Cooperative AF Relaying in Spectrum-Sharing Systems: Outage Probability Analysis under Co-Channel Interferences and Relay Selection

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Abstract—For cooperative amplify-and-forward (AF) relaying in spectrum-sharing wireless systems, secondary users share spectrum resources originally licensed to primary users to communicate with each other and, thus, the transmit power of secondary transmitters is strictly limited by the tolerable interference powers at primary receivers. Furthermore, the received signals at a relay and at a secondary receiver are inevitably interfered by the signals from primary transmitters. These co-channel interferences (CCIs) from concurrent primary transmission can significantly degrade the performance of secondary transmission. This paper studies the effect of CCIs on outage probability of the secondary link in a spectrum-sharing environment. In particular, in order to compensate the performance loss due to CCIs, the transmit powers of a secondary transmitter and its relaying node are respectively optimized with respect to both the tolerable interference powers at the primary receivers and the CCIs from the primary transmitters. Moreover, when multiple relays are available, the technique of opportunistic relay selection is exploited to further improve system performance with low implementation complexity. By analyzing lower and upper bounds on the outage probability of the secondary system, this study reveals that it is the tolerable interference powers at primary receivers that dominate the system performance, rather than the CCIs from primary transmitters. System designers will benefit from this result in planning and designing next-generation broadband spectrum-sharing systems.

Index Terms—Amplify-and-forward (AF) relaying, co-channel interference (CCI), interference power constraints, outage probability, relay selection, spectrum sharing.

I. INTRODUCTION

SPECTRUM-SHARING cognitive radio is a promising wireless technique to address the increasing shortage of the indispensable electromagnetic spectrum resources [1]. In particular, as illustrated in Fig. 1-a), spectrum-sharing cognitive radio allows secondary users (SUs) to share spectrum resources originally licensed to primary users (PUs) as long as the interference power coming from SUs remains below a pre-defined tolerable threshold. Therefore, unlike other techniques with exclusive usage of spectrum resources, spectrum-sharing

cognitive radio is more practical to be deployed in next-generation broadband wireless systems. Nevertheless, since the transmit power of SUs is dominated by the tolerable interference power at PUs, the coverage of secondary transmission in spectrum-sharing systems is strictly limited.

In order to extend the coverage of secondary transmission and/or enhance system performance in spectrum-sharing context, cooperative relaying techniques can be further exploited. As shown in Fig. 1-b), an intermediate relaying node between a transmitter and its destination can be used to assist the communication process. In general, there are two relaying protocols: decode-and-forward (DF) and amplify-and-forward (AF). The performance of DF relaying in spectrum-sharing context is widely investigated in the literature, see e.g. [2]–[5] and references therein. However, it is well-known that AF relaying has simpler structure than DF relaying and it can be more easily implemented in practice. In our recent work [6]–[8], system performance of cooperative AF relaying in spectrum-sharing environment is explored extensively. In particular, when multiple AF relays are available in spectrum-sharing systems (cf. Fig. 1-c)), techniques of relay selection can be applied for higher spatial diversity gain [6], [8]. However, in practical systems, PUs and SUs can simultaneously transmit signals by sharing the same spectrum resources. As a result, the relays and secondary receiver inevitably suffer interferences from PUs. From the viewpoint of SUs, these interferences come in the form of co-channel interferences (CCIs) and their effect on system performance is important to be analyzed and quantified. This issue will be addressed in this paper.

Figure 1-d) illustrates the system model under study in this paper. Unlike the system in the absence of CCIs as shown in Fig. 1-c), there are two different categories of interference channels in Fig. 1-d). One pertains to the channels from the secondary transmitter or the relays to the primary receivers, which are indicated by red arrow lines in Fig. 1-d), i.e. $SU_t \rightarrow PU_{r1}$ and $R_i \rightarrow PU_{r2}$, $i = 1, \dots, N$. These interference channels are related to the tolerable interference powers at primary receivers. The other category concerns the channels from primary transmitters to the relays or the secondary receiver, which are sketched by blue arrow lines in Fig. 1-d), i.e. $PU_{t1} \rightarrow R_i$, $i = 1, \dots, N$ and $PU_{t2} \rightarrow SU_r$. These interference channels introduce CCIs to secondary link. Due to the presence of CCIs from concurrent primary transmission, the performance of secondary transmission in

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spectrum-sharing context will be degraded significantly, compared to that in Fig. 1–c). An explicit quantification of this degradation is critical for system designers. This is the main motivation for this paper.

In order to compensate the performance loss due to CCIs, techniques of optimal transmit power allocation at the secondary transmitter and the relays, and of relay selection are adopted in this paper. Specifically, when allocating transmit powers at either the secondary transmitter or the relays, the effects of desired channels, interference channels, tolerable interference powers at primary receivers, and CCIs from primary transmitters are all taken into account. On the other hand, since the transmit power of secondary transmitters is strictly limited in spectrum-sharing systems, techniques of relay selection can be exploited to obtain higher spatial diversity gain and improve system performance. In general, there are two relay-selection strategies: opportunistic relay selection [8] and partial relay selection [6]. The former chooses the relay that has the highest end-to-end (i.e. source $\rightarrow$ relay $\rightarrow$ destination) signal-to-interference-plus-noise ratio (SINR). The latter chooses the relay that achieves the highest SINR at either the source-relay hop or the relay-destination hop. Clearly, opportunistic relay selection outperforms partial relay selection since high SINR at one hop cannot guarantee high SINR at another hop.

The results of this study reveal that it is the tolerable interference powers at primary receivers that dominate the outage probability of the secondary system under consideration, rather than the CCIs from primary transmitters or the average signal-to-noise ratio (SNR) at the secondary transmitter. Furthermore, lower and upper bounds on the outage probability are developed in closed-form under single-relay and multi-relay scenarios. Specifically, for the single-relay case, outage probability of the system is tight with the lower bound. On the other hand, for the multi-relay scenario, opportunistic relay selection can be exploited to improve outage probability significantly and it is highly predictable from the upper bound.

The rest of this paper is organized as follows. Section II details the system model. Section III discusses the criteria for optimal power allocation and relay selection. The distribution functions of the signal-to-interference ratios (SIRs) at two consecutive hops are derived in Section IV. The outage probability of the system with either single relay or multiple relays is analyzed in Section V and, finally, concluding remarks are provided in Section VI.

II. SYSTEM MODEL

Figure 1–d) illustrates a spectrum-sharing cooperative relaying system in the presence of CCIs. In the system, the transmission between two pairs of PUs ($\text{PU}_{t1} \rightarrow \text{PU}_{r1}$ and $\text{PU}_{t2} \rightarrow \text{PU}_{r2}$) occurs simultaneously with the transmission between a pair of SUs ($\text{SU}_t \rightarrow \text{SU}_r$). Also, since the power of secondary transmitter SU_t is strictly limited, N intermediate SUs serve as AF relays (R_i , $i = 1, \dots, N$) to assist the communication process between SU_t and its destination SU_r . It is assumed that all terminals in the system are equipped with single omnidirectional half-duplex antenna each.

