Bayesian Recovery of Clipped OFDM Signals: A Receiver-based Approach

Thesis by
Abdullatif Rabah Al-Rabah

In Partial Fulfillment of the Requirements
For the Degree of
Masters of Science

King Abdullah University of Science and Technology, Thuwal,
Kingdom of Saudi Arabia

May, 2013
The thesis of Abdullatif R. Al-Rabah is approved by the examination committee

Committee Chairperson: Dr. Tareq Y. AlNaffouri
Committee Member: Prof. Mohamed-Slim Alouini
Committee Member: Dr. Basem Shihada
ABSTRACT

A Bayesian recovery of clipped OFDM signals

Abdullatif R. Al-Rabah

Recently, orthogonal frequency-division multiplexing (OFDM) has been adopted for high-speed wireless communications due to its robustness against multipath fading. However, one of the main fundamental drawbacks of OFDM systems is the high peak-to-average-power ratio (PAPR). Several techniques have been proposed for PAPR reduction. Most of these techniques require transmitter-based (pre-compensated) processing. On the other hand, receiver-based alternatives would save the power and reduce the transmitter complexity. By keeping this in mind, a possible approach is to limit the amplitude of the OFDM signal to a predetermined threshold and equivalently a sparse clipping signal is added. Then, estimating this clipping signal at the receiver to recover the original signal. In this work, we propose a Bayesian receiver-based low-complexity clipping signal recovery method for PAPR reduction. The method is able to i) effectively reduce the PAPR via simple clipping scheme at the transmitter side, ii) use Bayesian recovery algorithm to reconstruct the clipping signal at the receiver side by measuring part of subcarriers, iii) perform well in the absence of statistical information about the signal (e.g. clipping level) and the noise (e.g. noise variance), and at the same time iv) is energy efficient due to its low complexity. Specifically, the proposed recovery technique is implemented in data-aided based. The data-aided method collects clipping information by measuring reliable
data subcarriers, thus makes full use of spectrum for data transmission without the need for tone reservation. The study is extended further to discuss how to improve the recovery of the clipping signal utilizing some features of practical OFDM systems i.e., the oversampling and the presence of multiple receivers. Simulation results demonstrate the superiority of the proposed technique over other recovery algorithms. The overall objective is to show that the receiver-based Bayesian technique is highly recommended to be an effective and practical alternative to state-of-art PAPR reduction techniques.
ACKNOWLEDGEMENTS

First and foremost, I would like to acknowledge and thank The Almighty Allah for blessing and guiding me throughout this period. There are many people around me who deserve my thanks and gratitude, to only some of whom it is possible to give particular mention here.

Above all, I would like to express my deepest gratitude to my advisor Dr. Tareq Y. Al-Naffouri, for his constant guidance, support, motivation and patience!. His academic experience and guidance have been invaluable to me.

I am most grateful to my friend and colleague Mudassir Masood (PhD candidate) for being available to help and guide me in our publications. I always found his comments, suggestions and encouragement to be of great value. I thank Ebrahim Al-Safadi for his help in the beginning of my research. I also would like to acknowledge Erica Jolly and Marcos Bracchitta (the instructors of academic English SkillsLab) for their supported workshops and thesis consultation clinic. I would like to thank IEEE Student Branch at KAUST for their workshops on Latex.

It is my pleasure to thank the members of my thesis committee, Professors Mohamed-Slim Alouini and Basem Shihada.

To my parents, my brothers and my sister I say: thank you all for supporting and encouraging me throughout this period.

Last but not least, I would like to thank KAUST for giving me opportunity to complete my master degree. I wish a bright and successful future for my University.
TABLE OF CONTENTS

Examination Committee Approval 2
Copyright 3
Abstract 4
Acknowledgements 6
List of Figures 9
List of Tables 10

1 Introduction 11
  1.1 Notation Used .......................................................... 12
  1.2 Orthogonal Frequency-Division Multiplexing ...................... 12
  1.3 Peak-to-Average Power Ratio (PAPR) ............................. 15
    1.3.1 PAPR Reduction by Clipping .................................. 17
    1.3.2 Receiver-based Approaches ................................... 19
  1.4 Compressed Sensing & Sparse Signal Recovery .................... 21
  1.5 Thesis Contributions ................................................ 25

2 PAPR Reduction by Bayesian Tone Reservation (TR) 28
  2.1 Motivation .............................................................. 28
  2.2 Problem Formulation .................................................. 29
  2.3 Bayesian Recovery Algorithm ..................................... 34
  2.4 Simulation Results and Discussion ................................ 38
    2.4.1 Experiment 1: exact parameters ............................. 39
    2.4.2 Experiment 2: estimated parameters ......................... 40
  2.5 Conclusion ............................................................. 42

3 PAPR Reduction by Bayesian Data-Aided (DA) 43
  3.1 Motivation .............................................................. 43
4 Oversampling and Multiple Receivers

4.1 Oversampling and Block Sparsity

4.1.1 Oversampling in OFDM

4.1.2 Oversampling in Data-aided

4.1.3 Simulation Results and Discussion

4.2 Multiple Receivers

4.2.1 Simulation Results and Discussion

4.3 Conclusion

5 Conclusion & Future Work

5.1 Concluding Remarks

5.2 Ideas for Future Work

References

Appendices
## LIST OF FIGURES

1.1 High peaks cause problem due to nonlinearity of transmitter amplifiers. 16
1.2 Sparse signal in the time domain and its representation in the frequency domain. 22

