Robust Power Allocation for Multi-Carrier Amplify-and-Forward Relaying Systems

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Abstract

It has been shown that adaptive power allocation can provide a substantial performance gain in wireless communication systems when perfect channel state information (CSI) is available at the transmitter. However when only imperfect CSI is available, the performance may degrade significantly, and as such robust power allocation schemes have been developed to minimize the effects of this degradation. In this paper, we investigate power allocation strategies for multicarrier systems, in which each subcarrier employs single amplify-and-forward (AF) relaying scheme. Optimal power allocation schemes are proposed by maximizing the approximated channel capacity under aggregate power constraint (APC) and separate power constraint (SPC). By comparison with the uniform power allocation scheme and the best channel power allocation scheme, we confirm that both the APC and SPC schemes achieve a performance gain over benchmark schemes. In addition, the impact of channel uncertainty is also considered in this paper by modeling the uncertainty regions as bounded sets, and results show that the uncertainty can degrade the worst-case performance significantly.

I. INTRODUCTION

To improve the throughput and power efficiency of communication systems, the transmission strategy is often adapted according to the channel conditions. When perfect channel state information (CSI) is available at the transmitter, the transmission power can be adapted to optimize a certain objective function such as the transmission rate, the signal-to-noise ratio (SNR), or the channel capacity. For multiple parallel channels, the optimal power allocation strategy is given by the water filling algorithm [1]. However, perfect CSI requires reliable channel estimation and error-free feedback, which is usually unrealistic for practical

systems. In practice, the available CSI is often partial or subject to some kind of uncertainty. As such, the problem is how to design robust power allocation algorithms with partial or imperfect CSI.

The CSI uncertainty is typically modeled in two ways. The first is the stochastic framework, in which the uncertainty of the channel fading itself is assumed to be following a certain distribution, and the design of algorithms is based on the mean and covariance of the distribution [2, 3]. The deterministic framework, on the other hand, assumes that the channel uncertainty is deterministic and taking values from a known set, and the uncertainty regions lie around these nominal channel coefficients [4–8]. This framework does not require the statistical knowledge of the CSI as it is the case for the stochastic framework, but needs the estimation of the instantaneous CSI. The stochastic framework gives optimal algorithms with respect to the average performance based on the channel fading distributions, while the deterministic framework designs robust algorithms with respect to the uncertainty region based on instantaneous channel conditions. More specifically, the authors in [4] considered spherical and unbiased channel uncertainty model, and derived an optimal power allocation strategy by maximizing the received SNR. In [5], the band model, or the confidence interval based model, was used for spectral uncertainty, for robust equalization design and for robust matched filter design. In [6], a variety of uncertainty models, including the high dimensional band model (the cubic model), were investigated to design precoders by maximizing the worst-case received SNR for multiple-input and multiple-output (MIMO) systems. In addition, the band model was also adopted in [7] to design the Tomlinson-Harashima precoders for broadcast channels. In [8], the band model was used to derive robust power split between the transmitter and the relay in single-carrier relaying system.

For relay systems, the amplify-and-forward (AF) scheme [9] is often adopted due to its simplicity. In [10], robust power allocation schemes for coherent and non-coherent AF relay networks were designed under imperfect global CSI by maximizing the output SNR. The authors used ellipsoidal uncertainty sets for the imperfect CSI model. The same authors formulated in [11] the optimal power allocation problem by maximizing the achievable rate with perfect global CSI, and solved it in a straightforward manner by using a class of conic optimization theory. In [12], optimal transmit power allocations at the transmitter and relay were obtained based on the average SNR and average bit error rate (BER) in AF relaying systems with deep faded direct links. In [13], a number of computational strategies were presented to evaluate both the problems of minimizing power consumption with a rate constraint and maximizing the throughput with a power constraint in MIMO relay systems. The authors in [14] investigated a novel multicarrier-relay scheme in which the subcarrier power allocation is jointly optimized with the relay scaling coefficients of different subcarriers. A quadratical-complexity algorithm and a suboptimal

algorithm with linear complexity were proposed to solve the optimization problem.

In this paper, we consider the power allocation problem for multicarrier AF-relaying systems, in which each subcarrier performs data transmission via a single relay. In case of a large number of subcarriers, some techniques, such as subcarrier grouping [15, 16] and multi-antenna relays [17, 18], where one relay can be designed properly to serve multiple subcarriers, can be utilized to considerably reduce the number of relays needed. The power allocation strategies are approached by maximizing the approximated channel capacity under aggregate power constraint (APC) and separate power constraint (SPC), respectively. The CSI channel uncertainty in this paper follows a deterministic framework, but unlike the spherical model in [4] or the ellipsoidal model in [10], we use the band model, adopted also in [8], to allow the uncertainty of each link to fluctuate within a region around its actual values. In addition, we also assume that the direct links are in deep fading, but unlike [12], we discard the signal from the direct links and only consider the relay link transmission.

