Non Linear Dynamics of Memristor Based 3rd Order Oscillatory System

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Abstract

In this paper, for the first time, we have reported the nonlinear dynamics of three Memristors based phase shift oscillator which plausibly can be a solution for realizing parametric oscillation as an autonomous linear time variant system. Sustained oscillation is reported though oscillating resistance and time dependent poles are present. The Memristor based phase shift oscillator is explained further with different parameters to present the resistance of Memristor as a time varying parameter which may eliminate the need of external periodic forces to oscillate. Multi-Memristors are used simultaneously with similar and different parameters are investigated in this paper. Mathematical formulas for analyzing such oscillator are verified with simulation results which are in good agreement.

Keywords: Memristor, oscillating poles, stable oscillation memristor, dynamic poles

1. Introduction

The resistor, capacitor, and inductor which are the three basic elements of any circuit design have recently encountered the missing fourth element- 'Memristor', that relates between the flux-linkage (ϕ) and the charge (q). HP lab first reported an experimental version of Memristor in 2008 [1]. That two terminal physical model achieved a resistance depending on the history of current. Memristor is modeled as a thin semiconductor film (TiO_2) sandwiched between two metal contacts. The semiconductor film has a region with low resistance R_{on} (high concentration of dopants), and the remainder has higher resistance R_{off} (low concentration of dopants). The total resistance of the Memristor R_M , is the sum of the weighted resistances of the doped and undoped regions depending on the width of the regions. Before this establishment of the passive Memristor Leon O. Chua first envisioned its existence [2]. He proposed and built Memristor with active elements [2, 3]. During the long thirty years the theoretical Memristor was investigated independently or along with the other three passive components (R , L , and C) [4-6]. After the successful physical model implemented by HP, Memristor has enraptured the research world for its unprecedented behavior. Researchers have demonstrated its potential for being used as a memory switch which offers the replacement of conventional transistor [7-11]. The nonvolatile memory applications of Memristor lead to more dense architecture where this nanoscale element is being implemented in crossbar arrays to perform either as a latch or logic module [12-17]. Apart from the digital domain Memristor is shown to have pragmatic solution for chaotic system [18, 19]. The analog circuit behavior of Memristor has not been studied much [20-22] rather more works are involved for chaos and bifurcation in dynamic systems.

Oscillators are the widely used signal generators of sine waves with various frequencies. For the well-known oscillators, it is established that:

- 1) all the resistors of the sustained oscillatory system must present constant value,
- 2) all the poles of the oscillatory system will be fixed in s plane to give sustained oscillation,
- 3) sustained oscillation will have a single frequency component in the Fourier spectra.

But for the parametric oscillator the parameters of the oscillatory system are periodically changed. It is observed in mechanical system, optical system (mainly in lasers), and physiological system. For example, a person on a swing initiates the oscillation of the swing by changing his center of gravity periodically [23]. But this kind of oscillation only occurs when the periodical parameters are applied externally. Because of time varying nature of parameters, it is more theoretically familiar to the linear time variant system in control systems and mechanics. The system is modeled by Mathieu equation in which the parameters are periodic function of time to describe such instability phenomenon [24]. Though realizing such oscillation with electrical circuits has not been studied much, even such demonstration requires external force which makes the system non autonomous [25].

Recently [26, 27] we demonstrated the surprising behavior of Memristor when used in Wien family oscillators. We reported sustained oscillation even with the presence of oscillating resistance and time dependent oscillating poles. Four different Memristor-based Wien oscillators are studied with analytical model and Spice simulations. However, all these oscillators are 2nd order and only single Memristor was replaced at a time for each oscillator.

In this article, we will report sustained oscillation in the 3rd order phase shift oscillator by replacing three Memristors at a time either with similar or different R_{init} . This Memristor-based third order oscillatory system in spite of the presence of oscillating Memristor resistance and time dependent oscillating poles to demonstrate parametric oscillation with a linear time variant

autonomous system. Mathematical analyses will be carried out with simulation results to validate inherent parametric oscillation.