2.1 PAPR is reduced via simple clipping scheme by limiting the signal amplitude to certain threshold. Then, at the receiver and from the free subcarriers (black circles in the figure) we get information about the clipping signal. 31
2.2 BER versus $\gamma$ when using exact values of the recovery parameter. 40
2.3 BER versus $\gamma$ when using rough initial estimates of the recovery parameters. 41

3.1 The nearest neighbours of $\mathcal{X}$. We can see two examples of received symbols deviated from their original points. A reliability function is required to select which subcarriers are most reliable. 44
3.2 Data aided: BER versus the threshold $\gamma$. 50
3.3 Data aided: average run-time versus the threshold $\gamma$. 51
3.4 Data aided: BER versus the number of reliable carriers $M$. 52

4.1 Part of the clipping signal when the OFDM time domain signal is oversampled by a factor of $J=4$, shows the block sparsity structure. 56
4.2 BER versus threshold. The signal is oversampled by $J = 4$. 60
4.3 Clipping signal has same support in different receivers. 61
4.4 BER versus $\gamma$ for data-aided with $R_x = 4$. 63

A.1 The block diagram of OFDM system. 76
A.2 The transmission process is parallel in frequency domain. 78
LIST OF TABLES

1.1 Notations table .............................................................. 13
1.2 Abbreviations table .......................................................... 14

2.2 Average run-time when using estimated recovery parameters in experiment 2 ......................................................... 41
Chapter 1

Introduction

Orthogonal frequency-division multiplexing (OFDM) is a recent popular scheme in wireless and wired communications. Specifically, it is already part of many broadband communication standards. However, high peak-to-average power ratio (PAPR) is one of the main challenges associated with OFDM signals. High PAPR results in nonlinear distortion to OFDM signals caused by the transmitter’s power amplifier. Practical power amplifiers operate in a limited (finite) linear range of input amplitudes. Therefore, a sufficient back-off is necessary to prevent any saturation of high peaks. However, increased back-off leads to inefficient use of power amplifiers. Most of PAPR reduction techniques are transmitter-based where additional processing are carried out at the transmitter side. However, such pre-compensated techniques avoid high PAPR while sacrificing transmitter complexity. For many battery-powered devices, transmitter-based PAPR reduction techniques are not suitable solutions. In this thesis, we would like to devise an alternative technique for PAPR reduction at the receiver side. The PAPR is reduced by a simple clipping operation and equivalently a clipping signal is generated. The approach is based on treating the clipping as a sparse signal and recover it by partial observation of its spectrum. We pursue a Bayesian data-aided technique approach that makes use of many pieces of information including: the clipping statistics, the block sparse vector and the presence of multipath antennas.
This chapter is organized as follows:

1. We start by introducing OFDM and its system model. Section 1.2 illustrates the advantages and important features of OFDM. In addition, it provides a discrete representation of OFDM model and its fundamental properties. A design of the transmitter and the receiver is discussed.

2. Section 1.3 presents an overview of PAPR reduction techniques. Specifically, this section describes the underlying cause of the PAPR problem. In addition, it provides a literature review of PAPR reduction techniques with more focus on receiver-based ones.

3. Section 1.4 provides a brief review of compressed sensing. A discussion of different Bayesian sparse signal recovery methods is presented.

4. Then the Chapter ends by thesis organization and contributions.

Our starting point is to introduce notations and abbreviations used throughout the thesis.

### 1.1 Notation Used

Refer to Table 1.1 for different font types considered for different variables. In addition, Table 1.2 contains the abbreviations used throughout the thesis.

### 1.2 Orthogonal Frequency-Division Multiplexing

The growing demand of high speed wireless applications has spurred the development in many wireless communication schemes. Recently, Orthogonal frequency-division multiplexing (OFDM) is adopted for high-speed wireless communications due to its robustness against multipath fading or inter-symbol interference (ISI). One of the
main approaches to handle ISI is to use an equalizer (e.g. ZF, MMSE, Viterbi, etc.).
Practically, the equalizer is often a finite impulse response (FIR) filter with a number of coefficients related to channel’s length. When the data rate is increased for larger bandwidths, this implies an increase in multi-path intensity. In other words, there is a large delay spread of the channel when compared to symbol duration time. Thus, tackling the ISI in high data rate scenarios results in a significant increase in length of equalizers and high-speed signal processing, even for known channels. Nevertheless, in today’s typical wireless communications, there has been an increasing interest in providing high data rate services. Therefore, a new system that employs a high data rate scheme with ability to avoid ISI in an efficient way is essentially needed. OFDM transmits data in parallel, hence a simple 1-tap per subcarrier frequency domain equalizer is required to mitigate ISI. OFDM is multi-carrier scheme, which transforms a frequency-selective wide-band channel into a group of non-selective narrow-band channels which appear to be flat. Moreover, the sidebands of the individual carriers overlap and yet, due to orthogonality of OFDM subcarriers, the signals are received without interference. A sufficient cyclic prefix is required to ensure orthogonality of the system over time dispersive channels.