The rest of this paper is organized as follows. In section II, the multicarrier AF-relaying system is introduced, and the channel capacity is proposed as the objective function. Next, section III derives the power allocation schemes for the system under either an aggregate power constraint or a separate power constraint. In section IV, we consider the impact of the channel uncertainty on the power allocation and channel capacity. Finally, some numerical results are shown in section V and conclusions are drawn in the last section.

II. SYSTEM MODEL

We consider a multicarrier system with N subcarriers in total. Each subcarrier transmits data via a single relay employing AF scheme. For each subcarrier, we are in presence of a single-input and single-output (SISO) assuming that the direct transmitter-receiver link is negligible due to large path attenuation. This choice is motivated by the fact the relay link plays a much more important role when the channel condition of the direct link is of bad quality. The channel gains for the transmitter-relay link and relay-receiver link of the i^{th} subcarrier are denoted as h_i and g_i , respectively¹. For the i^{th} transmitter-relay

¹In practical systems, the channel coefficients $\tilde{h}_i$ and $\tilde{g}_i$ for each link are complex, such that $\tilde{z}_i = \tilde{h}_i x_i + \tilde{n}_{1i}$ and $\tilde{y}_i = \tilde{g}_i z_i + \tilde{n}_{2i}$. Let $\tilde{h}_i = h_i e^{j\phi_i}$ and $\tilde{g}_i = g_i e^{j\psi_i}$, where $h_i = |\tilde{h}_i|$ and $g_i = |\tilde{g}_i|$. We assume that the relay and receiver have perfect information of the phase of $\tilde{h}_i$ and $\tilde{g}_i$. This assumption serves at least as an approximation for the scenario when the relay/receiver CSI is of sufficient reliable quality. As the phase information ϕ_i or ψ_i are obtained, they are applied to cancel the phase in $\tilde{z}_i$ and $\tilde{y}_i$, such that $z_i = e^{-j\phi_i} \tilde{z}_i = h_i x_i + n_{1i}$ and $y_i = e^{-j\psi_i} \tilde{y}_i = g_i z_i + n_{2i}$, which are given as (1) and (2).

link, we have

$$z_i = h_i x_i + n_{1i}, \quad (1)$$

where x_i is the signal from the i^{th} transmitter, z_i is the signal received by the i^{th} relay, and n_{1i} is the additive white Gaussian noise (AWGN) at the i^{th} relay with zero mean and variance σ_1^2 . With a relay gain A_i , the i^{th} receiver finally gets

$$y_i = A_i g_i (h_i x_i + n_{1i}) + n_{2i}, \quad (2)$$

where n_{2i} is the AWGN at the i^{th} receiver with zero mean and variance σ_2^2 .

For the channel assisted relays, the relay gain is determined according to the CSI of the transmitter-relay link as [19]

$$A_i^2 = \frac{\bar{q}_i}{\bar{p}_i h_i^2 + \sigma_1^2}, \quad (3)$$

where $\bar{p}_i = |x_i|^2$ is the power allocated for the i^{th} transmitter, and $\bar{q}_i$ is the power allocated for the i^{th} relay. The SNR for the i^{th} subcarrier is then given by

$$\gamma_i = \frac{\bar{p}_i h_i^2 \bar{q}_i g_i^2}{\sigma_1^2 \sigma_2^2 + \sigma_2^2 \bar{p}_i h_i^2 + \sigma_1^2 \bar{q}_i g_i^2}. \quad (4)$$

Taking p_i and q_i as the normalized power, i.e. $p_i = \bar{p}_i / \sigma_1^2$ and $q_i = \bar{q}_i / \sigma_2^2$, so that the SNR for the transmitter-relay link and relay-receiver link of the i^{th} subcarrier are $\gamma_{1i} = p_i h_i^2$ and $\gamma_{2i} = q_i g_i^2$, respectively. Thus the SNR for the i^{th} subcarrier can be approximated by

$$\gamma_i = \frac{\gamma_{1i} \gamma_{2i}}{1 + \gamma_{1i} + \gamma_{2i}} \leq \frac{\gamma_{1i} \gamma_{2i}}{\gamma_{1i} + \gamma_{2i}} \triangleq \gamma'_i. \quad (5)$$

where γ'_i is a tight upper bound of γ_i , especially in high SNR environments when γ_{1i} and γ_{2i} are large. This tight bound is very useful and can translate to a tight lower bound to the average bit-error-rate and outage probability [20]. In such cases, the end-to-end SNR for each subcarrier is given by (5). If we adopt the natural-logarithm based channel capacity (nats/Hz), a tight upper bound of the channel capacity for i^{th} subcarrier with a given power allocation p_i and q_i yields

$$C_i(p_i, q_i) = \log(1 + \gamma'_i) = \log \left(1 + \frac{p_i h_i^2 q_i g_i^2}{p_i h_i^2 + q_i g_i^2} \right), \quad (6)$$

while the channel capacity for the whole multicarrier system is given by:

$$C_\Sigma(\{p_i\}, \{q_i\}) = \sum_{i=1}^N \log \left(1 + \frac{p_i h_i^2 q_i g_i^2}{p_i h_i^2 + q_i g_i^2} \right), \quad (7)$$

We choose the natural-logarithm based channel capacity in (7) as our objective function, and maximize (7) with given power constraints to find the optimal power allocation schemes.