In conventional cooperative relaying systems, either in the absence of CCI (cf. Fig. 1–b)) or in the presence of CCIs

[9], the power of a transmitter is generally limited by its own maximum output power. However, in spectrum-sharing cooperative relaying systems, SUs can communicate with each other only when the interference power at PUs remains below pre-defined tolerable levels. Hence, the transmit power of a SU and its relaying node must be dynamically optimized with respect to not only the desired channels (i.e. $\text{SU}_t \rightarrow R_i$ and $R_i \rightarrow \text{SU}_r$, $i = 1, \dots, N$, which are shown by the black arrow lines in Fig.1–d)) but also to the interference channels from the secondary transmitter and the relays to the primary receivers (i.e. $\text{SU}_t \rightarrow \text{PU}_{r1}$ and $R_i \rightarrow \text{PU}_{r2}$, shown by the red arrow lines in Fig.1–d)). On the other hand, unlike the spectrum-sharing system in the absence of CCIs (cf. Fig. 1–c)), the received signals of relay R_i in Fig. 1–d) are interfered by the transmit signals of primary user PU_{t1} , which are shown by the blue arrow lines in Fig.1–d). Similarly, the received signals of SU_r are interfered by the transmit signals of PU_{t2} . In this work, all transmission channels are supposed to be independent of each other and to be subject to block Rayleigh fading. That is, the channels remain invariant during each data transmission block but vary from one block to another.

With the assistance of multiple intermediate AF relays, the data transmission between SU_t and SU_r is performed in two consecutive phases. During the first-hop phase, secondary transmitter SU_t sends signal x with optimal transmit power P_1 (to be determined in the next section) to all relays and, meanwhile, primary transmitter PU_{t1} sends signal s_1 with fixed power P_{s1} to primary receiver PU_{r1} .¹ Consequently, when the CCI from PU_{t1} is considered, the received signal at the i^{th} relay (R_i , $1 \leq i \leq N$) is given by

$$y_i = \sqrt{P_1} f_i x + \sqrt{P_{s1}} q_i s_1 + n_i, \quad (1)$$

where f_i and q_i denote the coefficients of the desired channel from SU_t to R_i and of the interference channel from PU_{t1} to R_i respectively, and n_i is the additive white Gaussian noise (AWGN) at relay R_i with zero mean and variance σ_i^2 . For ease of notation and without loss of generality, it is assumed throughout the rest of the paper that the AWGNs at all relays and at the secondary receiver have the same variance σ^2 .

During the second-hop phase, the relay R_i amplifies its received signal with a power gain β_i and forwards it to the secondary receiver with optimal transmit power P_2 (to be determined in the next section). Meanwhile, primary transmitter PU_{t2} sends signal s_2 with fixed power P_{s2} to primary receiver PU_{r2} . As a consequence, when the CCI from PU_{t2} is taken into account, the received signal at secondary receiver SU_r is given by

$$y = \sqrt{P_2} g_i \beta_i \left(\sqrt{P_1} f_i x + \sqrt{P_{s1}} q_i s_1 + n_i \right) + \sqrt{P_{s2}} q_0 s_2 + n, \quad (2)$$

where g_i and q_0 designate the coefficients of the desired channel from R_i to SU_r and of the interference channel from

¹Since this paper aims at evaluating the performance of cooperative relaying transmission in spectrum-sharing context, the power of the secondary transmitter will be optimized whereas the powers of the primary transmitters are assumed to be fixed. Primary transmitters introduce CCIs at the relays and at the secondary receiver and their effects on the performance of the secondary link will be studied by setting different power values in the simulation experiments presented in Section V.

Fig. 1. The evolution of spectrum-sharing cooperative relaying systems.

PU_{t2} to SU_r respectively, and n stands for the AWGN at SU_r with zero mean and variance σ^2 . Also, without loss of generality, it is assumed that the amplitudes of all transmit signals (i.e. x , s_1 , and s_2) are identical. In the sequel, two different choices of the power gain β_i are made explicitly.

For AF relaying technique, the relay gain aims to invert the first-hop channel while limiting the output power of the relay when the channel-fading amplitude at the first hop is low. In general, there are two methods to determine the relay gain:

- 1) When full channel state information (CSI) is available at relay R_i , that is, when CSI of both the desired channel $SU_t \rightarrow R_i$ and the interference channel $PU_{t1} \rightarrow R_i$ are known at relay R_i , the relay gain is determined by

$$\beta_i = (P_1 |f_i|^2 + P_{s1} |q_i|^2 + \sigma^2)^{-\frac{1}{2}}, \quad (3)$$

where the operator $|x|$ stands for the amplitude of x .

- 2) When only partial CSI is available at relay R_i , that is, when only the CSI of the desired channel $SU_t \rightarrow R_i$ is known at relay R_i , the relay gain is determined by

$$\beta_i = (\sqrt{P_1} f_i)^{-1}. \quad (4)$$

Substituting (3) into (2) and performing some straightforward algebraic manipulations, we obtain the end-to-end ($SU_t \rightarrow R_i \rightarrow SU_r$) SINR, and it is shown to be given by

$$\gamma'_i = \frac{\gamma_{ai} \gamma_{bi}}{\gamma_{ai} + \gamma_{bi} + 1}, \quad (5)$$

where γ_{ai} and γ_{bi} stand for the received SINRs at the first hop ($SU_t \rightarrow R_i$) and at the second hop ($R_i \rightarrow SU_r$) respectively, defined by

$$\gamma_{ai} \triangleq \frac{P_1 |f_i|^2}{P_{s1} |q_i|^2 + \sigma^2}, \quad \gamma_{bi} \triangleq \frac{P_2 |g_i|^2}{P_{s2} |q_0|^2 + \sigma^2}. \quad (6)$$

Similarly, substituting (4) into (2) and performing some algebraic manipulations, we obtain the end-to-end signal-to-interference ratio (SIR) and it is shown to be given by

$$\gamma_i = \frac{\gamma_{1i} \gamma_{2i}}{\gamma_{1i} + \gamma_{2i}}, \quad (7)$$

where γ_{1i} and γ_{2i} represent the received SIRs at the first hop

and at the second hop respectively, defined by

$$\gamma_{1i} \triangleq \frac{P_1 |f_i|^2}{P_{s1} |q_i|^2}, \quad \gamma_{2i} \triangleq \frac{P_2 |g_i|^2}{P_{s2} |q_0|^2}. \quad (8)$$

When the CCIs from PUs are considered, system performance of secondary transmission in spectrum-sharing context is dominated by the interference rather than noise. In essence, the received SIR given by (7) is an upper bound on the SINR in (5) and it becomes tight when the interference-to-noise ratio is medium or high. Moreover, (7) has the advantage of mathematical tractability over (5). Therefore, hereafter the system performance analysis is based on the SIR expression in (7).

III. CRITERIA FOR POWER ALLOCATION AND RELAY SELECTION

In this section, the criteria for optimal transmit power allocation (at either secondary source SU_t or relaying node R_i) and for relay selection are established.

A. Criteria for Optimal Power Allocation

In this subsection, the criteria for optimal transmit powers (P_1 at secondary transmitter SU_t and P_2 at relaying node R_i) are developed. Provided that relay R_i is chosen out of the N available relays (the relay-selection criterion will be discussed in the next subsection), in order to achieve the channel capacity C_1 at the first hop ($SU_t \rightarrow R_i$), by recalling the received SIR γ_{1i} in (8), the optimal transmit power P_1 at SU_t is determined as the solution to the optimization problem:

$$C_1 = \max_{P_1 \geq 0} \mathcal{E}_{P_1, f_i, q_i} \left\{ \log_2 \left(1 + \frac{P_1 |f_i|^2}{P_{s1} |q_i|^2} \right) \right\} \quad (9a)$$

$$\text{s.t. } \mathcal{E}_{P_1, h_0} \{ P_1 |h_0|^2 \} \leq W. \quad (9b)$$

where the operator $\mathcal{E}_X \{ \cdot \}$ denotes mathematical expectation associated with X , h_0 is the coefficient of interference channel from the secondary transmitter to primary receiver PU_{r1} , and W is the average tolerable interference power at PU_{r1} . Clearly, the optimal transmit power P_1 of the secondary transmitter is a function of f_i , q_i , h_0 , P_{s1} , and W , which will be explicitly determined below.