Nevertheless, due to its success in high data rate transmissions, OFDM is cur-

---

1See abbreviations in Table 1.2
| Abbreviations |
|---------------|--------------------------------------------------|
| A/D           | Analog to digital convertor                      |
| AMMSE         | Approximate minimum mean-square error           |
| AWGN          | Additive white Gaussian noise                    |
| BER           | Bit error rate                                   |
| CS            | Compressed sensing                               |
| CVX           | A package for specifying and solving convex programs [1] |
| DAC           | Digital to analog convertor                      |
| FBMP          | Fast Bayesian matching pursuit [2]               |
| FFT           | Fast fourier transform                           |
| FIR           | Finite impulse response                          |
| HPA           | High power amplifier                             |
| i.i.d.        | Independent and identically distributed         |
| IFFT          | Inverse fast fourier transform                   |
| ISI           | Inter-symbol interference                        |
| LS            | Least-squares                                    |
| ML            | Maximum likelihood (decoder)                     |
| MMSE          | Minimum mean-square error                        |
| OFDM          | Orthogonal Frequency-Division Multiplexing      |
| OFDMA         | Orthogonal Frequency-Division Multiple Access    |
| P/S           | Parallel to series                               |
| PAPR          | Peak-to-average power ratio                      |
| PDF           | probability density function                     |
| S/MIMO        | Single/ multiple-input and multiple-output       |
| S/P           | Series to parallel                               |
| SABMP         | Support agnostic Bayesian matching pursuit [3]   |
| SER           | Symbol error rate                                |
| UUP           | Uniform uncertainty principle                    |
| ZF            | Zero forcing                                     |

Table 1.2: Abbreviations table
rently used in practical wireless and wireline systems i.e., wireless local area network (WLAN), high-speed digital subscriber line (xDSL), 4th generation (4G) cellular communications and digital broadcasting TV. Moreover, OFDM is adopted in many wireless network standards, such as LTE (long-term evolution), IEEE 802.16 (WiMAX), IEEE 802.11 (WiFi) and etc.

Appendix A presents the transceiver design and discrete representation of OFDM system.

1.3 Peak-to-Average Power Ratio (PAPR)

An OFDM signal is the sum of a large number of independent signals modulated onto orthogonal subcarriers with equal bandwidths. The envelope of the resulting signal fluctuates with considerable variance. This unavoidable and inherent feature of time domain OFDM signals leads to high peak-to-average power ratio (PAPR).

PAPR is one of the major drawbacks of OFDM. Specifically, due to the nonlinearity of the commonly employed high power amplifiers (HPA), the high PAPR leads to: in-band distortion (which degrades the BER) and out-of-band radiation (which interferes neighboring frequency bands). A high PAPR signal requires power amplifiers with linear response over a wide range and hence, expensive transmitters. This is one of the main reasons why in LTE the use of OFDMA in the uplink was avoided [4]. One is thus faced by either of two options: to back-off and let the power amplifier operate in the linear region resulting in power inefficiency, or to operate in the nonlinear region and subject the OFDM signal to distortion. Thus, PAPR reduction remains one of the most important research areas in OFDM systems. Effective mitigation of the PAPR problem requires a proper tradeoff between key performance metrics including: the amount of PAPR reduction, maintaining the linearity of HPA, data rate loss, computational complexity and BER performance.
Figure 1.1: High peaks cause problem due to nonlinearity of transmitter amplifiers.
PAPR reduction techniques can be classified according to several criteria. However, they can be classified according to whether the additional processing that reduces PAPR is carried out at the transmission side, or at the receiver side. This classification clarifies the lack of receiver-based solutions when compared to transmitter-based ones. Many transmitter-based techniques have been proposed to reduce the PAPR including: coding, interleaving, tone reservation (TR), tone injection (TI), selected mapping (SLM), partial transmit sequence (PTS) and active constellation extension (ACE); details about each technique can be found in [5–9]. The implicit drawback, however, is one of an increased transmitter complexity. In many applications which have limited source of power (e.g. satellites, mobile phones and tablets are famous examples), the transmitter complexity is the main concern. This reflects the need to look for more energy-efficient receiver-based alternatives.

1.3.1 PAPR Reduction by Clipping

A time domain OFDM signal $x$ is obtained by performing IDFT to the complex-valued data. The discrete time domain OFDM signal can be written as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \mathcal{X}(k)e^{j2\pi \frac{nk}{N}} \quad (1.1)$$

where $\mathcal{X}(k)$ is the data symbol at the $k$th subcarrier. A high PAPR is generated when a large number of independently modulated subcarriers are added up. The discrete time PAPR of the transmit signal is defined as

$$PAPR = \frac{\max|x(n)|^2}{\mathbb{E}[|x(n)|^2]} \quad (1.2)$$

where $\mathbb{E}[.]$ denotes expectation. This PAPR increases significantly for large number of subcarriers. Most of the PAPR reduction schemes consider reducing the peaks i.e.,
max|\(x(n)\)|, before submitting the signal to the transmitter amplifier.