2. Phase Shift Oscillator with Memristor

Phase shift oscillator is a simple kind of sine wave oscillator comprised with three resistors and three capacitors. For the simplest form of this oscillator it uses an opamp with another feedback resistor to set up the close loop system. The three resistors (R_1, R_2 , and R_3) and capacitors (C_1, C_2 , and C_3) constitute a looped network which as a whole works as filter circuit and gives the necessary condition for sustained oscillation. Each $R - C$ section gives phase shift of 60 degrees. So the whole section produces phase shift of 180 degrees. It shifts the output of inverting amplifier by 180 degrees to fulfill the Barkhausen stability criteria. It is a third order oscillator as it has three $R - C$ sections. To simplify the mathematical modeling of this third order system, all equal valued capacitance C is considered. The characteristic equation for this oscillator then can be expressed as:

$$as^3 + bs^2 + cs + d = 0 \quad (1)$$

$$a = R_1 R_2 R_3 C^3 (1 + k), b = 3R_1 R_2 C^2 + 2R_1 R_3 C^2 + R_3 R_2 C^2, c = 2R_1 C + 2R_2 C + R_3 C, d = 1 \quad (2)$$

For sustained oscillation, the frequency of oscillation (f) and the condition for the gain (k) can be derived as:

$$\omega = 2\pi f = \frac{1}{C \sqrt{(3R_1 R_2 + 2R_1 R_3 + R_3 R_2)}} \quad (3)$$

$$k = 8 + 6 \frac{R_2}{R_3} + 6 \frac{R_1}{R_3} + 4 \frac{R_1}{R_2} + 2 \frac{R_2}{R_1} + 2 \frac{R_3}{R_2} + \frac{R_3}{R_1} \quad (4)$$

For the conventional case (3) and (4) are simplified by taking $R_1 = R_2 = R_3 = R$ which gets the value of k as 29 and f as:

$$f = \frac{1}{2\pi RC\sqrt{6}} \quad (5)$$

The characteristic equation for this system has three roots, two of those are complex conjugate roots and the other is a real root. Traditionally it is well believed that those roots must be fixed to their distinct locations for the whole oscillation period. In addition to that the resistors must show constant value as any change in the value will make the oscillation unstable.

The next section will describe in detail about the Memristor based 3rd order system where all the three resistors are changed with Memristors as shown in Fig.1. The state space in the non-saturation conditions of this circuit can be written as follows

$$\begin{pmatrix} \frac{dV_{c1}}{dt} \\ \frac{dV_{c2}}{dt} \\ \frac{dV_{c3}}{dt} \end{pmatrix} = \frac{-1}{m+1} \begin{pmatrix} \frac{1}{C_1} \left(\frac{1}{R_{M1}} + \frac{1}{R_{M2}} + \frac{1}{R_{M3}} \right) & \frac{1}{C_1} \left(\frac{1}{R_{M2}} + \frac{1}{R_{M3}} - \frac{m}{R_{M1}} \right) & \frac{1}{C_1} \left(\frac{1}{R_{M3}} - \frac{m}{R_{M1}} - \frac{m}{R_{M2}} \right) \\ \frac{1}{C_2} \left(\frac{1}{R_{M2}} + \frac{1}{R_{M3}} \right) & \frac{1}{C_2} \left(\frac{1}{R_{M2}} + \frac{1}{R_{M3}} \right) & \frac{1}{C_2} \left(\frac{1}{R_{M3}} - \frac{m}{R_{M2}} \right) \\ \frac{1}{R_{M3}C_3} & \frac{1}{R_{M3}C_3} & \frac{1}{R_{M3}C_3} \end{pmatrix} \begin{pmatrix} V_{c1} \\ V_{c2} \\ V_{c3} \end{pmatrix} \quad (6)$$

For simplicity, let us assume C_1, C_2 , and C_3 are equal valued C ($1\mu F$). The SPICE model in [28] is used to simulate the effect of Memristor resistance R_M in phase shift oscillator. R_{on} is changed to $0.5K\Omega$ (for Wien oscillator it was $0.1K\Omega$). The different value of R_{on} for the two oscillators does not alter any result. Only the factor n gets changed to 50000 (for Wien oscillator it was symbolized with k and the value was 10000 as in [27]). R_{init} is the only variable parameter and it is changed from $6.1K\Omega$ to $6.9K\Omega$ for simulation.

The analytical models for the Memristor resistance in case of periodic input and DC waveforms were described in [29, 30]. These formulas were provided for both symmetrical and non-symmetrical periodic inputs. In [27], we showed that for oscillating R_M , the maximum (R_{max}) and minimum (R_{min}) can be calculated if R_{init} is known. The amplitude swing ($R_{max} - R_{min}$) of R_M can be expressed as:

$$R_{max} - R_{min} \cong \frac{V_M n (R_{off} - R_{on})}{\pi R_{init} f_M} \quad (7)$$

f_M is the frequency of oscillation when R_M replaces any resistor of the circuit. V_M is the voltage across the Memristor.