In general, the tolerable interference power at PUs can be defined by means of average interference power, peak interference power, or both [10], [11]. The average interference power constraint has low feedback overhead and it applies to non-real time applications where the quality-of-service (QoS) depends upon the average output SNR. The peak interference power constraint has high feedback overhead since it requires instantaneous CSI of interference channels to determine the transmit power of SUs, and it is appropriate for real-time applications. Moreover, the transmit power of a SU is physically limited by its own maximum output power, which is essentially equivalent to the peak interference power constraint. Nevertheless, it was previously demonstrated that peak interference-power constraint will not affect system performance when it is large enough [11], [12]. Therefore, we consider only the average interference power constraint in the optimization problem above.

Applying the Lagrangian optimization technique to (9a)-(9b), it is straightforward to show that the optimal transmit power of the secondary transmitter is given by

$$P_1 = \left[\frac{\lambda_1}{|h_0|^2} - P_{s1} \frac{|q_i|^2}{|f_i|^2} \right]^+, \quad (9c)$$

where the operator $[x]^+ \triangleq \max(0, x)$ and the parameter λ_1 is determined by the average interference power constraint satisfying the equality in (9b), such that

$$\mathcal{E}_{h_0, f_i, q_i} \left\{ \left[\lambda_1 - P_{s1} \frac{|h_0|^2 |q_i|^2}{|f_i|^2} \right]^+ \right\} = W. \quad (9d)$$

It is worth pointing out that the optimal power allocation in (9c) performs like the well-known water-filling power allocation algorithm constrained by the maximum transmit power [13]. The parameter λ_1 is known as water-level and it will be explicitly determined in Section IV-A below. The operator $[x]^+$ implies that the transmit power P_1 is zero if the strength of the desired channel $|f_i|^2 \leq \frac{P_{s1}}{\lambda_1} |h_0|^2 |q_i|^2$, namely, no data will be transmitted in such case. In other words, only when $|f_i|^2 > \frac{P_{s1}}{\lambda_1} |h_0|^2 |q_i|^2$ can SU_t transmit to all relays, which makes the cooperative relaying transmission in spectrum-sharing context completely different from conventional cooperative relaying systems where the transmitter can send data to its relay at any time, regardless of the channel conditions between them.

During the second-hop phase, the relay R_i that is chosen out of the N available relays forwards the received signal with optimal transmit power P_2 , such that it achieves the channel capacity at the second hop while constrained by the average tolerable interference power W at primary receiver PU_{r2} . It is worthwhile mentioning that, in this paper, the average tolerable interference powers at primary receivers PU_{r1} and PU_{r2} are assumed to be symmetric (i.e. $W_1 = W_2 = W$), because this system outperforms that with non-symmetric interference power constraint (i.e. $W_1 \neq W_2$) provided that the sum of W_1 and W_2 is fixed [8]. Therefore, according to the received SIR γ_{2i} in (8), P_2 satisfies the optimization problem:

$$C_2 = \max_{P_2 \geq 0} \mathcal{E}_{P_2, g_i, q_0} \left\{ \log_2 \left(1 + \frac{P_2 |g_i|^2}{P_{s2} |q_0|^2} \right) \right\} \quad (10a)$$

$$\text{s.t. } \mathcal{E}_{P_2, h_i} \{ P_2 |h_i|^2 \} \leq W, \quad (10b)$$

where h_i denotes the complex coefficient of the interference-channel from relay R_i to primary receiver PU_{r2} . Using a similar approach as in the first-hop phase, P_2 is found to be given by

$$P_2 = \left[\frac{\lambda_2}{|h_i|^2} - P_{s2} \frac{|q_0|^2}{|g_i|^2} \right]^+, \quad (10c)$$

where the parameter λ_2 is determined by

$$\mathcal{E}_{q_0, h_i, g_i} \left\{ \left[\lambda_2 - P_{s2} \frac{|q_0|^2 |h_i|^2}{|g_i|^2} \right]^+ \right\} = W. \quad (10d)$$

It is noteworthy that the instantaneous CSI of interference channels, namely, h_0 and q_i in (9c) and q_0 and h_i in (10c), can be obtained at the SUs by periodic sounding of the pilot signals transmitted by the PUs. Although the acquisition of these CSI requires additional cost at the SUs, it enables them to strictly abide by the interference power constraints dictated by the PUs and to optimize system performance of the secondary transmission.

B. Criteria for Relay Selection

When multiple relays are available to cooperate between secondary transmitter SU_t and its receiver SU_r , multi-relay transmission in parallel incurs severe interference at PUs and they have to work on orthogonal channels in order to avoid interference with each other. Therefore, techniques of relay selection can be further exploited in practical systems to leverage spatial diversity gain while alleviating interference at PUs. That is, only the “best” relay is chosen to assist SU_t in transmitting to SU_r while the other relays stay silent.

In general, there are mainly two relay-selection strategies [8], [14], [15]. One is opportunistic relay selection, which is based on the end-to-end SIRs. The other is partial relay selection, which is based on single-hop SIRs (either the first hop or the second hop). It was previously demonstrated that opportunistic relay selection achieves the diversity gain $G_d = N$ whereas partial relay selection has diversity gain $G_d = 1$ [8]. Therefore, in this paper, opportunistic relay selection is adopted to attain better system performance. Mathematically, the index $\hat{i}$ of the chosen relay is determined by

$$\hat{i} = \arg \max_{i=1, \dots, N} \{ \gamma_i \}, \quad (11)$$

where γ_i is given by (7).

IV. DISTRIBUTIONS FUNCTIONS OF THE SIRs AT CONSECUTIVE HOPS

In this section, we first determine the optimal transmit powers at the secondary transmitter and the relays. Once the power allocation completed, the distribution functions of the SIRs at two consecutive hops are developed in closed-form.

A. Optimal Transmit Powers P_1 and P_2

Based on the power-allocation criteria described in Section III-A, in order to determine the optimal transmit powers P_1 and P_2 , first we need to determine the optimal power allocation parameters λ_1 and λ_2 . By virtue of (9d), in order

to determine the value of λ_1 we derive the probability density function (PDF) of $V \triangleq \frac{|h_0|^2 |q_i|^2}{|f_i|^2}$. Since all channels are supposed to be subject to Rayleigh fading with the same mean $\bar{\gamma}$, the PDFs of $|h_0|^2$, $|q_i|^2$, and $|f_i|^2$ are in the form of

$$f_X(x) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{x}{\bar{\gamma}}\right), \quad X \in \{|h_0|^2, |q_i|^2, |f_i|^2\}. \quad (12)$$

After some tedious but straightforward algebraic derivations, the PDF of V is obtained as in the following lemma.

Lemma 1: If three independent channel gains $|h_0|^2$, $|q_i|^2$, and $|f_i|^2$ are of Rayleigh distribution with the same mean $\bar{\gamma}$, then the PDF of $V = \frac{|h_0|^2 |q_i|^2}{|f_i|^2}$ is given by

$$f_V(x) = \frac{1}{\bar{\gamma}} \Psi\left(2, 1; \frac{x}{\bar{\gamma}}\right), \quad (13)$$

where $\Psi(a, b; x)$ denotes the Tricomi confluent hypergeometric function [16, Eq.(9.210.2)].

