reciprocity equations of correlation and convolution types to create virtual diffractions and enhance their SNR is similar to that of Mallinson et al. (2011), except diffraction energy is enhanced rather than refraction energy. I will assume an acoustic medium with an arbitrary velocity distribution with constant density, and wideband sources with unity amplitude at each frequency.

4.2.1 Reciprocity Equations of Correlation Type

Assume a source at $\mathbf{x}$ in Figure 4.2 and receivers at $\mathbf{A}$ and $\mathbf{B}$. The reciprocity theorem of correlation type (Wapenaar and Fokkema, 2006) states that the virtual

Green's function $G(\mathbf{B}|\mathbf{A})^{virt.}$ is given by the reciprocity theorem of correlation type:

$$\mathbf{B}, \mathbf{A} \in V_0; \quad 2iIm[G(\mathbf{B}|\mathbf{A})^{virt.}] = \int_{top} [G^*(\mathbf{B}|\mathbf{x}) \frac{\partial_x G(\mathbf{A}|\mathbf{x})}{\partial n} - G(\mathbf{A}|\mathbf{x}) \frac{\partial_x G^*(\mathbf{B}|\mathbf{x})}{\partial n}] d^2x, \quad (4.1)$$

where $\frac{\partial_x G(\mathbf{A}|\mathbf{x})}{\partial n} = \nabla G(\mathbf{A}|\mathbf{x}) \bullet \hat{n}$ for the outward point unit normal $\hat{n}$ on the boundary. Here, Green's function solves the Helmholtz equation for an arbitrary velocity distribution with a constant density and I follow the notation from Schuster (2009). The integration path is only over the *top* path as the half-circle path is neglected by the Wapenaar anti-radiation condition.

Now I want the diffractions to be reinforced so $G(\mathbf{A}|\mathbf{B})$ is replaced by the diffraction term defined as $\mathcal{G}(\mathbf{A}|\mathbf{B})$ to give, under the far-field approximation,

$$Im[\mathcal{G}(\mathbf{B}|\mathbf{A})^{virt.}] \approx k \int_{top} \mathcal{G}(\mathbf{A}|\mathbf{x})^* \mathcal{G}(\mathbf{B}|\mathbf{x}) d^2x, \quad (4.2)$$

where k is the average wavenumber and $\mathcal{G}(\mathbf{B}|\mathbf{A}) = G(\mathbf{B}|\mathbf{A})^{diff.}$ represents the diffraction contribution in the Green's function for a point scatter.

This approximation is analogous to that used in model-based redatuming of reflection data to a new datum, except in model-based datuming $\mathcal{G}(\mathbf{A}|\mathbf{x})^*$ is a model-based extrapolation Green's function that only accounts for direct arrivals, and $\mathcal{G}(\mathbf{B}|\mathbf{x})$ represents the reflection data devoid of direct waves and multiples.

According to the ray diagram shown in Figure 4.1(a), the correlated trace $\mathcal{F}^{-1}[\mathcal{G}(\mathbf{A}|\mathbf{x})^* \mathcal{G}(\mathbf{B}|\mathbf{x})]$ ($\mathcal{F}^{-1}$ denotes the temporal inverse Fourier transform) for a source at $\mathbf{x}$ has the same traveltimes $\tau_{A'B} - \tau_{A'A}$ for any source location $\mathbf{x}$. Such source locations are considered to be at stationary points, and similar to surface wave interferometry (Xue et al., 2009) or refraction wave interferometry (Dong et al., 2006), the summation of the correlated records over source positions tend to enhance the

SNR of the virtual diffraction arrival by a factor of $\sqrt{N}$. Here, N represents the number of source positions that generate the diffractions.