Reduction of high PAPR could be achieved through a simple clipping scheme by subjecting the signal to a amplitude limiter as follows

\[
x_p(i) = \begin{cases} 
\gamma e^{j\theta_{x(i)}} & \text{if } |x(i)| > \gamma \\
x(i) & \text{otherwise}
\end{cases}
\] (1.3)

where \(x_p(i)\) is the \(i\)th element of the signal after clipping, \(\gamma\) is the limiting threshold and \(\theta_{x(i)}\) is the phase of \(x(i)\). Clipping is performed at the transmitter side, hence, it generates in-band distortion which degrades BER performance. If this clipping is performed, however, no additional distortion is introduced by the HPA. Therefore, the HPA can be efficiently utilized. At the receiver side and in order to recover the original signal, it is required to estimate locations and amplitudes of the clips to restore the system performance. In [10], a method to reallocate clipped power to other unclipped parts of OFDM signal, thus while eliminates high peaks, it increases the average power. However this does not solve the problem but alleviates it. The other challenge associated with clipping is out-of-band radiation which interferes adjacent channels and reduces spectral efficiency. This major problem of out-of-band radiation can be solved by filtering. However, filtering may cause some peaks regrowth where the signal, after filtering, may exceed the clipping threshold. The overall peak regrowth can be reduced by repeating the clipping and filtering especially for oversampled signals, however this comes at a cost of increased computational complexity [11, 12]. Many different techniques have been proposed to provide less computational complexity [12–16], to name a few.

Reduction of PAPR by clipping suffers from unsolved in-band distortion. In this study, we are able to mitigate this disadvantage of clipping at the receiver. By that, clipping and clipping recovery can be classified now as a PAPR distortionless reduction approach.
1.3.2 Receiver-based Approaches

Clipping operation can be seen as adding a clipping (peak-reducing) signal that eliminates high peaks (which are above a certain threshold). In practice, a few peaks of OFDM signal are of high peaks. Therefore, the clipping signal can be considered a \textit{sparse} signal. This soft clipping is equivalent to adding a \textit{sparse} signal $c$ to the original time domain signal $x$ with active elements only where clipping has occurred, i.e.,

$$x_p = x + c$$ (1.4)

As we can see, the problem is transformed from PAPR reduction to mitigation of a nonlinear distortion (clipping). This mitigation is carried out at the receiver side. We know that the signal to be recovered at the receiver side is a sparse signal. Thus, a compressive sensing (CS) can be adopted for such a problem.

An early receiver-based nonlinear distortion mitigation techniques, which can be used to mitigate clipping signal, are [17–19]. These technique uses iterative decoders to estimate data symbols and use these estimates to mitigate nonlinear distortion then iteratively update data symbols. This process continues in this repetitive manner for several iteration until some convergence criteria is met. Thus it suffers from relatively high complexity. More methods based on previous ones with some modifications have been proposed. Same procedure in [19] is used in [20] but to estimate and cancel the clipping noise in frequency domain and in [21] with two different equalizers to estimate clipped and clipping signals. In [22], a modification is added to the method in [18] to used conditional Quasi-ML decoder that relay on reliable ML decision. A method that considers MIMO-OFDM systems to mitigate clipping distortion is proposed in [23]. This method employs the method in [21] for space-time and space-frequency block codes (STBC/SFBC) MIMO-OFDMs. A method similar to [23] is proposed in [24] to
incorporate channel state information (CSI) in the iterative estimation of nonlinear distortion.

Several approaches adopt compressed sensing (CS) from the fact that the clipping signal is a sparse signal. In [25] a CS approach that uses reserved tones as sensing carriers to recover the clipping signal. In similar procedure, the channel estimation pilots are adopted to avoid more data rate loss in [26]. In [27], a hybrid approach that optimizes transmitter-side clipping and employs CS for receiver-side recovery is presented. This technique adds complexity to transmitter and divides the reserved tones between the transmitter-side and receiver-side processes, thus limits its ability for severe clipping scenario. However, most of CS techniques are based on convex $\ell_1$-optimization, hence relatively of high complexity. In addition, they do not make full use of available prior information that could enhance the performance. A CS approach that uses advanced method for reliable data selection is proposed in [28]. This reliable data subset is used to mitigate nonlinear distortion in OFDM signals. This method uses clipping to reduce PAPR and mitigate the clipping signal at the receiver side based on reliable carriers. This method, however, adopts enhanced convex $\ell_1$-optimization programming which relatively of high complexity. Moreover, it does not deal with oversampled OFDM signals which is the case in practical communication.

This work proposes receiver-based low complexity Bayesian approach to reduce PAPR. While the approach is Bayesian (thus acknowledging the sparsity of sparse clipping signal and the Gaussianity of the additive noise), it is agnostic to the distribution of the sparse signal support and robust to uncertainty in noise variance and sparsity rate. The approach is enhanced by utilizing prior information about the clipping signal from the received signal including the phase information and probable locations where clipping has occurred. Hence, on the one hand, our approach is robust against the uncertainties of clipping signal statistics, while on the other hand the approach utilizes the received signal to extract information that assists in a robust
recovery of clipping signal. In addition, sparsity recovery is achieved via a greedy low-complexity Bayesian matching pursuit method. More details about Bayesian approach can be found in Section 1.4.

In practice, the clipping is carried out after oversampling OFDM signal since the transmitted signal is obviously analog. Oversampling OFDM signals results in block clipping signal\(^2\). As has been discussed previously, out-of-band distortion can be avoided by repeating clipping-and-filtering until the overall peak regrowth is reduced. It has been shown that, filtering an oversampled signal causes an insignificant peak regrowth, thus practical systems should require oversampling before clipping to avoid considerable peak regrowth \([29,30]\). In Chapter 4, we study clipping signal recovery when oversampling is applied and Bayesian block-sparsity technique is used \([31]\).