Proof: See Appendix A. ■

Before proceeding further, we introduce the following integral equality associated with the Meijer's G -function, which will be applied repeatedly in the sequel.

Lemma 2: For $c, \alpha > 0$, the following integral equality holds

$$\int_0^c x^{\alpha-1} G_{p,q}^{m,n} \left[x \left| \begin{matrix} (a_p) \\ (b_q) \end{matrix} \right. \right] dx = G_{p+1,q+1}^{m,n+1} \left[c \left| \begin{matrix} 1, (a_p + \alpha) \\ (b_q + \alpha), 0 \end{matrix} \right. \right], \quad (14)$$

where $G[x|\cdot]$ denotes the Meijer's G -function [16, Eq.(9.301)].

Proof: By virtue of [17, vol.3, Eq.(2.24.2.2)] and the scaling property of Meijer's G -function [17, vol.3, Eq.(8.2.2.15)], (14) can be derived in a straightforward manner. ■

After obtaining the PDF of V in (13) and substituting it into (9d), the power allocation parameter λ_1 can be shown to be determined by (15) at the top of the next page, where $c_1 \triangleq \frac{\lambda_1}{\bar{\gamma} P_{s1}}$. Subsequently, by expressing Tricomi confluent hypergeometric function in the integrand of (15) in terms of Meijer's G -function [17, vol.3, Eq.(8.4.46.1)]:

$$\Psi(a, b; x) = \frac{1}{\Gamma(a)\Gamma(a-b+1)} G_{1,2}^{2,1} \left[x \left| \begin{matrix} 1-a \\ 0, 1-b \end{matrix} \right. \right], \quad (16)$$

(15) can be rewritten as (17)–(19) at the top of the next page, where (14) was used to derive (18), and the definition of c_1 immediately after (15) was substituted into (18) and the scaling property of Meijer's G -function [17, vol.3, Eq.(8.2.2.15)] was exploited to reach (19). It is noteworthy that (19) implies that, when the average tolerable interference power W at PUs is fixed, the value of the power-allocation parameter λ_1 depends only upon the product of $\bar{\gamma}$ and P_{s1} . That is, increasing the mean $\bar{\gamma}$ of desired-channel gain is equivalent to decreasing the power P_{s1} of CCI, and vice versa. This provides high flexibility to improve system performance of secondary transmission in spectrum-sharing context.

Although the power-allocation parameter λ_1 in (19) cannot be explicitly expressed in closed-form, it can be easily obtained in a numerical way because Meijer's G -function is a built-in function in popular mathematical softwares, such as Matlab and Mathematica. Moreover, since the tolerable

interference powers at the primary receivers are symmetric (i.e. $W_1 = W_2 = W$), it is straightforward that the power allocation parameter at the second hop is identical to that at the first hop, that is, $\lambda_2 = \lambda_1$. With the values of λ_1 and λ_2 established, the optimal transmit powers P_1 and P_2 can be readily determined by using (9c) and (10c), respectively. Afterwards, with the resultant P_1 and P_2 , we derive the distribution functions of the SIRs at consecutive hops in the next subsection.

B. Distribution Functions of the SIRs γ_{1i} and γ_{2i}

Substituting P_1 in (9c) into the expression of γ_{1i} in (8), the SIR pertaining to the first hop can be given by

$$\gamma_{1i} = \left[\frac{\lambda_1}{P_{s1}} \frac{|f_i|^2}{|h_0|^2 |q_i|^2} - 1 \right]^+ = \left[\frac{\lambda_1}{P_{s1}} V' - 1 \right]^+, \quad (20)$$

where $V' \triangleq \frac{|f_i|^2}{|h_0|^2 |q_i|^2} = \frac{1}{V}$. Upon using a similar approach as in the proof of Lemma 1, the PDF of V' can be shown to be expressed as

$$f_{V'}(x) = \frac{1}{\bar{\gamma} x^2} \Psi\left(2, 1; \frac{1}{\bar{\gamma} x}\right). \quad (21)$$

In view of (20)–(21) and performing some tedious mathematical derivations by means of the powerful Meijer's G -function, the distribution functions of γ_{1i} can be obtained in compact forms and they are given in the following theorem.

Theorem 1: For the spectrum-sharing cooperative AF relaying system in the presence of CCIs, the PDF and CDF of the received SIR at the first hop, i.e. γ_{1i} in (20), are given by

$$f_{\gamma_{1i}}(\gamma) = \frac{1}{(\gamma+1)^2 G_1} \Psi\left(2, 1; \frac{c_1}{\gamma+1}\right) \quad (22)$$

and

$$F_{\gamma_{1i}}(\gamma) = 1 - \frac{1}{c_1 G_1} G_{2,3}^{2,2} \left[\frac{c_1}{\gamma+1} \left| \begin{matrix} 1, 0 \\ 1, 1, 0 \end{matrix} \right. \right], \quad (23)$$

respectively, where the parameter G_1 is determined by

$$G_1 \triangleq G_{2,3}^{2,2} \left[c_1 \left| \begin{matrix} 0, -1 \\ 0, 0, -1 \end{matrix} \right. \right]. \quad (24)$$

Proof: See Appendix B. ■

Using a similar approach as in the proof of Theorem 1, the distribution functions of γ_{2i} can be derived and are summarized in the following theorem.

Theorem 2: For the spectrum-sharing cooperative AF relaying system in the presence of CCIs, the PDF and CDF of the received SIR at the second hop, i.e. γ_{2i} in (8), are given by

$$f_{\gamma_{2i}}(\gamma) = \frac{1}{(\gamma+1)^2 G_2} \Psi\left(2, 1; \frac{c_2}{\gamma+1}\right) \quad (25)$$

and

$$F_{\gamma_{2i}}(\gamma) = 1 - \frac{1}{c_2 G_2} G_{2,3}^{2,2} \left[\frac{c_2}{\gamma+1} \left| \begin{matrix} 1, 0 \\ 1, 1, 0 \end{matrix} \right. \right], \quad (26)$$

respectively, where the parameters c_2 and G_2 are defined as

$$c_2 \triangleq \frac{\lambda_2}{\bar{\gamma} P_{s2}}, \quad (27)$$

$$\begin{aligned}
W &= \int_0^{\frac{\lambda_1}{P_{s1}}} (\lambda_1 - x P_{s1}) f_V(x) dx \\
&= \frac{\lambda_1}{\bar{\gamma}} \int_0^{\frac{\lambda_1}{P_{s1}}} \Psi\left(2, 1; \frac{x}{\bar{\gamma}}\right) dx - \frac{P_{s1}}{\bar{\gamma}} \int_0^{\frac{\lambda_1}{P_{s1}}} x \Psi\left(2, 1; \frac{x}{\bar{\gamma}}\right) dx \\
&= \lambda_1 \int_0^{c_1} \Psi(2, 1; x) dx - \bar{\gamma} P_{s1} \int_0^{c_1} x \Psi(2, 1; x) dx,
\end{aligned} \tag{15}$$

$$W = \lambda_1 \int_0^{c_1} G_{1,2}^{2,1} \left[x \middle| \begin{matrix} -1 \\ 0, 0 \end{matrix} \right] dx - \bar{\gamma} P_{s1} \int_0^{c_1} x G_{1,2}^{2,1} \left[x \middle| \begin{matrix} -1 \\ 0, 0 \end{matrix} \right] dx \tag{17}$$

$$= \lambda_1 G_{2,3}^{2,2} \left[c_1 \middle| \begin{matrix} 1, 0 \\ 1, 1, 0 \end{matrix} \right] - \bar{\gamma} P_{s1} G_{2,3}^{2,2} \left[c_1 \middle| \begin{matrix} 1, 1 \\ 2, 2, 0 \end{matrix} \right] \tag{18}$$

$$= \bar{\gamma} P_{s1} \left\{ G_{2,3}^{2,2} \left[\frac{\lambda_1}{\bar{\gamma} P_{s1}} \middle| \begin{matrix} 2, 1 \\ 2, 2, 1 \end{matrix} \right] - G_{2,3}^{2,2} \left[\frac{\lambda_1}{\bar{\gamma} P_{s1}} \middle| \begin{matrix} 1, 1 \\ 2, 2, 0 \end{matrix} \right] \right\}, \tag{19}$$

$$G_2 \triangleq G_{2,3}^{2,2} \left[c_2 \middle| \begin{matrix} 0, -1 \\ 0, 0, -1 \end{matrix} \right]. \tag{28}$$