III. POWER ALLOCATION WITH PERFECT CHANNEL ESTIMATION

In this section, we assume reliable estimation and error-free feedback of the channel conditions, so that perfect CSI of both the transmitter-relay link and relay-receiver link are available to the transmitters. Performance gain can then be achieved by adapting the power allocation across subcarriers for the transmitters and relays according to the CSI.

A. Power Allocation with Aggregate Power Constraints

The total power available for the whole system is often limited. Therefore, we constrain the aggregate power available for all the transmitters and relays within S , i.e. $\sum_{i=1}^N (\bar{p}_i + \bar{q}_i) < S$. Our resulting optimization problem can thus be written as

$$\begin{aligned} & \max_{\{p_i, q_i\} \in \mathcal{S}} C_{\Sigma}(\{p_i\}, \{q_i\}), \quad \text{where} \\ & \mathcal{S} = \left\{ \{p_i, q_i\} : \sum_{i=1}^N (\sigma_1^2 p_i + \sigma_2^2 q_i) \leq S, p_i > 0, q_i > 0 \right\}. \end{aligned} \quad (8)$$

As the power constraint in $\mathcal{S}$ forms a convex set, and the channel capacity in (7) is concave on $\{p_i, q_i\}^2$, the maximization is convex, and can be rewritten as a Lagrangian dual problem

$$\begin{aligned} & \min_{\lambda > 0} \max_{\{p_i, q_i\}} L_1(\{p_i\}, \{q_i\}, \lambda), \quad \text{where} \\ & L_1(\{p_i\}, \{q_i\}, \lambda) = \sum_{i=1}^N \log \left(1 + \frac{p_i h_i^2 q_i g_i^2}{p_i h_i^2 + q_i g_i^2} \right) + \lambda \left(S - \sum_{i=1}^N (\sigma_1^2 p_i + \sigma_2^2 q_i) \right). \end{aligned} \quad (9)$$

The optimal power allocation $\{p_i^*, q_i^*\}$ for the inner maximization of (9) can be achieved by directly taking the first partial derivatives of $L_1(\{p_i\}, \{q_i\}, \lambda)$ over p_i and q_i , respectively, i.e. $\frac{\partial L_1}{\partial p_i} = 0$ and $\frac{\partial L_1}{\partial q_i} = 0$, which are equivalent to

$$\begin{aligned} & \frac{h_i^2 (q_i g_i^2)^2}{(p_i h_i^2 + q_i g_i^2 + p_i h_i^2 q_i g_i^2)(p_i h_i^2 + q_i g_i^2)} = \lambda \sigma_1^2 \\ & \frac{g_i^2 (p_i h_i^2)^2}{(p_i h_i^2 + q_i g_i^2 + p_i h_i^2 q_i g_i^2)(p_i h_i^2 + q_i g_i^2)} = \lambda \sigma_2^2. \end{aligned} \quad (10)$$

From (10), it is obvious to see that $p_i h_i \sigma_1 = q_i g_i \sigma_2$, which shows that the status of the transmitter and relay in one subcarrier are always the same. Taking this simple relation into (10), we could derive the

²It is easy to prove that the Hessian matrix of (7) is negative semidefinite when $p_i > 0$ and $q_i > 0$, so the channel capacity (7) is concave on $\{p_i, q_i : p_i > 0, q_i > 0, i = 1, 2, \dots, N\}$.

optimal assignment of power to each transmitter and relay as ³

$$p_i^* = \frac{1}{\sigma_1 h_i} \left(\frac{\alpha_i}{\lambda} - \frac{1}{\alpha_i} \right)^+, \quad q_i^* = \frac{1}{\sigma_2 g_i} \left(\frac{\alpha_i}{\lambda} - \frac{1}{\alpha_i} \right)^+, \quad (11)$$

where $\alpha_i = \frac{h_i g_i}{\sigma_2 h_i + \sigma_1 g_i}$, and $(\cdot)^+ = \max(\cdot, 0)$. The results are similar to the water filling strategy. For each subcarrier, it is active only if the channel conditions of this subcarrier are good enough to satisfy $\alpha_i^2 > \lambda$, and α_i is determined by the channel gain and noise level of both links.

From (11), we see that whether one subcarrier is active or not depends on the Lagrangian multiplier λ , and it is determined by the outer minimization in (9). First, the channel capacity in (7) can be simplified to

$$C_\Sigma(\{p_i^*\}, \{q_i^*\}, \lambda) = \sum_{\mathcal{T}_1} \log \left(\frac{\alpha_i^2}{\lambda} \right), \quad (12)$$

where $\mathcal{T}_1 = \{t_1, t_2, \dots, t_{N_1}\}$ is the index set in which the subcarriers are in active mode, and N_1 is the cardinality of the set $\mathcal{T}_1$. Considering that the power assigned to each subcarrier is $\sigma_1^2 p_i^* + \sigma_2^2 q_i^* = \left(\frac{1}{\lambda} - \frac{1}{\alpha_i^2} \right)^+$, we then have

$$L_1(\{p_i^*\}, \{q_i^*\}, \lambda) = \lambda S + \sum_{\mathcal{T}_1} \left\{ \log \left(\frac{\alpha_i^2}{\lambda} \right) - 1 + \frac{\lambda}{\alpha_i^2} \right\}. \quad (13)$$