3. Memristors-based phase shift oscillator with equal R_{init}

In this section we will demonstrate the effect of one Memristor replacing all the resistors of the three $R - C$ sections of the conventional phase shift oscillator. In this case R_1, R_2 , and R_3 are replaced with R_M (Fig. 1). As R_M is found to be oscillating then R_M can be represented as:

$$R_{Mi} = R_{avg i} \pm \Delta R_{Mi} \sin(2\pi f_M t), i = 1, 2, 3 \quad (8)$$

Here ΔR_M is the sinusoidal amplitude and R_{avg} is the average value of the oscillation.

3.1 Oscillation in RM:

From the simulation result, sustained oscillation is found even with oscillating behavior of R_M . Though R_{on} ($0.5K\Omega$) and R_{off} ($16K\Omega$) are supposed to be the most likely parameters for R_M , sustained oscillation with an average value of R_{avg} is achieved. In Fig. 2, the simulation result of R_{M1}, R_{M2} , and R_{M3} are shown when $R_{init} = 6.5K\Omega$. It is observed that R_{M1} oscillates from $6.05K\Omega$ to $6.95K\Omega$ with R_{avg1} exactly falls on $R_{init} = 6.5K\Omega$. It is to be noted that V_{M1} (voltage across R_{M1}) is approximated as V_1 (voltage across R_1) to compute the values of R_{M1} . From the simulation, it is observed that $V_1 = 234mV$ (when $R_1 = 6.5K\Omega$) and V_{M1} is found as $233mV$ for $R_{init} = 6.5K\Omega$. For other values of R_{M1} , V_{M1} can be well estimated as V_1 (for different R_1 but in correspondence to R_{M1}) with a maximum error of less than 1%. When R_{init} is changed from $6.1K\Omega$ to $6.9K\Omega$, the maximum error between simulation and calculation is calculated to be 1.22%. It is found that R_{M2} is also oscillating

with a maximum amplitude of $6.55K\Omega$ and minimum amplitude of $6.28K\Omega$. R_{avg2} is found to be $6.415K\Omega$ which is shifted down from R_{init} by $0.085K\Omega$. The amplitude swing of R_{M2} (ΔR_{M2}) from simulation is $0.27K\Omega$ and if we use (7) then the swing is calculated to be $0.27K\Omega$. From the simulation it is found that V_2 (voltage across R_2) = $69.7mV$ (when $R_2 = 6.5K\Omega$) and V_{M2} (voltage across R_{M2}) is $69.4mV$ for $R_{init} = 6.5K\Omega$. Maximum error in calculating R_{M2} using (7) is then found as 6.3%. In the case of R_{M3} , it oscillates in different magnitude because of V_{M3} has different amplitude. V_{M3} (voltage across R_{M3}) in this case is very close to V_3 (voltage across R_3). For $R_{init} = 6.5K\Omega$, V_{M3} is $25mV$ and for $R_3 = 6.5K\Omega$, V_3 is $26mV$. For other values of R_{init} both V_{M3} and V_3 are approximately equal with a maximum difference of $1.03mV$ (when $R_{init} = 6.9K\Omega$). Thus V_3 can be a good estimation of V_{M3} to calculate R_{M3} (when $R_{init} = 6.5K\Omega$). R_{M3} is found to be oscillating from $6.37K\Omega$ to $6.47K\Omega$. R_{avg3} gets shifted up by $0.08K\Omega$. The simulated values of R_{M3} and calculated values using (7) are well matched with a maximum error of 5%. Table 1 compares the result for ΔR_M for phase shift oscillator with Wien Oscillator. The negligible difference between the calculation and simulation confirms (8) as a generalized model for calculating R_M in both second order and third order oscillatory systems. These oscillating R_M can be the good candidate for inherently oscillating parameter and thus Memristor based phase shift oscillator plausibly develops autonomous linear time variant system to realize parametric oscillation.