### 1.4 Compressed Sensing & Sparse Signal Recovery

Compressed sensing, also known as compressive sensing, is a novel sampling (sensing) paradigmatic technique for finding solutions of an under-determined system. In \([32–35]\), a new algorithm is developed to recover sparse signals (where the majority of elements are zeros) from what is for normal signal incomplete information. In signal processing, unlike traditional recovery methods which follow Shannon/Nyquist criterion\(^3\), CS method can recover sparse signals from much fewer measurements. In other words, CS intends to reduce the number of measurements while maintain the necessary information for desire signal recovery.

However, using this technique relies on satisfying two fundamental principles:

1. **Sparsity**: which pertains to desired signals where most of its coefficients are zeros and few of them are non-zero. Hence, this signals are compressible and the information in time domain, for example, can be covered by a sensing rate

\(^2\)Block sparse signals are signals have a structure where the few active elements appear in groups.

\(^3\)The sampling rate should be at least twice the highest frequency of the signal.
much smaller than what its bandwidth indicates. In other words, CS exploits the redundancy in the bandwidth.

2. Incoherence: which pertains to the measurements domain. Measurement samples should be taken from a basis which is different from (and incoherent to) basis of the sparse signal. It has been shown that projections over random basis can satisfy this incoherence feature\(^4\). Therefore, a few measurements from different domain are enough to recover a sparse signal.

In summary, CS is an efficient technique to reconstruct a sparse signal by sensing what appears to be incomplete measurements from an incoherent domain. In our work, the sparse signal is in the time domain and the measurements are taken from the frequency domain. In Fig. 1.2, an example of sparse signal in time domain and we can see that its representation in the frequency domain spreads out in all frequencies. This means that a sparse signal in the time domain impacts all subcarriers of an OFDM system. It also shows the redundancy in the frequency domain.

![Figure 1.2: Sparse signal in the time domain and its representation in the frequency domain](image)

Let us now illustrate the fundamental concept of CS model. Let \( N \)-dimensional sparse vector \( \mathbf{c} \), with \( S \ll N \) non-zero entries\(^5\), be the desired solution of an under-

\(^4\)See the discussion following Equation (1.6)

\(^5\)Locations of non-zero entries (or active coefficients) of sparse signal are called the support.
determined system defined by
\[ y = Ax + n \] (1.5)
where \( y \) is an \( M \times 1 \) measurements vector, \( A \) is an \( M \times N \) measurement matrix and \( n \) is a noise vector. Here, the number of measurements satisfy \( M \ll N \) where \( N \) is number of unknowns, creating the under-determined system. A simple mathematical solution is to use the least-squares (LS) method to solve this problem. However, the system is ill-conditioned, therefore the LS criterion has infinitely many solutions.

According to [36], if the measurement matrix \( A \) obeys the Uniform Uncertainty Principle (UUP), then, the sparse vector \( x \) can be reconstructed via minimizing \( \ell_1 \)-optimization problem
\[
\min \|x\|_{\ell_1} \quad \text{s.t.} \quad \|Ax - y\|_2^2 < \epsilon
\] (1.6)
where \( \epsilon \) is a small noise-dependent parameter. Fortunately, partial Fourier measurements matrices with random row-selections have been shown to obey the UUP\(^6\) [33, 36]. The condition required to correctly estimate the \( S \)-sparse signal is that the number of measurements \( M \) has to follow the inequality
\[
M \geq S \log \left( \frac{N}{S} \right).
\]
Hence, by satisfying sufficient conditions on \( A, n \) and sparsity of \( x \), the convex \( \ell_1 \)-optimization in (1.6) provides the unique solution. The use of \( \ell_1 \) regularization has received widespread of applications e.g., data compression [33, 37], digital photography [38, 39]. A review of other applications in medical imaging, error correction, analog-to-digital conversion and sensor networks can be found in [40].

In contrast to CS-based approaches which use basis pursuits that utilize only the sparsity information. Bayesian approaches acknowledge further statistical informa-

---

\(^6\)It will be the case for our study as we can see later.
tion of sparse signals and noise. Bayesian matching pursuit methods are recursive procedures that provide an approximate minimum-mean square error (AMMSE) estimate of sparse signal. Moreover, Bayesian estimating algorithms are known of being relatively very fast when compared to CS techniques which suffer from high complexity.

Define a linear regression model similar to (1.5) but with unit norm columns in $A$, we have

$$y = Ax + n$$

(1.7)

In [2], a fast Bayesian matching pursuit (FBMP) is proposed to find an AMMSE estimate of $x$, i.e., $\hat{x}_{\text{ammse}} \sim \mathbb{E}[x|y]$. However, several assumptions are required. Active elements of sparse vector $x$ are assumed to follow zero-mean Gaussian distribution. In addition, the active elements of $x$ are activated through i.i.d. Bernoulli model with sparsity rate (probability of success) $\rho$. In other words, an element $x(i)$ is activated with probability of $\rho$ and its value is drawn from a Gaussian mixture. The noise $n$ is assumed to be i.i.d. AWGN. The procedure to evaluate AMMSE estimate of $x$, i.e., $\hat{x}_{\text{ammse}}$ can be summarized in two steps:

1. Build a basis selection metric to specify which elements of $x$ are most likely active\(^7\). A greedy algorithm is developed to evaluate the basis selection metric for large number of possible support.