With the distribution functions of γ_{1i} and γ_{2i} established, in the next section we analyze the outage probability of the end-to-end SIR at the secondary receiver.

V. ANALYSIS OF OUTAGE PROBABILITY

Due to the high complexity of the distribution functions of the SIRs at consecutive hops as stated in *Theorems 1 and 2* above, it is mathematically intractable to derive the exact distribution functions of the end-to-end SIR given in (7). Therefore, in this section we analyze the lower and upper bounds on the outage probability for single-relay and multi-relay scenarios, so as to gain insight into system performance.

A. Single-Relay Scenario

We start by looking at the lower and upper bounds of the end-to-end SIR γ_i . In view of (7), we recognize that [18]

$$\underbrace{\frac{1}{2} \min\{\gamma_{1i}, \gamma_{2i}\}}_{\gamma_{i-\text{low}}} \leq \gamma_i = \frac{\gamma_{1i}\gamma_{2i}}{\gamma_{1i} + \gamma_{2i}} \leq \underbrace{\min\{\gamma_{1i}, \gamma_{2i}\}}_{\gamma_{i-\text{up}}}, \tag{29}$$

where the lower bound is achieved when γ_{1i} and γ_{2i} are symmetric (i.e. $\gamma_{1i} = \gamma_{2i}$) and the upper bound is achieved when γ_{1i} and γ_{2i} are non-symmetric (i.e. $\gamma_{1i} \gg \gamma_{2i}$ or $\gamma_{1i} \ll \gamma_{2i}$). Notice that, for notational consistency, we still use γ_i to express the end-to-end SIR even when there is only a single relay.

The outage probability is defined as the probability that the instantaneous end-to-end SIR falls below a predefined threshold value γ_{th} . Mathematically, the outage probability is expressed as

$$P_{\text{outage}}^{\text{single}}(\gamma_{\text{th}}) = \Pr\{\gamma_i < \gamma_{\text{th}}\} = F_{\gamma_i}(\gamma_{\text{th}}), \tag{30}$$

where the superscript “single” of $P_{\text{outage}}^{\text{single}}(\gamma_{\text{th}})$ stands for the single-relay case. Intuitively, (30) means that the outage

probability can be obtained by evaluating the CDF of the end-to-end SIR at the threshold γ_{th} . Moreover, since outage probability is a monotonically decreasing function of the end-to-end SIR, substituting (29) into (30) yields the lower and upper bounds on outage probability. That is,

$$F_{\gamma_{i-\text{up}}}(\gamma_{\text{th}}) \leq P_{\text{outage}}^{\text{single}}(\gamma_{\text{th}}) \leq F_{\gamma_{i-\text{low}}}(\gamma_{\text{th}}). \tag{31}$$

Since the SIRs γ_{1i} and γ_{2i} at consecutive hops are independent of each other, by recalling the theory of order statistics, it is straightforward to show that the CDF of $\gamma_{i-\text{up}} \triangleq \min\{\gamma_{1i}, \gamma_{2i}\}$ is given by

$$\begin{aligned}
F_{\gamma_{i-\text{up}}}(\gamma) &= 1 - [1 - F_{\gamma_{1i}}(\gamma)][1 - F_{\gamma_{2i}}(\gamma)] \\
&= F_{\gamma_{1i}}(\gamma) + F_{\gamma_{2i}}(\gamma) - F_{\gamma_{1i}}(\gamma)F_{\gamma_{2i}}(\gamma)
\end{aligned} \tag{32}$$

On the other hand, in light of (29), it is evident that $\gamma_{i-\text{low}} = \frac{1}{2}\gamma_{i-\text{up}}$. Hence, by virtue of (32), the CDF of $\gamma_{i-\text{low}}$ can be expressed as

$$F_{\gamma_{i-\text{low}}}(\gamma) = F_{\gamma_{1i}}(2\gamma) + F_{\gamma_{2i}}(2\gamma) - F_{\gamma_{1i}}(2\gamma)F_{\gamma_{2i}}(2\gamma). \tag{33}$$

Consequently, substituting (32) and (33) into (30) gives us the lower and upper bounds on outage probability and they are

$$\begin{aligned}
&F_{\gamma_{1i}}(\gamma_{\text{th}}) + F_{\gamma_{2i}}(\gamma_{\text{th}}) - F_{\gamma_{1i}}(\gamma_{\text{th}})F_{\gamma_{2i}}(\gamma_{\text{th}}) \\
&\leq P_{\text{outage}}^{\text{single}}(\gamma_{\text{th}}) \leq \\
&F_{\gamma_{1i}}(2\gamma_{\text{th}}) + F_{\gamma_{2i}}(2\gamma_{\text{th}}) - F_{\gamma_{1i}}(2\gamma_{\text{th}})F_{\gamma_{2i}}(2\gamma_{\text{th}}).
\end{aligned} \tag{34}$$

It will be shown below that simulation results of outage probability is very tight with the lower bound computed by (34). Furthermore, in order to gain insights into system performance, we investigate the effects of tolerable interference power, average SNR and CCI on the outage probability by deriving the diversity order and the coding gain as follows. Specifically, by virtue of (34), the CDF of the received SIR can be approximately given by the lower bound and it is dominated by the worse link between consecutive hops, that is, $F_{\gamma_i}(\gamma) \approx \max\{F_{\gamma_{1i}}(\gamma), F_{\gamma_{2i}}(\gamma)\}$. Without loss of generality, assuming $F_{\gamma_{1i}}(\gamma) > F_{\gamma_{2i}}(\gamma)$ leads to $F_{\gamma_i}(\gamma) \approx F_{\gamma_{1i}}(\gamma)$. Thus,

the PDF of the received SIR can be approximately given by $f_{\gamma_i}(\gamma) \approx f_{\gamma_{1i}}(\gamma)$. Then, according to [20], if the limit of the PDF of the received SIR can be expressed as:

$$\lim_{\gamma \rightarrow 0} f_{\gamma_i}(\gamma) = a\gamma^t + o(\gamma^{t+\epsilon}), \quad (35)$$

where $a, \epsilon > 0$ and $o(\cdot)$ pertain to the Landau notation, then the diversity order G_d is given by

$$G_d = t + 1 \quad (36)$$

and the coding gain is proportional to $a^{-\frac{1}{t+1}}$, i.e.