With the optimal power allocation p_i^* and q_i^* , the optimization problem in (9) then yields a minimization problem $\min_{\lambda > 0} L_1(\{p_i^*\}, \{q_i^*\}, \lambda)$, and the optimal Lagrangian multiplier λ can be obtained via the first order optimality condition of (13) as

$$\lambda^* = \frac{N_1}{S + \sum_{\mathcal{T}_1} \frac{1}{\alpha_i^2}}. \quad (14)$$

As the optimal λ^* in (14) depends on the active user set $\mathcal{T}_1$, while $\mathcal{T}_1$ is decided jointly by the channel conditions $\{h_i, g_i\}$ and the threshold λ^* , the value of λ^* is solved iteratively with (14) by supposing that all the users are initially active.

B. Power Allocation with Separate Power Constraints

In this subsection, we consider a more rigid situation in which the power available for transmitters and relays has separate constraints. The optimization problem in this case then yields $\max_{\{p_i\} \in \mathcal{P}, \{q_i\} \in \mathcal{Q}} C_\Sigma(\{p_i\}, \{q_i\})$, where the power constraints are given by

$$\mathcal{P} = \left\{ \sum_{i=1}^N \sigma_1^2 p_i \leq P, p_i > 0 \right\}, \quad \mathcal{Q} = \left\{ \sum_{i=1}^N \sigma_2^2 q_i \leq Q, q_i > 0 \right\}. \quad (15)$$

³Please note that the power allocations given in (11) for APC and (18) for SPC are only optimal to the channel capacity with approximated SNR, but may not optimal to the capacity with accurate SNR. In section V-B, we discuss the impact of the SNR approximation on the optimal power allocation.

The power for all transmitters and all relays are limited within P and Q , respectively, and the power constraints $\mathcal{P}$ and $\mathcal{Q}$ are convex sets, so the optimization problem $\max_{\{p_i\} \in \mathcal{P}, \{q_i\} \in \mathcal{Q}} C_\Sigma(\{p_i\}, \{q_i\})$ is convex and can be written with two Lagrangian multipliers u^2 and v^2 as

$$\min_{u, v > 0} \max_{\{p_i, q_i\}} L_2(\{p_i\}, \{q_i\}, u, v), \text{ with} \quad (16)$$

$$L_2(\{p_i\}, \{q_i\}, u, v) = \sum_{i=1}^N \log \left(1 + \frac{p_i h_i^2 q_i g_i^2}{p_i h_i^2 + q_i g_i^2} \right) + u^2 \left(\frac{P}{\sigma_1^2} - \sum_{i=1}^N p_i \right) + v^2 \left(\frac{Q}{\sigma_2^2} - \sum_{i=1}^N q_i \right).$$

In the same way, the inner maximization gives the equations that $\{p_i\}, \{q_i\}$ must satisfy:

$$\frac{h_i^2 (q_i g_i^2)^2}{(p_i h_i^2 + q_i g_i^2 + p_i h_i^2 q_i g_i^2)(p_i h_i^2 + q_i g_i^2)} = u^2 \quad (17)$$

$$\frac{g_i^2 (p_i h_i^2)^2}{(p_i h_i^2 + q_i g_i^2 + p_i h_i^2 q_i g_i^2)(p_i h_i^2 + q_i g_i^2)} = v^2.$$

Note that it is very similar to (10), and we also have $p_i h_i u = q_i g_i v$, which indicates that the transmitter and relay in the same subcarrier are always in the same mode, either active or inactive. Solving the pair of equations (17) with $p_i h_i u = q_i g_i v$ leads to the optimal power allocations for all the transmitters and relays as

$$p_i^* = \frac{1}{u h_i} \left(\beta_i - \frac{1}{\beta_i} \right)^+, \quad q_i^* = \frac{1}{v g_i} \left(\beta_i - \frac{1}{\beta_i} \right)^+, \quad (18)$$

where $\beta_i = \frac{h_i g_i}{v h_i + u g_i}$. The subcarrier with channel condition satisfying $\beta_i > 1$ will be able to transmit, while others are suspended to save power for the active subcarriers. Also note that β_i is not only determined by the channel gain h_i and g_i , but also by the Lagrangian multipliers u and v .