4.2.2 Reciprocity Equations of Convolution Type

It is assumed that the virtual data $G(\mathbf{B}|\mathbf{A})^{virt.}$ can be extrapolated to get $G(\mathbf{x}'|\mathbf{A})^{virt.}$ for $\mathbf{x}'$ along the horizontal dashed line in Figure 4.2(b); similarly, the field data can be extrapolated to get $G(\mathbf{x}'|\mathbf{B})$. In this case, the reciprocity theorem of convolution type (Schuster, 2009) can then be used to obtain the super-virtual data

$$G(\mathbf{B}|\mathbf{A})^{super} \approx \int_{hydro} [G(\mathbf{B}|\mathbf{x}') \frac{\partial_{\mathbf{x}'} \mathbf{G}(\mathbf{A}|\mathbf{x}')}{\partial \mathbf{n}'} - \mathbf{G}(\mathbf{A}|\mathbf{x}') \frac{\partial_{\mathbf{x}'} \mathbf{G}(\mathbf{B}|\mathbf{x}')}{\partial \mathbf{n}'}] d^2 \mathbf{x}', \quad (4.3)$$

where the integration is along the *hydro* dashed line in Figure 4.2(b). Under the far-field approximation and setting $G(\mathbf{A}|\mathbf{x}') \rightarrow \mathcal{G}(\mathbf{A}|\mathbf{x}')$ and $G(\mathbf{B}|\mathbf{x}') \rightarrow \mathcal{G}(\mathbf{B}|\mathbf{x}')^{virt.}$, I get

$$\mathcal{G}(\mathbf{B}|\mathbf{A})^{super} \approx 2ik \int_{hydro} \mathcal{G}(\mathbf{B}|\mathbf{x}')^{virt.} \mathcal{G}(\mathbf{A}|\mathbf{x}') d^2 x', \quad (4.4)$$

where $\mathcal{G}(\mathbf{B}|\mathbf{A})^{super}$ represents the super-virtual data obtained by convolving the recorded data $\mathcal{F}^{-1}[\mathcal{G}(\mathbf{A}|\mathbf{x}')] with the virtual data $\mathcal{F}^{-1}[\mathcal{G}(\mathbf{B}|\mathbf{x}')^{virt.}]$. Here, the SNR of the reconstructed diffraction arrival is enhanced by the factor $\sqrt{N}$. However, practical considerations such as artifacts associated with limited recording apertures, discrete source and receiver sampling, windowing of the diffracted waves, and the far-field approximation will likely prevent the attainment of this ideal enhancement.$

In the next section, I will use the example of diffractions that have been windowed

from the original data so that $G(\mathbf{A}|\mathbf{x}) \approx G(\mathbf{A}|\mathbf{x})^{diff}$.

4.3 Synthetic Data Example

To demonstrate the effectiveness of the proposed method, super-virtual diffraction arrivals are extracted from synthetic shot gathers computed with a 2-4 FD forward modeling code for part of the BP2004 model. Three diffractors are placed under the salt body as shown in Figure 4.3. The goal is to extract the diffraction arrivals associated with these diffractors. Four hundred shot gathers are generated with a 20-Hz Ricker wavelet and a 20 m interval. All the shots are recorded by the same 800 receivers with a 10 m interval. The sources and receivers are placed at the depth of 10 m, and the free surface condition is not implemented. A common shot gather is shown in Figure 4.4(a). The diffractions associated with the point diffractor on the right of the salt body are indicated with dashed red lines. These diffractions are identified by comparison against the predicted diffraction traveltimes for that diffractor, and Figure 4.4(b) shows these time-windowed diffractions. In order to eliminate other coherent events in the time window, a median filter is applied to these data in Figure 4.4(b) along the diffraction moveout curve (Moser et al., 1999) and the result is shown in Figure 4.4(c), where the diffractions are enhanced. However, Figure 4.4(c) contains strong artifacts from other coherent signals, because within the time window in Figure 4.4(b) the amplitudes of the direct waves are an order of magnitude greater than the amplitudes of the diffraction events. Figure 4.4(d) shows the super-virtual diffraction with improved SNR compared to the result after median filtering in Figure 4.4(c).