3.2 Time dependant oscillating poles:

In [26, 27] we showed the Wien oscillator can have oscillating poles which are time dependent. Even for phase shift oscillator similar time dependent oscillating poles are found. The Memristor-based phase shift characteristic equation is time dependent as in (9a) which is

similar to the linear time variant (LTV) parametric oscillation system. Using some algebraic simplifications, these coefficients can be obtained as in (9b) to (9d) where they are function of time. The solution of (9a) will have three time dependent roots. These roots will have one real root and the other two are complex conjugates. These roots are defined in terms of the coefficients (a, b, c , and d). The coefficients in (2) can be derived as:

$$a(t)s^3 + b(t)s^2 + c(t)s + d = 0 \quad (9a)$$

$$a = \{R_{avg1}(R_{avg2} R_{avg3} + R_{avg2} \Delta R_{M3} + R_{avg3} \Delta R_{M2}) + R_{avg3} R_{avg2} \Delta R_{M1}\}C^3(1 + k) \quad (9b)$$

$$b = 3R_{M1}R_{M2}C^2 + 2R_{M1}R_{M3}C^2 + R_{M3}R_{M2}C^2 \quad (9c)$$

$$c = [(2R_{avg1} + 2R_{avg2} + R_{avg3}) + (2\Delta R_{M1} + 2\Delta R_{M2} + \Delta R_{M3})]C, \quad d = 1 \quad (9d)$$

From (9), it is found that except d the other three coefficients are completely dependent on R_M . As poles of such system are dependent on R_M and because of the oscillating behavior of R_M , the poles will follow the time dependent oscillation as R_M . The real pole is found to have little effect for this case as it is located far from the imaginary axis. But this real pole shows oscillating behavior with time dependent. The two conjugate poles are also oscillating with time. As a result the poles will be oscillating in the s plane. This is surprising to have all the poles of this third order system to be oscillating with time and yet sustained oscillation is achieved in the output. In Fig. 4, the time dependent oscillating poles are shown for the three cases. One of the complex conjugate poles for this system is displayed in Fig. 3(a). Here we can see that the pole is not fixed but oscillating in different planes with different amplitude for the three cases. Mirror image is found for the conjugate pole. Fig. 3(b) represents the oscillating real pole of the system. This instability phenomenon of dynamic poles results in sustained oscillation only because of the automatic adjustment by the oscillating R_M parameter.

3.3 Frequency of oscillation:

The frequency of oscillation for this system f_M can be modified from (3) as:

$$f_M = \frac{1}{2\pi C \sqrt{(3R_{M1}R_{M2} + 2R_{M1}R_{M3} + R_{M3}R_{M2})}} \quad (10)$$

As all three Memristors are oscillating, one may expect that the frequency of oscillation will have a range rather be single valued as in conventional case. But from the simulation the output is found oscillating presenting an impulse on FFT. Though the width of the impulse is little broader than the impulse attained from the traditional case. If we plot R_M 's using (7) versus f_M (in Fig.5 for $R_{init} = 6.5K\Omega$), R_M 's will be seen to merge on R_{init} at higher frequency. This in fact the case with Memristor as at higher frequency Memristor works as a simple resistor. Now assuming that R_{avg1} , R_{avg2} , and R_{avg3} are equal to R_{init} (though R_{avg2} and R_{avg3} are shifted from R_{init} by $0.085K\Omega$ and $0.08K\Omega$ respectively), then replacing R_M by $R_{init} \pm \Delta R_M$ in (10) we can again plot R_{init} vs f_M as shown in Fig.4. This curve will intersect each R_M 's curves (zoomed in Fig.4) at two frequency points which will give the valid frequency region for this Memristor based third order system to oscillate. As there are three frequency ranges the oscillatory system chooses the common region of operation. Though the system has a frequency range to operate, sustained oscillation can only be possible at that frequency which is obtained from the intersection point of R_{init} vs f_M curve with R_{init} line. For $R_{init} = 6.5K\Omega$, f_M from Fig.4 is found $10.03Hz$ whereas from simulation f_M is observed to be $10Hz$. It is necessary to point out here that all the poles of this system are found to be oscillating in s plane. But the dominating conjugate pole crosses the imaginary axis when $R_M = R_{init}$ ($\approx R_{avg}$). This might be the crucial reason for the system to settle down at one intersecting frequency to give sustained oscillation. Nevertheless the range of frequency of oscillation gives the hint of possible region for

sustained oscillation. Apart from the graphical method of estimating f_M , using (8) and (10) f_M can be analytically expressed as:

$$f_M = \frac{1}{2\pi C \sqrt{(6R_{init}^2 \pm 5(\Delta R_{M1} + 4\Delta R_{M2} + 3\Delta R_{M3}))}} \quad (11)$$

Using (11), f_M can be calculated for this system. Fig.5 compares the simulated and the calculated values of f_M for different R_{init} . The maximum error in approximating f_M is 0.67% which is very negligible.