With the optimal power allocation in (18), the channel capacity in (7) is then simplified to

$$C_\Sigma(\{p_i^*\}, \{q_i^*\}, u, v) = \sum_{\mathcal{T}_2} \log(\beta_i^2), \quad (19)$$

where $\mathcal{T}_2 = \{t_1, t_2, \dots, t_{N_2}\}$ is the index set in which the subcarriers are active. Note that $u^2 p_i^* + v^2 q_i^* = \left(1 - \frac{1}{\beta_i^2}\right)^+$, and we can get $L_2(\{p_i^*\}, \{q_i^*\}, u, v)$ as

$$L_2(\{p_i^*\}, \{q_i^*\}, u, v) = u^2 \frac{P}{\sigma_1^2} + v^2 \frac{Q}{\sigma_2^2} + \sum_{\mathcal{T}_2} \left\{ \log(\beta_i^2) - 1 + \frac{1}{\beta_i^2} \right\}. \quad (20)$$

By minimizing (20) with respect to u and v , the solution for u^* and v^* can be determined by numerically solving the set of two equations

$$\sum_{\mathcal{T}_2} \frac{g_i}{u g_i + v h_i} - v \sum_{\mathcal{T}_2} \frac{1}{h_i g_i} = u \left(\frac{P}{\sigma_1^2} + \sum_{\mathcal{T}_2} \frac{1}{h_i^2} \right), \quad (21)$$

$$\sum_{\mathcal{T}_2} \frac{h_i}{u g_i + v h_i} - u \sum_{\mathcal{T}_2} \frac{1}{h_i g_i} = v \left(\frac{Q}{\sigma_2^2} + \sum_{\mathcal{T}_2} \frac{1}{g_i^2} \right).$$

Multiplying the first equation in (21) by u and the second equation by v , and then adding them, we have

$$\left(\frac{P}{\sigma_1^2} + \sum_{\mathcal{T}_2} \frac{1}{h_i^2}\right) u^2 + \left(\frac{Q}{\sigma_2^2} + \sum_{\mathcal{T}_2} \frac{1}{g_i^2}\right) v^2 + 2uv \sum_{\mathcal{T}_2} \frac{1}{h_i g_i} = N_2 \quad (22)$$

which means the solution (u, v) for (21) lies on an ellipse in the $u-v$ plane in a form of $au^2 + bv^2 + 2cuv = d$, with which we can solve u from (21) and get the corresponding v from (22) iteratively.

IV. CHANNEL UNCERTAINTY

For practical communication systems, the perfect CSI at the transmitter side is often unavailable. Due to the erroneous, outdated, or quantized feedback, the CSI obtained by the transmitters is typically subject to uncertainty. In this section, we investigate the error analysis of the CSI at the transmitter side. The channel conditions are modeled deterministically as

$$h_i = h_{ti} + h_{ui}, \quad g_i = g_{ti} + g_{ui}, \quad (23)$$

where h_{ti} and g_{ti} are the actual channel gains for the transmitter-relay link and relay-receiver link, while h_{ui} and g_{ui} are the associated channel uncertainty for each link meeting the requirements that $|h_{ui}| < h_{ti}$ and $|g_{ui}| < g_{ti}$. The uncertainty h_{ui} 's and g_{ui} 's are the errors that cause the mismatch of the CSI obtained at the transmitters and the instantaneous actual CSI. The uncertainty regions for h_{ui} and g_{ui} are given as bounded sets

$$\mathcal{U}_h = \{h_{ui} : h_{ui} \in [-\epsilon_{1i}, \epsilon_{1i}]\}, \quad \mathcal{U}_g = \{g_{ui} : g_{ui} \in [-\epsilon_{2i}, \epsilon_{2i}]\}. \quad (24)$$

Note that this uncertainty model is based on the notion of confidence interval. A quantity estimated from some measurements cannot be perfectly accurate, however, we can define a “confidence interval”, just like the sets $\mathcal{U}_h$ and $\mathcal{U}_g$ in (24), within which the actual quantity lies with a very high probability based on the measurement/estimation quality. This notion of confidence interval works regardless of the type of estimator employed.

Under such channel uncertainty, the optimal power allocation $\{p_i^*, q_i^*\}$ can be determined by solving the following minimax problem

$$\{p_i^*, q_i^*\} = \arg \max_{\{p_i, q_i\}} \min_{\{h_{ui}\} \in \mathcal{U}_h, \{g_{ui}\} \in \mathcal{U}_g} C_\Sigma(\{p_i\}, \{q_i\}), \quad (25)$$

where the inner minimization over $\mathcal{U}_h$ and $\mathcal{U}_g$ gives the worst-case channel capacity for the multicarrier system. For such an inner minimization, it is easy to show that $\frac{\partial C_\Sigma}{\partial h_{ui}} > 0$ and $\frac{\partial C_\Sigma}{\partial g_{ui}} > 0$ within the constraints $\mathcal{U}_h$ and $\mathcal{U}_g$. In this way, the inner minimization in (25) gives the uncertainty $(h_{ui}, g_{ui}) =$

$(-\epsilon_{1i}, -\epsilon_{2i})$, so that worst case happens only when the h_{ui} 's and g_{ui} 's reach the lower bounds of their uncertainty regions. In this case, the worst-case channel capacity is given by

$$C_{\Sigma-wc} = \sum_{i=1}^N \log \left(1 + \frac{p_i(h_{ti} - \epsilon_{1i})^2 q_i(g_{ti} - \epsilon_{2i})^2}{p_i(h_{ti} - \epsilon_{1i})^2 + q_i(g_{ti} - \epsilon_{2i})^2} \right). \quad (26)$$