Another synthetic example is shown to illustrate that the super-virtual diffraction can be used to estimate the source and receiver statics. Synthetic data are generated with the same acquisition geometry as in the previous example for the Figure 4.5

velocity model. Random noise and random statics are added in the common shot gather in Figure 4.6(a), where the red lines outline the diffraction arrivals for the left most diffractor. In Figure 4.6(b), the diffraction energy is almost invisible because of the random noise, so that median filtering fails when it is applied along the predicted hyperbolic moveout (Figure 4.6(c)). Since the diffraction arrivals are temporally isolated from other events, the super-virtual diffraction is obtained without median filtering and shown in Figure 4.6(d). The actual moveout curve of the diffraction is preserved and plotted as the blue line in Figure 4.7. In this figure, the red line indicates the predicted arrival time of the diffraction without considering the source and receiver statics. The source and receivers statics can now be estimated from the difference between the blue and red lines or by a phase closure principle (Sheng et al., 2005). In addition, the moveout curve can be used to represent the Green's function $G(\mathbf{B}|\mathbf{x}_o)$ for a point source at $\mathbf{x}_o$, which can be used as the natural migration operator $G(\mathbf{B}|\mathbf{x}_o)^*G(\mathbf{A}|\mathbf{x}_o)^*$ (Schuster, 2002).

4.4 Field Data Example

In this section, the proposed method is applied to the Friendswood cross-well data to extract super-virtual diffraction arrivals. This data set was collected at Exxon's test site located near Friendswood, Texas. The source and receiver intervals are both 3.05 m and the distance between the two wells is 182.9 m. There are 98 shot gathers with 96 traces each in this data set. Figure 4.8(a) shows a raw common shot gather and the target diffraction is outlined with 2 red lines. A zoom view of the diffraction is shown in Figure 4.8(b). In this example, the moveout of the diffraction is manually picked for the purpose of median filtering and Figure 4.8(c) shows the result after applying a median filter along the moveout curve. It is clear that the coherent noise is effectively removed by median filtering. To further improve the result, the super-

virtual diffraction method is applied to the median-filtered events and the result is shown in Figure 4.8(d), which is of much higher SNR compared to Figure 4.8(c).

4.5 Conclusion

I presented the general theory of super-virtual diffraction interferometry where the signal-to-noise ratio (SNR) of diffraction arrivals can be theoretically increased by the factor $\sqrt{N}$, where N is the number of receiver and source positions associated with the recording of the diffractions. There are two steps to this methodology: correlation and summation of the data to generate traces with virtual diffraction arrivals, followed by the convolution and stacking of the data with the virtual traces to create super-virtual diffractions. This method is valid for any medium that generates diffraction arrivals due to isolated subwavelength scatterers. There are at least three benefits with this methodology: 1). the diffraction arrivals can be used as migration operators (Schuster, 2002; Brandsberg-Dahl et al., 2007; Sinha et al., 2009); 2). the diffraction arrivals can be used for estimating source and receiver statics; 3). estimation of velocities by traveltimes tomography or MVA.

The problem with this method is that there will be artifacts associated with coherent events and quality degradation due to a limited recording aperture and a coarse spacing of the source and receivers.

Super-virtual Diffraction Interferometry

a) Crosscorrelate and stack to generate virtual diffractions

b) Convolve to generate virtual diffractions

c) Stack super-virtual diffractions to increase SNR

Figure 4.1: The steps for creating super-virtual diffraction arrivals. (a) Correlation of the recorded trace at $\mathbf{A}$ with that at $\mathbf{B}$ for a source at $\mathbf{x}$ to give the correlated trace $\phi_x(\mathbf{A}, \mathbf{B}, t)$ with the virtual diffraction having traveltime denoted by $\tau_{A'B} - \tau_{A'A}$. This arrival time will be the same for all source positions x , so stacking $\sum_x \phi_x(\mathbf{A}, \mathbf{B}, t)$ will enhance the SNR of the virtual diffraction by $\sqrt{N}$. (b) Similar to that in (a) except the virtual diffraction traces are convolved with the actual diffraction traces and stacked for different geophone positions x' to give the (c) super-virtual trace with an enhanced SNR. Here, N denotes the number of coincident source and receiver positions.

Figure 4.2: (a) Geometry for computing virtual Green's functions $G(\mathbf{B}|\mathbf{A})$ from the recorded data $G(\mathbf{A}|\mathbf{x})$ and $G(\mathbf{B}|\mathbf{x})$ using the reciprocity theorem of correlation type in an arbitrary acoustic medium of constant density. (b) Geometry for computing super-virtual Green's functions $G(\mathbf{B}|\mathbf{A})^{super}$ from the recorded data $G(\mathbf{A}|\mathbf{x}')$ and the virtual data $G(\mathbf{B}|\mathbf{x}')^{virt.}$ using the reciprocity theorem of convolution type.