3.4 Gain (k) adjustment:

For the traditional phase shift oscillator the necessary condition for sustained oscillation is to have $k = 29$. For this Memristor based third order system, gain k can be derived as:

$$k = 8 + \frac{6R_{M2} + 6R_{M1}}{R_{M3}} + \frac{4R_{M1} + 2R_{M3}}{R_{M2}} + \frac{2R_{M2} + R_{M3}}{R_{M1}} \quad (12)$$

As R_M is oscillating so k is expected to behave as sinusoid which is modeled in (11). From simulation it is observed that sustained oscillation is possible only for a certain value of k . When $R_{init} = 6.5K\Omega$, the simulated value of k for this case is found as 28.75. To mathematically calculate the value of k , three different methods are taken: (i) As R_{init} is very close to R_{avg} so R_{init} can be directly put in place of R_M in (12). k is found to be exactly 29 in this straightforward method. (ii) Another way can be, if we replace the sinusoidal R_M in (11) using (8), it will give k against as a function of t which is plotted in Fig. 6. The average value of this curve can be computed by integrating the curve for the half cycle of R_M and dividing it by the half time period of R_M . The average value of k becomes as 29.0862. (iii) The third method is that, k can be plotted against R_M using (12) for different values of R_{init} (for this R_{init} is changed from $6.1K\Omega$ to $6.9K\Omega$) as in Fig. 7. In addition, by integrating

this curve and averaging under the limit of integration, the calculated value of k can be found as 29.087. All of these three methods give a very close value of k with the simulated one. Even when R_{init} is changed to any value (from $6.1K\Omega$ to $6.9K\Omega$) all these methods are well applicable with a maximum error of 1.03%, 0.99%, and 1.01% respectively for the three methods.

4. Memristors-based phase shift oscillator with unequal R_{init}

In this case, R_{init} is put differently for the three Memristors. In previous section R_{init} was same for all R_M . The other parameters in the SPICE sub circuit is kept similar as before. Even with different R_{init} , all the three R_M are found to be oscillating and output oscillation is also sustained. More importantly irrespective of whether the three R_M have same or different R_{init} , they keep the exact amplitude swing in R_M with same R_{avg} as compared to the swing of R_M in section 3. As for example, when R_{init1} is $6.5K\Omega$, R_{M1} oscillates from $5.65K\Omega$ to $6.55K\Omega$ independent of the R_{init} values of R_{M2} and R_{M3} . Similar things happen to R_{M2} and R_{M3} . Table 2 shows the results in details. Here colored labels are shown for each R_M to present the observation of fixed swing of R_M . Even the shifting of R_{avg} from R_{init} is fixed. The reason behind this incident can be the fact that, V_M is almost constant for a certain R_{init} and V_M of one Memristor does not depend on the R_{init} values of other R_M s'. If one examines the colored labels in Table 2, it can be seen that V_{M1} , V_{M2} , and V_{M3} are constant depending on the R_{init} of the corresponding R_M and they do not alter values because of the similar or different R_{init} of others. It should be pointed out here that even the phase differences of the three R_M are constant and so based on these observations and using (8), the three R_M (in Ω) can be expressed as

$$R_{M1} \approx R_{init1} \pm 450 \sin(2\pi f_M t)$$

$$R_{M2} \approx (R_{init2} - 80) \pm 130 \sin(2\pi f_M t + 19\pi/360) \quad (13)$$