2. Evaluate an approximate MMSE estimate using the dominant support from step 1. Computations of the dominant posterior from step 1 can be incorporated in evaluating AMMSE estimate, hence, making the algorithm computationally of low-complexity.

A similar approach is in [41] with different way to evaluate the dominant support. More details about similarity between these two algorithms is discussed in [2].

\(^7\)Henceforth will be called the dominant support.
In [3] a support agnostic Bayesian matching pursuit (SABMP) is proposed. This approach is similar to FBMP but with the following advantages:

1. The approach does not assume Gaussian statistics to the sparse signal and it can be performed even for unknown signal prior.

2. Does not require estimates of signal statistics and it is agnostic with regard to statistics variation.

3. It can estimate priori statistics of the additive noise and the sparsity rate when they are not available.

4. It is fast and simple due to its greedy approach and order-recursive nature.

Block version of the above algorithm is is proposed in [31]. This method develops more advanced greedy approach to find the dominant block support in order-recursive manner. Then, the method proceeds to evaluate AMMSE estimate of desired sparse signal by using previous computations used to find the dominant support, thus the approach enjoys low-complexity.

1.5 Thesis Contributions

This thesis contributes in many aspect of PAPR problem. The following are the main contributions.

1. The work of the thesis attempts to reconstruct sparse clipping signals via a Bayesian approach which evaluates the AMMSE estimate of the desired clipping signals. Reduction of OFDM signal is carried out via simple clipping operation which is equivalently generates the sparse clipping signal. Therefore, the problem is transformed from PAPR reduction to sparse signal recovery. Bayesian recovery methods allow us to make full use of prior information about clipping
signal. Therefore, these pieces of information is incorporated to enhanced and modified the Bayesian approach.

2. Our proposed technique is robust against the following uncertainties, therefore our approach employs least possible assumptions and relies only on the information at the receiver side. The uncertainties include:

- Unknown statistics of desired signal. The adopted Bayesian approach does not assume any distribution for clipping signal nor requires variance of clipping signal.

- Unknown recovery parameters (sparsity rate, noise variance and clipping threshold). The approach does not require estimates of the recovery parameters. Rather, rough estimates of the recovery parameters can be evaluated from received information only. A refinement process is used to refine these parameters iteratively to enhance overall estimate of clipping signal.

3. In Chapter 2, tone reservation (TR) technique is used to sense information about clipping signal at the receiver. Problem formulation of tone reservation in OFDM systems is provided then Bayesian recovery of the clipping signal is presented. In this Chapter, our approach uses only received information to recovery the original OFDM signal. The Chapter then ends by providing simulation experiments.

4. Data-aided (DA) technique is presented in Chapter 3. Tone reservation consumes part of data spectrum, hence reduces the data rate. However, data-aided technique allows us to sense clipping information from data subcarriers, thus no loss is introduced in data rate. The Chapter first introduces a reliability function that selects reliable subcarriers. Therefore, from these reliable subcarriers we are able to establish an under-determined system from which the
clipping signal is recovered. Similar Bayesian approach used in Chapter 2 is implemented to DA technique to recover the clipping signal. Then Chapter 3 concludes with simulation results.

5. Practically, clipping is performed in analog OFDM signals. To get analog signals, OFDM signals need to be oversampled. Oversampling OFDM signals results on block sparsity of sparse clipping signals. We adopt Bayesian approach to recover the block sparse signal. Chapter 4 demonstrates this feature.

6. Receivers at the base station (BS) are usually equipped with multiple antennas. To utilize this feature, we concatenate received signals in one system. Chapter 4 explains how could this idea improves estimating clipping signals.

The thesis ends by concluding remarks and discussion of future scope.
Chapter 2

PAPR Reduction by Bayesian Tone Reservation (TR)

2.1 Motivation

Tone reservation is mostly used as a distortionless technique to efficiently reduce the PAPR of OFDM signals. TR is a scheme where a small set of subcarriers are optimized to compute the values for these subcarriers such that the PAPR is minimized. First work towards this direction was done by Tellado [42] where TR is used as transmitter-based technique. The objective is to find the optimized time domain signal such that when it is added to the OFDM signal, the PAPR is reduced. This technique which is based on convex optimization, provides a significant reduction to PAPR especially for large number of reserved tones. To reduce the complexity of this technique, a gradient approach is proposed in [43]. Similar tone reservation techniques are proposed in [17, 44–46].

In [25], a PAPR reducing method which uses the reserved tones as sensing carriers to recover the clipping signal via CS approach. The channel estimation pilots are adopted for this purpose in [26]. A hybrid approach based on TR and CS which composed of transmitter-side and receiver-side CS is presented in [27]. These CS-based techniques reduce PAPR at the transmitter side by clipping then at the receiver,
they mitigate the clipping based on CS approach. However, techniques which are based on convex relaxation regularized $\ell_1$-optimization suffer from relatively high complexity. In addition, they do not make full use of a priori information that could enhance the performance.

In this chapter, we will propose a Bayesian receiver-based low-complexity clipping signal recovery method. This method is able to i) reduce PAPR via a simple clipping scheme, ii) use a Bayesian recovery algorithm to reconstruct the clipping signal. The Bayesian recovery can be enhanced by utilizing all prior information.