Figure 4.3: Part of the BP2004 velocity model with three diffractors below the salt body.

Figure 4.4: Synthetic data results for part of the BP2004 model. (a) A common shot gather with a source at offset 6 km. Red lines indicate the time window and the moveout of the diffraction event. (b) The diffraction event within a small time window. (c) The result after median filtering and (d) after processing the median filtered data to get the super-virtual diffraction.

Figure 4.5: Velocity model with a fault and two diffractors.

Figure 4.6: Synthetic data results for the fault model. (a) A common shot gather with a source at offset 36 m. Red lines indicate the time window of the diffraction event. (b) The diffraction event within a small time window. (c) The result after median filtering and (d) after processing the raw data to get the super-virtual diffraction.

Figure 4.7: The super-virtual diffraction. In this figure, the red line indicates the predicted diffraction arrival times and the blue line indicates the picked arrival times.

Figure 4.8: Friendswood cross-well data example. (a) A common shot gather with a source at depth of 36.6 m. Red lines indicate the time window and the moveout of the diffraction event. (b) The diffraction event within a small time window. (c) The result after median filtering and (d) the super-virtual diffraction.

Chapter 5

Conclusions

Chapter 2 presents the novel technique of multisource least-squares reverse time migration to improve the quality of RTM images and increase the computational efficiency. For the 2D examples, the quality of the LSRTM images depends on the accuracy of the migration velocity model and the number of sub-supergathers in the input file. The best image for the 2D HESS VTI model is obtained by migrating eight supergathers with 30 iterations, which shows balanced amplitudes, is almost free of migration artifacts and crosstalk noise, and demonstrates a speedup of 3.75 compared to conventional RTM. The 3D examples illustrate the advantages of least-squares migration: balancing the reflector amplitudes, improving the spatial resolution, and reducing migration artifacts. The empirical examples show that the multisource LSRTM can produce images of better quality with similar computation cost. In the future, the multisource LSRTM algorithm can be tested with more complex physical models such as elastic medium, TTI medium, or viscoelastic medium, than the acoustic medium, and 3D field data tests will further validate the proposed new method.

In Chapter 3, I implemented a frequency-selection encoding strategy to speed up the least-squares reverse time migration of marine data. Traditional random phase encoding method is not applicable to marine data due to the mismatch in acquisition geometry between the observed data and the calculated synthetic data. With

frequency-selection encoding, all the shots are encoded with encoding functions that are orthogonal to each other in the frequency domain, so the calculated synthetic data can be effectively decoded at the receiver locations for comparison with the observed data. Because of the data redundancy in the frequency domain, the frequency sampling rate can be large, which leads to significant computational savings. Numerical tests on part of the Marmousi2 model and a field dataset from Gulf of Mexico show that the frequency-selection encoding can significantly improve the efficiency of the LSRTM and reduce its cost to the level of conventional shot domain RTM. Empirical results suggest that the LSRTM with frequency-selection encoding is an efficient method to produce better images than conventional RTM. 3D application of this method is very significant with an additional degree of freedom. How to optimally choose a subset of shots from thousands of shots is an interesting topic for future research.

In Chapter 4, I presented the general theory of super-virtual diffraction interferometry where the signal-to-noise ratio (SNR) of diffraction arrivals can be theoretically increased by the factor $\sqrt{N}$, where N is the number of receiver and source positions associated with the recording of the diffractions. There are two steps to this methodology: correlation and summation of the data to generate traces with virtual diffraction arrivals, followed by the convolution and stacking of the data with the virtual traces to create super-virtual diffractions. This method is valid for any medium that generates diffraction arrivals due to isolated subwavelength scatterers. There are at least three benefits with this methodology: 1). the diffraction arrivals can be used as migration operators (Schuster, 2002; Brandsberg-Dahl et al., 2007; Sinha et al., 2009); 2). the diffraction arrivals can be used for estimating source and receiver statics; 3). estimation of velocities by traveltime tomography or MVA

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