$$R_{M3} \approx (R_{init3} - 80) \pm 50 \sin(2\pi f_M t - 19\pi/360)$$

f_M here can be computed using (10) and R_M can be replaced with R_{init} . As all the R_M do not have common R_{init} so the graphical approach (Fig. 4) of estimating the frequency of oscillation is not appropriate in this case. But frequency of oscillation can be computed using (10). In Table 2 both the simulation and calculated values of frequency of oscillation are mentioned and it is found that the error is very negligible. It is to be pointed out here that dynamic poles are also observed in this scheme. To compute the gain, similar methods are applicable as mentioned in previous section. But due to uncommon R_{init} and as the R_M have slight phase differences (in previous section the phase differences are not considered), estimating the gain will not be a good choice. Gain of the Memristor based phase shift is defined by R_4/R_{M1} , as R_{M1} is oscillating so gain values will not be constant but will inversely follow R_{M1} . It does not mean that we are manually tuning the gain but it is more an intuitive observation as shown in Fig. 8 for $R_{init1} = 6.9K\Omega$, $R_{init2} = 6.5K\Omega$, and $R_{init3} = 6.1K\Omega$. To incorporate the phase difference, R_M is used as (13). From Fig. 8, it is observed that gain of this circuit oscillates to adjust with R_{M1} in order to have an average gain of 26.5 and from the conventional case the gain is found 28. The more sensible approach would be to compare R_4 as R_4 is tuned to get the sustained oscillation. We therefore used (12) with $R_M = R_{init}$ and compared with simulation and for all cases the values of R_4 are listed in Table 2. It is found that R_4 using (12) gives matched result with simulation with a very negligible error less than 1%.

In above discussions we change the parameter R_{init} in the SPICE model. We have the freedom to change the R_{on} and R_{off} parameters also. When either R_{on} or R_{off} is changed, it

is observed that none of them have any effect on sustained oscillation. All the values listed in Table 2 remain exactly the same whatever R_{on} or R_{off} is, except the swing of R_M gets changed without any deviation in phase of R_M as mentioned in (13). If we see (7), the swing of R_M is found to be linearly dependent on $(R_{off} - R_{on})$. So as the difference between R_{off} and R_{on} is decreased, the swing of R_M decreases and vice versa. This observation is also evident in the SPICE simulation. Moreover the swings of R_M from the simulation does match with the calculated swing using (7) with negligible error. But it should be kept in mind that the difference between R_{off} and R_{on} should not be as low to restrict the complete swing with respect to R_M .

5. Conclusion

We reported the unprecedented characteristics of Memristor in conventional phase shift oscillator. The effect of using Memristor in place of resistor have been described mathematically which are verified by the simulation results. The model for R_M is generalized for Memristor based both second and third order oscillatory system. It is now definite that sustained oscillation can be achieved in despite of the presence of the oscillating resistance of Memristor and time dependent oscillating poles of the system which in other words a good example for parametric oscillation. The unconventional properties of Memristor are not only limited to second order oscillator (Wien oscillator) but also for third order oscillator (phase shift oscillator) similar behavior of Memristor is observed. As a result these Memristor based systems demonstrate autonomous parametric oscillatory system where Memristor sets its resistance as a periodically changing parameter. The nonlinear dynamics of the Memristor based 3rd order system are thoroughly discussed with equal R_{init} , and unequal R_{init} . The effect of R_{on} and R_{off} parameters on sustained oscillation are also reported in this article.

The oscillating resistance, operational frequency range, and dynamic poles of the Memristor based third order system can surely help to redefine the conventional concept. Challenges remain with the availability of passive Memristor and nonlinearity of both Memristor and opamp. As the order of the system increases there are added complexities to the mathematical modeling. Simpler and more generalized model is required to describe the unconventional characteristics of Memristor in oscillation. The unconventional results can have impact in designing neural network, oscillatory circuits, signal generation, sensitive control systems, amplitude and frequency modulation, etc. As for now the proposed models and mathematical reasoning can help circuit designers to implement Memristor in developing autonomous linear time variant system to exhibit parametric oscillation.

References

- [1] D. B. Strukov, G. S. Snider, D. R. Stewart, and R. S. Williams, "The missing memristor found," *NATURE*, vol. 453, pp. 80-83, May 1, 2008.
- [2] L. O. Chua, "Memristor-The Missing Circuit Element," *IEEE Trans. Circuit Theory*, vol. CT-18, no. 5, pp. 507, Sep. 1971.
- [3] L. O. Chua and S. M. Kang, "Memristive Devices and Systems," *Proc. of the IEEE*, vol. 64, no. 2, pp. 20, Feb. 1976.
- [4] G. F. Oster and D. M. Auslander, "The Memristor: A new bondgraph element," *Trans ASME on Dynamical Syst. Meas Contr.*, vol. 94, no. 3, pp. 249-252, 1972.
- [5] G. F. Oster, "A note on memristor," *IEEE Trans. Circuits Syst.*, vol. CAS-21, pp. 152, Jan. 1974.
- [6] L. O. Chua and C. W. Tseng, "A memristive circuit model for P-N junction diodes," *Int. J. Circuit Theory and Appl.*, vol. 2, pp. 367-389, Dec. 1974.