$$G_c \sim a^{-\frac{1}{t+1}}. \quad (37)$$

In light of (22), it is straightforward that

$$\begin{aligned} \lim_{\gamma \rightarrow 0} f_{\gamma_{1i}}(\gamma) &= \frac{1}{G_1} \Psi(2, 1; c_1) \\ &= \frac{-1 + (1 + c_1) e^{c_1} \Gamma(0, c_1)}{e^{c_1} \Gamma(0, c_1)} \\ &= 1 + c_1 - \frac{e^{-c_1}}{\Gamma(0, c_1)}. \end{aligned} \quad (38)$$

At high SNR (i.e. large $\bar{\gamma}$), $c_1 = \frac{\lambda_1}{\bar{\gamma} P_{s1}} \rightarrow 0$, and we have $\Gamma(0, c_1) \sim -\ln c_1$ [21, Eq. (3.42)] and $e^{-c_1} \sim 1 - c_1$. Substituting them into (38) leads to

$$\begin{aligned} \lim_{\gamma \rightarrow 0} f_{\gamma_{1i}}(\gamma) &= 1 + c_1 + \frac{1 - c_1}{\ln c_1} \\ &\sim 1 - c_1. \end{aligned} \quad (39)$$

Consequently, applying (35)-(37) to (39) yields

$$G_d = 1 \quad (40)$$

and

$$G_c \sim 1 + \frac{\lambda_1}{\bar{\gamma} P_{s1}}, \quad (41)$$

where the value of c_1 shown immediately after (15) was substituted into (39) to reach (41). Clearly, (41) shows that the coding gain increases with λ_1 while decreases with $\bar{\gamma}$ and P_{s1} . This implies that the power-allocation parameter λ_1 has more significant effect on coding gain than the average SNR ($\bar{\gamma}$) and CCI (P_{s1}), which will be illustrated further by simulation results below.

In order to confirm the efficiency of the preceding analysis, extensive simulation experiments are performed to compare the simulation results of outage probability with the analytical results of the above lower and upper bounds. In the Monte-Carlo simulations, all channels are subject to Rayleigh fading with unit mean and the variance of AWGNs at all nodes is set to unity. The power-allocation parameters at the secondary transmitter and the relay are off-line computed as per (19). Furthermore, for each channel realization, the optimal transmit powers at the secondary transmitter and the relay are determined by using (9c) and (10c), respectively.

On the one hand, the outage probability of the secondary receiver versus tolerable interference powers at primary receivers is illustrated in Figs. 2 and 3, where the average SNR at the secondary transmitter is fixed at 20dB and the outage threshold is set to 0dB. In particular, Fig. 2 focuses on the scenarios with symmetric CCIs. That is, the CCI at

Fig. 2. Outage probability versus tolerable interference powers at primary receivers for the single-relay scenario with symmetric co-channel interferences.

Fig. 3. Outage probability versus tolerable interference powers at primary receivers for the single-relay scenario with non-symmetric co-channel interferences, in comparison with the scenario with symmetric interferences.

the relay (CCI_1) is identical to that at the secondary receiver (CCI_2), i.e. $CCI_1 = CCI_2$. It is observed from Fig. 2 that the simulation results of outage probability are very tight with the lower bound computed by (34). Moreover, outage probability decreases with increasing tolerable interference power (W) at primary receivers, since larger value of W allows higher transmit power at the secondary transmitter. It is also attested that outage probability deteriorates with higher CCI from concurrent primary transmission, which is in agreement with the observation in [19]. Figure 3 focuses on the scenario with non-symmetric CCIs, in contrast to the symmetric case with same sum of CCI_1 and CCI_2 . It is observed that the simulation results are still close to the lower bound. Furthermore, it is seen that the non-symmetric scenario under-performs the symmetric scenario in terms of outage probability, since the end-to-end SIR is dominated by the SIR of the worse link between two consecutive hops, as suggested by (29).

On the other hand, the outage probability of the secondary receiver versus the average SNR at the secondary transmitter is illustrated in Figs. 4 and 5, where the tolerable interference powers at primary receivers are symmetric and are fixed at 20dB, and the outage threshold is set to 0dB. Specifically, Fig. 4 focuses on the scenarios with symmetric CCIs. It is observed that the simulation results are very tight with the lower bound computed by (34). Furthermore, the outage probability deteriorates with increasing CCIs introduced by primary transmitters, as expected. Figure 5 treats the scenario with non-symmetric CCIs, in contrast to the symmetric case with the same sum of CCI_1 and CCI_2 . It is found that

Fig. 4. Outage probability versus average SNR at the secondary transmitter for the single-relay scenario with symmetric co-channel interferences.

Fig. 5. Outage probability versus average SNR at the secondary transmitter for the single-relay scenario with non-symmetric co-channel interferences, in comparison with the scenario with symmetric interferences.

the simulation results are still close to the lower bound. Moreover, the non-symmetric scenario performs worse than the symmetric scenario, due to the same reason as described above.

Subsequently, Fig. 6 demonstrates the effect of CCIs from concurrent primary transmission on outage probability of the secondary receiver, where the average SNR at the secondary transmitter is fixed at 20dB and the tolerable interference powers at primary receivers are set to $W_1 = W_2 = 5\text{dB}$ and $W_1 = W_2 = 20\text{dB}$. In this figure, the abscissa stands for the symmetric CCIs from primary transmitter (i.e. $P_{s1} = P_{s2}$) and it decreases from 35dB to 5dB. It is observed that the simulation results of outage probability are tight with the lower bound computed by (34). Also, it is seen that outage probability improves with decreasing CCIs, as expected.

In the last but not the least, we compare Fig. 2, Fig. 4, and Fig. 6. It is observed that the curves in Fig. 2 demonstrate larger coding gain than the curves in Fig. 4 and Fig. 6. This means that the tolerable interference powers at primary receivers have more significant effect on outage probability than the average SNR at the secondary transmitter and the CCIs from primary transmitters. This is because the tolerable interference powers determine not only the transmit powers of the secondary transmitter and the relay, but also the gain of optimal power allocation. Furthermore, this observation is consistent with the preceding analysis on coding gain.

In summary, when there is only a single relay in the system under study, we conclude that, 1) it is the tolerable interference powers at primary receivers that dominate system

Fig. 6. Outage probability versus co-channel interferences for the single-relay scenario with symmetric tolerable interference powers at primary receivers.

performance, rather than the CCIs from concurrent primary transmission or the average SNR at the secondary transmitter; 2) the scenario with symmetric CCIs at the relay and the secondary receiver outperforms that with non-symmetric CCIs in terms of outage probability; 3) outage probability of the system is highly predictable from the lower bound that is analytically provided in (34) together with (23) and (26).