Denote $\hat{h}_i = h_{ti} - \epsilon_{1i}$ and $\hat{g}_i = g_{ti} - \epsilon_{2i}$, then the worst-case $C_{\Sigma-wc}$ yields

$$C_{\Sigma-wc} = \sum_{i=1}^N \log \left(1 + \frac{p_i \hat{h}_i^2 q_i \hat{g}_i^2}{p_i \hat{h}_i^2 + q_i \hat{g}_i^2} \right), \quad (27)$$

which takes the same form as the channel capacity in (7). To this end, we can obtain robust power allocation schemes for APC and SPC by optimizing the worst-case channel capacity in (27) for the uncertainty case. By the same procedure as in Section III-A and Section III-B, the robust power allocation are still given by (11) and (18), only with h_i and g_i replaced by $\hat{h}_i = h_{ti} - \epsilon_{1i}$ and $\hat{g}_i = g_{ti} - \epsilon_{2i}$, respectively.

To show that the power allocation given by (25) is robust, we must confirm that the minimax theorem holds for the optimization problem in (25). For the problem $\min_x \max_y f(x, y)$, the minimax theorem holds if $\min_x \max_y f(x, y)$ and $\max_y \min_x f(x, y)$ lead to an identical solution (x^*, y^*) , or equivalently $f(x^*, y) \leq f(x^*, y^*) \leq f(x, y^*)$ [21]. The point (x^*, y^*) is called a saddle point of $f(x, y)$, so the minimax theorem holds if and only if the saddle point exists. We write the channel capacity given in (7) with $h_i = h_{ti} + h_{ui}$ and $g_i = g_{ti} + g_{ui}$ as $C_{\Sigma}(\{p_i, q_i\}, \{h_{ui}, g_{ui}\})$, or for short C_{Σ} . We now show that its saddle point exists, and this saddle point is $(x^*, y^*) = (\{-\epsilon_{1i}, -\epsilon_{2i}\}, \{p_i^*, q_i^*\})$.

For the max-min problem $\max_{\{p_i, q_i\}} \min_{\mathcal{U}_h, \mathcal{U}_g} C_{\Sigma}$, it is easy to show that $\frac{\partial C_{\Sigma}}{\partial h_{ui}} > 0$ and $\frac{\partial C_{\Sigma}}{\partial g_{ui}} > 0$ with h_{ui} and g_{ui} constraint by $\mathcal{U}_h$ and $\mathcal{U}_g$, so the channel capacity C_{Σ} is monotonically increasing with respect to h_{ui} and g_{ui} , i.e. the inner minimization $\min_{\mathcal{U}_h, \mathcal{U}_g} C_{\Sigma}$ gives $x^* = \{-\epsilon_{1i}, -\epsilon_{2i}\}$. Then consider the outer maximization $\max_{\{p_i, q_i\}} C_{\Sigma}(\{p_i, q_i\}, \{-\epsilon_{1i}, -\epsilon_{2i}\})$. Using the same process as in Section III, we have the optimal power allocation $y^* = \{p_i^*, q_i^*\}$ given by (11) for APC and (18) for SPC, with $h_i = h_{ti} - \epsilon_{1i}$ and $g_i = g_{ti} - \epsilon_{2i}$.

On the other hand, we now show that the min-max problem $\min_{\mathcal{U}_h, \mathcal{U}_g} \max_{\{p_i, q_i\}} C_{\Sigma}$ also gives the result $(x^*, y^*) = (\{-\epsilon_{1i}, -\epsilon_{2i}\}, \{p_i^*, q_i^*\})$. First, the inner maximization gives the optimal power allocation $y^*(\{h_{ui}, g_{ui}\}) = \{p_i^*(\{h_{ui}, g_{ui}\}), q_i^*(\{h_{ui}, g_{ui}\})\}$, where the values of optimal power allocation $\{p_i^*, q_i^*\}$ depend on the uncertainty levels $\{h_{ui}, g_{ui}\}$. Now we consider the following three cases:

- Case 1: The channel uncertainty is $x_1 = \{h_{ui,1}, g_{ui,1}\}$, and the corresponding optimal power allocation is given by $y_1^* = \{p_i^*(x_1), q_i^*(x_1)\}$ with the channel capacity $C_{\Sigma}(x_1, y_1^*)$.

- Case 2: The channel uncertainty is $x_2 = \{h_{ui,2}, g_{ui,2}\}$, and the optimal power allocation regarding to x_2 is $y_2^* = \{p_i^*(x_2), q_i^*(x_2)\}$, while the channel capacity is $C_\Sigma(x_2, y_2^*)$. We suppose the channel conditions of x_2 is a little worse than that of x_1 , denoted as $x_2 < x_1$, which means $h_{ui,2} \leq h_{ui,1}$ and $g_{ui,2} \leq g_{ui,1}$ for all the index $i = 1, 2, \dots, N$, but the equality does not hold for all i .
- Case 3: The channel uncertainty is $x_1 = \{h_{ui,1}, g_{ui,1}\}$, but the given power allocation is $y_2^* = \{p_i^*(x_2), q_i^*(x_2)\}$, and then the corresponding channel capacity is $C_\Sigma(x_1, y_2^*)$.