- [7] G. Snider, “Architecture and methods for computing with reconfigurable resistor crossbars (Patent style)”, U. S. Patent 7203789, April 2007.
- [8] Z. B. Yan, S. Z. Li, K. F. Wang, J. –M. Liu, “Unipolar resistive switching effect in YMn1- δ O3 thin films.” Appl. Phys. Lett, vol. 96, no. 1, pp. 012103 - 012103-3, Jan. 2010
- [9] H. Manem, G. S. Rose, X. He, and W. Wang, “Design Considerations for Variation Tolerant Multilevel CMOS/Nano Memristor Memory,” Proceedings of the ACM Great Lakes Symposium on VLSI, Providence, Rhode Island, USA, pp. 287-292, May 2010.
- [10] Wang, F.Z., Na Helian, Sining Wu, Mian-Guan Lim, Yike Guo, and Parker, M.A., "Delayed Switching in Memristors and Memristive Systems," Electron Device Letters, IEEE , vol. 31, no. 7, pp. 755-757, July 2010.
- [11] Prodromakis, T., Michelakis, K., and Toumazou, C., "Switching mechanisms in microscale memristors," Electronics Letters, vol. 46, no. 1, pp. 63-65, January 7 2010.
- [12] Rajendran, J., Manem, H., Karri, R., and Rose, G.S., "Memristor based programmable threshold logic array," Nanoscale Architectures (NANOARCH), 2010 IEEE/ACM International Symposium on, pp.5-10, June 2010.
- [13] S. Hyun Jo, T. Chang, I. Ebong, Bhavitavya B. B., P. Mazumder, and Wei Lu, "Nanoscale Memristor Device as Synapse in Neuromorphic Systems," Nano Letters, vol: 10, no. 4, pp 1297–1301, March 2010.
- [14] B. L. Mouttet, “Programmable crossbar signal processor (Patent style)”, U.S. Patent 7302513, Nov 2007.
- [15] B. L. Mouttet, “Crossbar control circuit (Patent style)”, U.S. Patent 7609086, Oct. 2009.

- [16] X. Liu, M. T. Mayer, W. Dunwei, "Negative differential resistance and resistive switching behaviors in Cu₂S nanowire devices," Appl. Phys. Lett, vol. 96, no. 22, pp. 223103 - 223103-3, 2010.
- [17] J. H. Young, J. Y. Lee, S. Y. Choi, J. W. Kim, "Microscopic origin of bipolar resistive switching of nanoscale titanium oxide thin films," Appl. Phys. Lett, vol. 95, no. 16, pp. 162108 - 162108-3, 2009.
- [18] Bao, B.C., Liu, Z., and Xu, J.P., "Steady periodic memristor oscillator with transient chaotic behaviours," Electronics Letters, vol. 46, no. 3, pp. 237-238, Feb. 2010.
- [19] Iu, H. H. C., Yu, D. S., Fitch, A. L., Sreeram, V., and Chen, H., "Controlling Chaos in a Memristor Based Circuit Using a Twin-T Notch Filter," Circuits and Systems I: Regular Papers, IEEE Transactions on, vol. PP, no. 99, pp. 1, Jan 2011.
- [20] Pershin, Y.V., Di Ventra, M., "Practical Approach to Programmable Analog Circuits With Memristors," Circuits and Systems I: Regular Papers, IEEE Transactions on, vol. 57, no. 8, pp. 1857-1864, Aug. 2010.
- [21] Petras, I., "Fractional-Order Memristor-Based Chua's Circuit," Circuits and Systems II: Express Briefs, IEEE Transactions on, vol. 57, no. 12, pp. 975-979, Dec. 2010.
- [22] Di Ventra M., Pershin Y.V., and Chua L.O., "Circuit elements with memory: memristors, memcapacitors and meminductors", Proceedings of IEEE, vol. 97, iss. 10, pp. 1717-1724, Oct 2009.
- [23] William B. Case, "The pumping of a swing from the standing position," Am. J. Phys., vol. 64, pp. 215-220, 1996.
- [24] G. Searby, and D. Rochwerger, "A parametric acoustic instability in premixed flames," Journal of Fluid Mechanics, vol. 231, pp. 529-543, 1991.