B. Multi-Relay Scenario

When there are $N > 1$ relays available, opportunistic relay selection described in Section III-B can be exploited to achieve higher spatial diversity gain so as to improve the outage performance of secondary link without extra transmit power. Specifically, the relay R_i , $i = 1, \dots, N$, that has the maximum end-to-end SIR will be chosen to cooperate between the secondary transmitter and its receiver, while the other $N - 1$ relays keep silent. Mathematically, the maximum end-to-end SIR is given by

$$\gamma_{\max} = \max_{i=1, \dots, N} \gamma_i. \quad (42)$$

Substituting (29) into (42) yields the lower and upper bounds on $\gamma_{\max}$, given by

$$\underbrace{\max_{i=1, \dots, N} \gamma_{i-\text{low}}}_{\gamma_{\text{low}}} \leq \gamma_{\max} \leq \underbrace{\max_{i=1, \dots, N} \gamma_{i-\text{up}}}_{\gamma_{\text{up}}}. \quad (43)$$

Notice that, when there are multiple relaying nodes and opportunistic relay selection is adopted, the end-to-end SIRs are not independent of each other because of the common interference-power constraint at secondary transmitter and the common CCI at the secondary receiver (cf. h_0 and q_0 in Fig. 1-d)). Due to potential correlation effect, exact analysis of the lower and upper bounds given by (43) is mathematically intractable.² In order to proceed, we relax the lower bound and approximate the upper bound shown in (43). In particular, the approximated upper bound will be shown to be very tight with simulation results. More specifically, the CDF of the lower

²As pointed out by one of the anonymous reviewers, the correlation between received SNRs in spectrum-sharing systems with relay selection was recently addressed in [12], [22], [23]. In such case, however, as stated by the authors of [12], [22], accurate analysis for the outage probability of the highest received SNR is mathematically intractable. [23] deals with the selection between direct link and relaying link with no relay selection.

Fig. 7. The effect of opportunistic relay selection on outage probability of the secondary receiver.

bound γ_{low} shown in (43) can be further bounded by

$$F_{\gamma_{\text{low}}}(\gamma) < F_{\gamma'_{\text{low}}}(\gamma) \\ = [F_{\gamma_{1i}}(2\gamma) + F_{\gamma_{2i}}(2\gamma) - F_{\gamma_{1i}}(2\gamma)F_{\gamma_{2i}}(2\gamma)]^N, \quad (44)$$

where $\gamma'_{\text{low}} \triangleq \max_{i=1, \dots, N} \gamma_{i-\text{low}}$ in which $\gamma_{i-\text{low}}$, $i = 1, \dots, N$ are assumed to be independent of each other, and (32) was exploited to derive the equality in (44). Similarly, the CDF of the upper bound γ_{up} given in (43) can be further bounded by

$$F_{\gamma_{\text{up}}}(\gamma) < F_{\gamma'_{\text{up}}}(\gamma) \\ = [F_{\gamma_{1i}}(\gamma) + F_{\gamma_{2i}}(\gamma) - F_{\gamma_{1i}}(\gamma)F_{\gamma_{2i}}(\gamma)]^N, \quad (45)$$

where $\gamma'_{\text{up}} \triangleq \max_{i=1, \dots, N} \gamma_{i-\text{up}}$ in which $\gamma_{i-\text{up}}$, $i = 1, \dots, N$ are assumed to be independent of each other, and (33) was exploited to derive the equality in (45). As a result, recalling the fact that outage probability is a monotonically decreasing function of the end-to-end SIR, and substituting (44) and (45) into (43) yields the lower and upper bounds on the outage probability of the end-to-end SIR, given by

$$[F_{\gamma_{1i}}(\gamma_{\text{th}}) + F_{\gamma_{2i}}(\gamma_{\text{th}}) - F_{\gamma_{1i}}(\gamma_{\text{th}})F_{\gamma_{2i}}(\gamma_{\text{th}})]^N \\ < P_{\text{outage}}^{\text{multi}}(\gamma_{\text{th}}) \lesssim \\ [F_{\gamma_{1i}}(2\gamma_{\text{th}}) + F_{\gamma_{2i}}(2\gamma_{\text{th}}) - F_{\gamma_{1i}}(2\gamma_{\text{th}})F_{\gamma_{2i}}(2\gamma_{\text{th}})]^N, \quad (46)$$

where the superscript “multi” of $P_{\text{outage}}^{\text{multi}}(\gamma_{\text{th}})$ stands for the multi-relay case and the operator $\lesssim$ means “approximately less than”.

Figure 7 illustrates the effect of opportunistic relay selection on outage probability versus the average SNR at the secondary transmitter. In the simulations, the tolerable interference powers at primary receivers are set to 20dB (i.e. $W_1 = W_2 = 20\text{dB}$) and the CCIs from primary transmitters are symmetric with $\text{CCI}_1 = \text{CCI}_2 = 5\text{dB}$ on the left-hand side panel and $\text{CCI}_1 = \text{CCI}_2 = 10\text{dB}$ on the right-hand side panel. It is seen that, when the number of relays increases, outage probability decreases significantly due to higher spatial diversity gain. Essentially, by use of a similar approach as in [8], it is easy to show that the diversity order is N when there are N relays and opportunistic relay selection is adopted in the communication process between the secondary transmitter and its receiver. These different diversity orders correspond to the slopes of the curves associated with different number of relays in Fig. 7.

On the other hand, it is observed from Fig. 7 that, when there are $N = 2$ or 3 relays, the simulation results of outage probability are very tight with the approximate upper bound offered by (46) in closed-form. However, when there are $N = 4$ relays, the simulation results are slightly larger than the approximate upper bound. This implies that when the number of relays increases and opportunistic relay selection is adopted, the effect of the correlation between the end-to-end SIRs becomes more evident. In practice, an increased number of relays (N) costs larger feedback overhead ($\log_2 N$) and, thus, it is widely accepted that up to four relays with 2-bit feedback overhead is feasible in real-world wireless systems with relay selection. Finally, it is remarkable that, when the number of relays increases from $N = 1$ to $N > 1$ and opportunistic relay selection is adopted, the observation that the outage probability is tight with the lower bound for the single-relay case and moves closer and closer to the upper bound when there are more and more relays available is not surprising. The reason behind this is as follows. In the single-relay case, the end-to-end SIR approaches its upper bound shown in (29) due to the random fluctuations of channel fading, i.e. $\gamma_{1i} \neq \gamma_{2i}$. However, when there are more and more relays, the maximum end-to-end SIR approaches its lower bound shown in (43) since γ_{1i} and γ_{2i} are symmetric. A similar phenomenon can be observed, for example in [24, Fig. 6] and [25, Fig. 1].

In summary, since the transmit power at SUs in spectrum-sharing systems is strictly limited and the CCIs from concurrent primary transmission are inevitable, opportunistic relay selection can be exploited to improve outage probability of secondary link significantly, and this probability is well predictable from the approximate upper bound analytically provided in (46).

VI. CONCLUSIONS

In spectrum-sharing cooperative AF relaying systems, the effect of CCIs from concurrent primary transmission on the outage probability of secondary transmission was analytically investigated in this paper. In order to combat the CCIs, the transmit powers of a secondary transmitter and its relays were optimized with respect to both the average tolerable interference powers at primary receivers and the CCIs from primary transmitters. Subsequently, the lower and upper bounds on outage probability of the secondary receiver were explicitly derived. In particular, when there is only a single relay, the outage probability closely approaches the lower bound. When there are multiple relays, opportunistic relay selection can be exploited to further improve system performance and the outage probability is very tight with the upper bound. By analyzing these bounds on outage probability, this study reveals that it is the tolerable interference powers at PUs that dominate system performance of secondary transmission in spectrum-sharing context, even though the CCIs from PUs will degrade it.

APPENDIX A PROOF OF LEMMA 1

Since the channel gains $|h_0|^2$, $|q_i|^2$, and $|f_i|^2$ are assumed to be independent of each other, we can rewrite $V = \frac{|h_0|^2 |q_i|^2}{|f_i|^2}$

as $V = X|q_i|^2$, where $X \triangleq \frac{|h_0|^2}{|f_i|^2}$. In the sequel, we first derive the PDF of X and then the PDF of V .