Compare case 1 and case 3, they have the same channel uncertainty but different power allocation. As y_1^* is the optimal power allocation with associated channel uncertainty x_1 , so we have $C_\Sigma(x_1, y_1^*) > C_\Sigma(x_1, y_2^*)$. Then comparing case 2 and case 3, note that they have the same power allocation. As shown above, the channel capacity C_Σ is monotonically increasing with h_{ui} and g_{ui} when the power allocation are fixed. As $x_1 > x_2$, then $C_\Sigma(x_1, y_2^*) > C_\Sigma(x_2, y_2^*)$. In this way, we have $C_\Sigma(x_1, y_1^*) > C_\Sigma(x_1, y_2^*) > C_\Sigma(x_2, y_2^*)$ or $C_\Sigma(x_1, y_1^*) > C_\Sigma(x_2, y_2^*)$. Comparing case 1 and case 2, we see that the case 2 has worse channel conditions ($x_2 < x_1$), and the maximized channel capacity is also less. Then it is easy to say that the channel capacity for the system C_Σ with the power allocation $\{p_i^*(h_{ui}, g_{ui}), q_i^*(h_{ui}, g_{ui})\}$ reaches its minimum at $\{h_{ui}, g_{ui}\} = \{-\epsilon_{1i}, -\epsilon_{2i}\}$. In this way, the outer minimization of the min-max problem gives the results $(x^*, y^*) = (\{-\epsilon_{1i}, -\epsilon_{2i}\}, \{p_i^*, q_i^*\})$.

Now we have shown that the max-min problem $\max_{\{p_i, q_i\}} \min_{\mathcal{U}_h, \mathcal{U}_g} C_\Sigma$ and the min-max problem $\min_{\mathcal{U}_h, \mathcal{U}_g} \max_{\{p_i, q_i\}} C_\Sigma$ lead to the identical solution $(x^*, y^*) = (\{-\epsilon_{1i}, -\epsilon_{2i}\}, \{p_i^*, q_i^*\})$, and this indicates that the minimax theorem holds for the optimization in (25) and (x^*, y^*) is a saddle point of $C_\Sigma(\{p_i, q_i\}, \{h_{ui}, g_{ui}\})$, which means that the power allocation in (25) is robust.

V. SELECTED NUMERICAL RESULTS

In this section, we show some selected numerical results for our proposed power allocation schemes. The channel gains of each subcarrier are supposed to be deterministic with some uncertainty. The interference between any two subcarriers, and between the transmitter-relay link and relay-receiver link are ignored, in order to focus on the impact of the channel uncertainty.

A. Comparison of Different Allocation Scheme

In Fig. 1, we ignore the channel uncertainty and compare the performance of different power allocation schemes. These power allocation schemes include:

- APC scheme: The aggregate power is constraint within S , and the power allocated for each transmitter and relay follows (11).

- SPC scheme: The power constraints for the transmitters and relays are P and Q with $P = Q = S/2$. The optimal power allocation follows (18).
- Uniform Power Allocation (UPA) scheme: The power is uniformly distributed among each transmitter and relay, i.e. $p_i = q_i = S/(2N)$.
- Best Channel Power Allocation (BCPA) scheme: The total power is allocated to the subcarrier with the best channel conditions, and the best channel k^* is defined as

$$k^* = \operatorname{argmax} \left\{ \frac{h_i^2 g_i^2}{h_i^2 + g_i^2}, i = 1, 2, \dots, N \right\}, \quad (28)$$

which is the channel with the best normalized SNR when the transmitter and relay in the same subcarrier receive equal power. The power allocation to the selected subcarrier is $p_{k^*} = q_{k^*} = S/2$. The channels for the transmitter-relay links and the relay-receiver links are supposed to be subject to independent and identically distributed (i.i.d.) Rayleigh fading with the average channel gain $E[h_i]$ and $E[g_i]$. The Gaussian noise variance are set to unity, i.e. $\sigma_1^2 = \sigma_2^2 = 1$. The number of subcarriers in Fig. 1 is given as $N = 100$.

From the subfigures, we see that the APC and SPC schemes performs almost the same when the transmitter-relay links and the relay-receiver links have equal average channel gains. When the discrepancy between the average channel gains $E[h_i]$ and $E[g_i]$ increases, the performance gap between the APC and SPC schemes becomes larger, and meanwhile, the performance of BCPA gradually exceeds that of UPA. In addition, it is not surprising to see that the APC and SPC schemes outperform UPA and BCPA, and it is necessary to point out that for SPC, it is also of importance to split the power reasonably between transmitters and relays when the total power budget ($P + Q$) is fixed. More power should be distributed to the links with better channel conditions, so that the SPC scheme performs closer to the APC scheme.