### 2.2 Problem Formulation

The time domain OFDM signal is characterized by larger signal peakiness leads to high peak-to-average-power ratio (PAPR). In a simple clipping scheme, reduction of PAPR is achieved by subjecting the signal to a magnitude limiter to a certain threshold $\gamma$, hence,

$$x_p(i) = \begin{cases} \gamma e^{j\theta_x(i)} & \text{if } \|x(i)\| > \gamma \\ x(i) & \text{otherwise} \end{cases}$$

where $i = 0, 1, ..., N - 1$ (2.1)

where $x_p(i)$ is the $i$th element of the signal after clipping, $\gamma$ is the limiting threshold and $\theta_x(i)$ is the phase of $x(i)$.

Since a few peaks of $x$ are larger than the threshold $\gamma$, this soft clipping is equivalent to adding a sparse signal $c$ to the original time domain signal $x$. This sparse signal $c$ has active elements only where clipping has occurred, i.e.,

$$x_p = x + c$$

(2.2)

Noteworthy that this clipping scheme assures that, i) the phase of $c$ is exactly
opposite to that of $x_p$, and $ii$) no distortion to the phase of $x_p$ is introduced. It is very important to make sure that the phase of the clipped signal is undistorted. This undistorted phase is utilized in the recovery of clipping signal i.e. $c$, as will be witnessed later.

Equation (2.2) can be equivalently rewritten as

$$x_p = F^H \mathcal{X} + c.$$  

(2.3)

this clipped signal to be transmitted and easily estimated at the receiver.

Since the clipped signal is transmitted through the channel and corrupted by the noise, it is better to process this signal in the frequency domain. Hence at the receiver, recall from discrete representation of the OFDM system in Appendix A, the received signal in frequency domain can be written as

$$\mathcal{Y} = \Lambda \mathcal{X}_p + \mathcal{Z},$$

(2.4)

where $\mathcal{Z} = F z$ and $z$ is a zero-mean i.i.d. complex Gaussian noise with variance $\sigma_n^2$, and $\Lambda = \text{diag}(\mathcal{H})$, where $\mathcal{H}$ is a diagonal matrix with channel impulse response in frequency domain being in the diagonal. In this thesis we consider a dispersive fading frequency-selective channel with known impulse response.

Let us assume that the OFDM system has $N$ subcarriers (tones), out of which $K$ subcarriers are used for data transmission and the remaining $M \ll K$ subcarriers\footnote{These few subcarriers are enough to estimate $c$. For more details in sparse signal reconstruction from incomplete frequency information, see Section 1.4.} are reserved for sparse signal recovery at the receiver. Let, $S_c$ denotes an $N \times N$ binary selection matrix with 1’s only at $M$ locations along the diagonal determined according to the reserved subcarriers (free subspace).

We proceed by projecting $\mathcal{Y}$ onto the reserved subcarriers selection matrix ($S_c$).
Figure 2.1: PAPR is reduced via simple clipping scheme by limiting the signal amplitude to certain threshold. Then, at the receiver and from the free subcarriers (black circles in the figure) we get information about the clipping signal.
This gives us

\[ S_c \mathbf{y} = S_c (\Lambda (\mathbf{x} + \mathbf{c}) + \mathbf{z}) \]
\[ = S_c \Lambda \mathbf{c} + S_c \mathbf{z}, \]

or

\[ \mathbf{y}' = \Psi \mathbf{c} + \mathbf{z'}, \tag{2.5} \]

where \( \mathbf{y}' = S_c \mathbf{y} \), \( \Psi = S_c \Lambda \mathbf{F} \), and \( \mathbf{z}' = S_c \mathbf{F} \mathbf{z} \). Note that, projecting \( \mathbf{x} \) onto \( S_c \) results in a zero vector. The only unknown in the resulting equation is \( \mathbf{c} \). Note that, since \( S_c \) is a diagonal selection matrix, it contains \( (K = N - M) \) zero rows corresponding to data subspace. Therefore, there are \( K \) zero entries in \( \mathbf{y}' \) which we remove to get a new \( M \)-dimensional vector \( \mathbf{y}'_m \). Similarly the new \( M \times N \) measurements matrix is \( \Psi_m \). The \( \mathbf{z}'_m \) is a \( M \times 1 \) white Gaussian noise vector with \( \mathbf{z}'_m \sim \mathcal{CN}(0, \sigma^2_n \mathbf{I}) \). Therefore (2.5) becomes

\[ \mathbf{y}'_m = \Psi_m \mathbf{c} + \mathbf{z}'_m, \tag{2.6} \]

which is a set of \( M \) equations and thus represents projection of the \( N \)-dimensional sparse signal onto a basis of dimension much smaller (\( M \ll N \)).