- [25] Komine, V., Galliou, S., and Makarov, A., "A parametric quartz crystal oscillator," *Ultrasonics, Ferroelectrics and Frequency Control*, IEEE Transactions on , vol. 50, no. 12, pp. 1656- 1661, Dec. 2003.
- [26] A. Talukdar, A. G. Radwan, K. N. Salama, "Time Domain Oscillating Poles: Stability Redefined in Memristor Based Wien-Oscillators," *ICM 2010*, Cairo, Egypt, pp. 288-291, 2010.
- [27] A. Talukdar, A. G. Radwan, K. N. Salama, "Generalized model for Memristor-based Wien family oscillators," *Microelectron. J* (2011), doi:10.1016/j.mejo.2011.07.001
- [28] Z. Biolek, D. Boilek, V. Biolkova, "SPICE Model of Memristor with Nonlinear Dopant Drift," *RADIOENGINEERING*, vol. 18, no. 2, pp. 210, June 2009.
- [29] A. Radwan, M. A. Zidan, and K. N. Salama, "On the mathematical modeling of Memristors," *IEEE International Conference on Microelectronics (ICM)*, Cairo, Egypt, pp. 284-287, Dec 2010.
- [30] A. G. Radwan, M. A. Zidan, and K. N. Salama, "HP Memristor Mathematical Model for Periodic Signals and DC," *IEEE International Midwest Symposium on Circuits and Systems (MWSCAS)*, Seattle, USA, pp. 861-864, Aug 2010.

Fig. 1: Schematics of Memristor based phase shift oscillator

Fig. 2: Simulation result of R_{M1} , R_{M2} , and R_{M3} for $R_{init}=6.5K\Omega$

TABLE I
Comparison between Wien and Phase shift oscillator

Replaced Resistor with R _M	Wien Oscillaor [R _{init} =5KΩ]			Phase shift Oscillator [R _{init} =6.5KΩ]	
	(R _{max} – R _{min}) using (6)				
	Simulation	Calculation		Simulation	Calculation
	R ₁	0.70		0.728	0.90
R ₂	NA		0.27	0.27	
R ₃			0.10	0.0974

Fig. 3: a) Oscillating complex pole with time, b) Oscillating real pole with time

Fig. 4: Frequency response of R_{M1} , R_{M2} , and R_{M3}

Fig. 5: Comparison between simulated and calculated f_M for different R_{init} .

Fig. 6: Mathematical relationship between k and t when $R_{init} = 6.5K\Omega$

Fig. 7: k as a function of R_M

Fig. 8: Transient response of RM1, RM2, RM3, and gain when $R_{init1}=6.9K\Omega$, $R_{init2}=6.5K\Omega$, $R_{init3}=6.1K\Omega$, $R_{on}=0.5K\Omega$ and $R_{off}=16K\Omega$

TABLE II
Simulation results of three Memristor based phase shift oscillator for different Rinit

R_{init1}	R_{init2}	R_{init3}	R_4	R_4 using (12)	V_{out}	V_{M1}	V_{M2}	V_{M3}	R_{M1}	R_{M2}	R_{M3}	f	f using (10)
$K\Omega$					$Volts$				$K\Omega$			Hz	
6.1	6.5	6.9	192.2	183.4	.779	.229	.0692	.028	5.65- 6.55	6.29- 6.55	6.77- 6.87	10.17	10.11
	6.9	6.5	185.2	181.7	.757	.222	.0693	.026	5.65- 6.55	6.69- 6.94	6.37- 6.47	10.12	10.06
	6.1	6.1	175.27	176.9	.763	0.233	0.0698	0.0263	5.65- 6.55	5.88- 6.16	5.97- 6.07	10.73	10.65
6.5	6.1	6.9	194.5	190.9	.794	.242	.0698	.0279	6.05- 6.96	5.88- 6.16	6.77- 6.87	10.12	10.05
	6.9	6.1	180	186.9	.739	.225	.0693	.0248	6.08- 6.93	6.7- 6.95	5.97- 6.07	10	9.95
	6.5	6.5	186.88	188.5	.765	0.233	0.0698	0.0264	6.06- 6.94	6.28- 6.55	6.37- 6.47	10.06	9.99
6.9	6.1	6.5	189.3	196.1	.79	.214	.0713	.0269	6.46- 7.35	5.88- 6.17	6.37- 6.47	10.04	9.95
	6.5	6.1	181.85	193.8	.76	.238	.071	.025	6.47- 7.33	6.29- 6.56	5.97- 6.07	9.96	9.9
	6.9	6.9	198.49	200.1	.767	0.233	0.0698	0.0264	6.47- 7.34	6.69- 6.95	6.77- 6.87	9.47	9.4