B. Approximated SNR v.s. Accurate SNR

In section II, we mentioned that approximated SNR γ'_i (not γ_i) in (5) is used for the channel capacity as our objective function. However, the optimal power allocation derived from the objective function with approximated SNR and accurate SNR may be different. In Fig. 2, we investigate the average channel capacity for different power constraint level with approximated SNR and accurate SNR. In this figure, we investigate three cases, the approximated case (the objective function uses the approximated SNR), the accurate case (the objective function uses the accurate SNR), and the accurate/approximated case (the channel capacity is measured with accurate SNR, but the power allocation is from the approximated case). The channel for each link is assumed i.i.d. Rayleigh fading with unit average channel gain.

In the figure, we plot the performance for the APC scheme of two-carrier system, the SPC scheme of two-carrier system and the APC scheme for single carrier system. As we can see, the approximated channel capacity is a tight bound of the accurate capacity, and as the power budget increases (i.e. the SNR also increases), the gap between the approximated and accurate capacity is smaller. Comparing the dash curves (the accurate case) and the circles/squares (the accurate/approximated case), we find that in the low SNR region (small power budget), there exists small discrepancy between them, while at high SNR region (large power budget), this performance discrepancy almost disappears. This confirms that the power allocation derived for the approximated case is suboptimal in comparison to the accurate case, but the performance difference between these two power allocation schemes is so tiny that it can be almost ignored, and the optimal power allocation for the approximated case can also be regarded as “optimal” as the accurate case.

C. Impact of Uncertainty on Channel Capacity

In Fig. 3, we consider the impact of uncertainty on the worst-case channel capacity. The uncertainty for each link is measured by ρ_{ji} , which is defined as

$$\rho_{1i} = \epsilon_{1i}/h_{ti}, \quad \rho_{2i} = \epsilon_{2i}/g_{ti}. \quad (29)$$

For simplicity, we take equal normalized uncertainty level for all links, i.e. $\rho_{1i} = \rho_{2i} = \rho$ for all i . When $\rho = 0$, there is no channel uncertainty. On the other hand, $\rho = 1$ corresponds to maximum uncertainty that leads to the worst-case channel capacity dropping to zero. In this figure, the channels are subject to Rayleigh fading with in $E[h_i] = 2$ and $E[g_i] = 1/2$. The noise level is the same as in Fig. 1. For the SPC, we equally distributed the total power to the transmitters and relays, i.e. $P = Q = S/2$. The uncertainty level of $\rho = 0.2$, $\rho = 0.5$ and $\rho = 0.8$ correspond to low uncertainty, mediate uncertainty and high uncertainty, respectively.

As we can see in this figure, the channel capacity rises as we provide more power. The uncertainty can greatly decrease the worst-case channel capacity. For the mediate level uncertainty, the worst-case capacity is less than 50% of the capacity without uncertainty, while for the high level uncertainty, the worse-case capacity is only about 10%. For the low or zero uncertainty, increasing power constraints achieves a remarkable improvement in the performance. Under the high uncertainty, however, providing more power also brings difference, but this difference seems fruitless when compared with the zero and low uncertainty. In addition, the gap between APC and SPC also diminishes when the uncertainty level increases. As the uncertainty level increases, the channel gains of all links will be smaller, and the

difference of the channel gains between the transmitter-relay links and the relay-receiver links decreases. For the APC, the total power allocated for all the transmitter will be close to that for all the relays, i.e. the power allocated for all the transmitter and all the relays will approach to be equal (the SPC scheme). In this case, the power allocation scheme for APC will be closer to that for SPC, so that the performance gap between APC and SPC diminishes.

D. Impact of Noise Variance on Active Carriers

In Fig. 4, we fix the noise variance of the transmitter-relay link as $\sigma_1^2 = 1$, but vary σ_2^2/σ_1^2 from 0.1 to 10. The channel gains for both links are randomly distributed from Rayleigh fading distribution with unit mean. The APC is employed with $S = 100$, and different levels of uncertainty are examined. In this figure, we investigate the percentage of active subcarriers over different uncertainty levels and noise levels. As expected, the percentage decreases either as the uncertainty level rises or the noise variance increases. The uncertainty exerts a great influence on the percentage of active subcarriers. When the variances are set to unity, more than 45% of subcarriers are kept active, while under high uncertainty level $\rho = 0.8$, only around 10% remain active.

VI. CONCLUSION

In this paper, optimal power allocation algorithms are proposed for multicarrier AF relaying systems with both APC and SPC by maximizing the approximated channel capacity. Some selected numerical results show that the performance of the APC and SPC schemes outperform considerably the UPA and BCPA schemes when the perfect CSI is available. When only imperfect CSI is obtained, the worst-case throughput is employed to develop robust power allocation schemes. The channel uncertainty levels are shown to have a substantial impact on the performance. When the uncertainty rises from low level to high level, a large fraction of the worst-case throughput is lost.

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Figure 1: Channel capacity for different power allocation schemes over Rayleigh fading ($N = 100$).

Figure 2: Channel capacity for different power allocation schemes over Rayleigh fading with $E[h_i] = 1$ and $E[g_i] = 1$.

Figure 3: Impact of uncertainty on channel capacity for APC and SPC over Rayleigh fading with $E[h_i] = 2$ and $E[g_i] = 1/2$ ($N = 100$).

Figure 4: Impact of noise variance on active subcarriers for APC over Rayleigh fading ($N = 100$).