Conditioning on $|f_i|^2$, the PDF of X can be given by

$$\begin{aligned} f_X(x) &= \int_0^\infty f_{X||f_i|^2}(x|y) f_{|f_i|^2}(y) dy \\ &= \int_0^\infty y f_{|h_0|^2}(xy) f_{|f_i|^2}(y) dy \end{aligned} \quad (47)$$

$$= \frac{1}{\bar{\gamma}^2} \int_0^\infty y \exp\left[-\frac{1}{\bar{\gamma}}(x+1)y\right] dy \quad (48)$$

$$= (x+1)^{-2}, \quad (49)$$

where (12) was substituted into (47) to derive (48), and [16, Eq.(3.351.3)] was used to reach (49). Moreover, using a similar approach as in the above, the PDF of V can be derived as

$$\begin{aligned} f_V(x) &= \int_0^\infty f_{V||q_i|^2}(x|y) f_{|q_i|^2}(y) dy \\ &= \int_0^\infty \frac{1}{y} f_X\left(\frac{x}{y}\right) f_{|q_i|^2}(y) dy \end{aligned} \quad (50)$$

$$= \frac{1}{\bar{\gamma}} \int_0^\infty \frac{y}{(x+y)^2} \exp\left(-\frac{y}{\bar{\gamma}}\right) dy \quad (51)$$

$$= \frac{1}{\bar{\gamma}} \Psi\left(2, 1; \frac{x}{\bar{\gamma}}\right), \quad (52)$$

where (12) was inserted again in (50) to derive (51), and [17, vol.1, Eq.(2.3.6.9)] was used to obtain (52). This leads to the intended result shown in (13).

APPENDIX B PROOF OF THEOREM 1

According to (20), the CDF of the SIR γ_{1i} at the first hop can be expressed by definition as

$$\begin{aligned} F_{\gamma_{1i}}(\gamma) &= \Pr\{\gamma_{1i} = 0\} + \Pr\{\gamma_{1i} < \gamma | \gamma_{1i} > 0\}, \gamma > 0 \end{aligned} \quad (53)$$

$$= \Pr\{\gamma_{1i} = 0\} + \frac{\Pr\{0 < \gamma_{1i} < \gamma\}}{\Pr\{\gamma_{1i} > 0\}} \quad (54)$$

$$\begin{aligned} &= \Pr\left\{V' \leq \frac{P_{s1}}{\lambda_1}\right\} \\ &\quad + \frac{\Pr\left\{V' < \frac{P_{s1}}{\lambda_1}(\gamma+1)\right\} - \Pr\left\{V' \leq \frac{P_{s1}}{\lambda_1}\right\}}{1 - \Pr\left\{V' \leq \frac{P_{s1}}{\lambda_1}\right\}} \end{aligned} \quad (55)$$

$$= F_{V'}\left(\frac{P_{s1}}{\lambda_1}\right) + \frac{F_{V'}\left(\frac{P_{s1}}{\lambda_1}(\gamma+1)\right) - F_{V'}\left(\frac{P_{s1}}{\lambda_1}\right)}{1 - F_{V'}\left(\frac{P_{s1}}{\lambda_1}\right)} \quad (56)$$

where the second term $\Pr\{\gamma_{1i} < \gamma | \gamma_{1i} > 0\}$ in the right-hand side of (53) denotes the conditional probability that $\gamma_{1i} < \gamma$ occurs given that $\gamma_{1i} > 0$ has occurred, and (20) was exploited to derive (55). Then, differentiating (56) with respect to γ yields the PDF of γ_{1i} , given by

$$f_{\gamma_{1i}}(\gamma) = \frac{\frac{P_{s1}}{\lambda_1} f_{V'}\left(\frac{P_{s1}}{\lambda_1}(\gamma+1)\right)}{1 - F_{V'}\left(\frac{P_{s1}}{\lambda_1}\right)}. \quad (57)$$

By virtue of (21), the term $F_{V'}\left(\frac{P_{s1}}{\lambda_1}\right)$ in the denominator

of (57) can be computed as

$$F_{V'}\left(\frac{P_{s1}}{\lambda_1}\right) = \int_0^{\frac{P_{s1}}{\lambda_1}} \frac{1}{\bar{\gamma} x^2} \Psi\left(2, 1; \frac{1}{\bar{\gamma} x}\right) dx \quad (58)$$

$$= \int_0^{\frac{P_{s1}}{\lambda_1}} \frac{1}{x^2} \Psi\left(2, 1; \frac{1}{x}\right) dx \quad (59)$$

$$= \int_{\frac{\lambda_1}{\bar{\gamma} P_{s1}}}^\infty \Psi(2, 1; x) dx \quad (60)$$

$$\begin{aligned} &= \int_0^\infty \Psi(2, 1; x) dx \\ &\quad - \int_0^{\frac{\lambda_1}{\bar{\gamma} P_{s1}}} \Psi(2, 1; x) dx \end{aligned} \quad (61)$$

$$= 1 - \frac{\lambda_1}{\bar{\gamma} P_{s1}} G_{2,3}^{2,2} \left[\frac{\lambda_1}{\bar{\gamma} P_{s1}} \middle| \begin{matrix} 0, -1 \\ 0, 0, -1 \end{matrix} \right] \quad (62)$$

where [16, Eq.(7.612.2)] was used to derive the first integral term in (61), and the second integral term in (61) was derived by using a similar approach as in (15)-(17). Consequently, substituting (62) into (57) yields

$$f_{\gamma_{1i}}(\gamma) = \frac{\bar{\gamma}}{G_1} \left(\frac{P_{s1}}{\lambda_1}\right)^2 f_{V'}\left(\frac{P_{s1}}{\lambda_1}(\gamma+1)\right), \quad (63)$$

where G_1 denotes the Meijer's G -function given in (62). Finally, substituting (21) into (63) and performing some algebraic manipulations yields the intended PDF expression shown in (22).

On the other hand, based on the resulting PDF above, the CDF of γ_{1i} can be explicitly derived by definition as

$$F_{\gamma_{1i}}(\gamma) = \int_0^\gamma \frac{1}{(x+1)^2 G_1} \Psi\left(2, 1; \frac{c_1}{x+1}\right) dx. \quad (64)$$

By introducing the change of variable $y = \frac{1}{x+1}$, (64) can be rewritten as

$$F_{\gamma_{1i}}(\gamma) = \frac{1}{G_1} \int_{\frac{1}{\gamma+1}}^1 \Psi(2, 1; c_1 y) dy \quad (65)$$

$$= \frac{1}{c_1 G_1} \int_{\frac{c_1}{\gamma+1}}^{c_1} \Psi(2, 1; x) dx \quad (66)$$

$$= \frac{1}{c_1 G_1} \int_{\frac{c_1}{\gamma+1}}^{c_1} G_{1,2}^{2,1} \left[x \middle| \begin{matrix} -1 \\ 0, 0 \end{matrix} \right] dx \quad (67)$$

$$\begin{aligned} &= \frac{1}{G_1} G_{2,3}^{2,2} \left[c_1 \middle| \begin{matrix} 0, -1 \\ 0, 0, -1 \end{matrix} \right] \\ &\quad - \frac{1}{c_1 G_1} G_{2,3}^{2,2} \left[\frac{c_1}{\gamma+1} \middle| \begin{matrix} 1, 0 \\ 1, 1, 0 \end{matrix} \right], \end{aligned} \quad (68)$$

where (16) was employed to derive (67), and (14) and the scaling property of Meijer's G -function [17, vol.3, Eq.(8.2.2.15)] were used to derive (68). Finally, applying (24) to (68) leads to the desired CDF expression shown in (23), which completes the proof.

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