Recall from (2.1) and (2.2), a clipping at the \( i \)-th entry is 
\( c(i) = -(x(i) - \gamma) \). This implies that the phase of the nonzero elements of \( \mathbf{c} \) is always opposite to that of the corresponding elements of the clipped signal \( \mathbf{x}_p \). Since the phase can be deduced from \( \mathbf{x}_p \), we need to estimate only the magnitudes and locations of the non-zero elements of the sparse signal. Therefore the modified version of (2.6) becomes (note that, henceforth, \( \mathbf{c} \) is a vector of magnitudes of \( \mathbf{c} \))

\[ \mathbf{y}'_m = \Psi_m \Theta \mathbf{c} + \mathbf{z}'_m, \tag{2.7} \]
where $\Theta_c$ is a matrix containing the anti-phases of signal $x_p$ along the diagonal, i.e.

$$\Theta_c = -\Theta_{x_p} = -\text{diag}\{e^{j\theta_{x_p}(1)}, e^{j\theta_{x_p}(2)}, \ldots, e^{j\theta_{x_p}(N)}\}.$$ 

Combining $\Psi_m$ and $\Theta_c$ we get

$$\mathbf{Y}'_m = \Phi_m c + \mathbf{Z}'_m,$$ 

(2.8) where $\Phi_m = \Psi_m \Theta_c$. Since all parameters except $c$ are complex in the above equation, we can split the complex equation into real and imaginary parts as follows:

$$\begin{bmatrix} \Re(\mathbf{Y}'_m) \\ \Im(\mathbf{Y}'_m) \end{bmatrix} = \begin{bmatrix} \Re(\Phi_m) \\ \Im(\Phi_m) \end{bmatrix} c + \begin{bmatrix} \Re(\mathbf{Z}'_m) \\ \Im(\mathbf{Z}'_m) \end{bmatrix},$$ or

$$\mathbf{Y} = \Phi c + \mathbf{Z}.$$ 

(2.9)

Unlike (2.8) which is a system of $M$ equations, we now have $2M$ equations to estimate a real unknown vector $c$ of dimension $N$, therefore a better estimation of $c$ would be achieved. We aim to recover $c$ using this under-determined system and to do so we pursue an MMSE estimate which is discussed in Section 2.3.

Once we evaluate an estimate of $c$, i.e., $\hat{c}$, we subtract it from the estimated clipped signal $\hat{x}_p$ to get an estimate of the original signal $x$, see (2.2). The time domain clipped signal can be estimated by equalizing the channel effect from (2.4).

\[ \text{diag}\{a, b\} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}. \]

\[ \text{Re}(a), \text{Im}(a) \text{ denote operations of extracting, respectively, the real and imaginary components of complex valued } a. \]
Thus, we have

\[ \hat{x}_p = F^H (x_p + \Lambda^{-1} Z) \]

\[ = x_p + e, \quad (2.10) \]

where \( e = F^H \Lambda^{-1} Z \) represents the error in estimating \( x_p \).

This explains the process of recovering the transmitted signal while avoiding the high PAPR problem. Now, we present the Bayesian recovery of sparse signal \( c \), which utilizes all prior information.

### 2.3 Bayseian Recovery Algorithm

The sparse signal \( c \) can be represented as \( c = c_v \odot c_b \) where \( \odot \) denotes element-by-element multiplication. For example

\[ c = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \odot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 2 \\ 0 \end{bmatrix} \]

\( c_v \) is composed of values of clipping signal while \( c_b \) is a binary vector with 1’s at active clipping locations and 0’s at others.

A tractable Bayesian approach to recover the amplitude of \( c \) from (2.9) would normally impose an assumption that the active elements of \( c_v \) are drawn from a Gaussian distribution. However, this is not the case for the clipping signal. Recall from (2.1) and (2.2) that \( c \) is composed of the difference between a constant value \( \gamma \) and a Rayleigh distributed signal (the amplitude of \( x \) follows Rayleigh distribution [5]).
Hence, the nonzero elements of $c$ are certainly not Gaussian. Therefore, we pursue a Bayesian approach for the estimation of $c$ which does not make any assumption about the statistics of the nonzero elements of $c$.

We proceed by finding a minimum mean square error (MMSE) estimate of $c$ given $\hat{Y}$ as follows:

$$
\hat{c}_{\text{mmse}} \triangleq \mathbb{E}[c|\hat{Y}] = \sum_S p(S|\hat{Y})\mathbb{E}[c|\hat{Y}, S], \quad (2.11)
$$

where the sum is evaluated for all $2^N$ possible support sets $S$ of $c$. If we know the actual support $S$, the linear model in (2.9) becomes,

$$
\hat{Y} = \Phi_S c_S + \tilde{Z},
$$

where $\Phi_S$ is a matrix formed by selecting columns of $\Phi$ indexed by the support $S$, similarly $c_S$ is formed by selecting entries of $c$ indexed by support $S$. It is impossible to compute $\mathbb{E}[c|\hat{Y}, S]$, since the distribution of $c$ is unknown. Therefore, instead the best linear unbiased estimate (BLUE) is used as follows

$$
\mathbb{E}[c|\hat{Y}, S] = (\Phi_S^H \Phi_S)^{-1} \Phi_S^H \hat{Y}.
$$

It remains to evaluate the posterior $p(S|\hat{Y})$ and the sum in (2.11). Using Bayes rule we can write

$$
p(S|\hat{Y}) = \frac{p(\hat{Y}|S)p(S)}{p(\hat{Y})}, \quad (2.12)
$$

note that $p(\hat{Y})$ is common to all posteriors and therefore, could be ignored. A normal Bayesian approach would consider that the elements of $c$ are activated according to a Bernoulli distribution with success probability $\rho$. However, note that, the closer the
clipped signal $x_p$ to the threshold the more probable it is to be clipped. Therefore, we see that using $\rho$ as success probability for each location is not a good idea. Thus, we modify this approach such that the probability of success of some entries are enhanced over the others. To do so, we define $w$ as the difference between the amplitude of estimated clipped signal $\hat{x}_p$ and threshold $\gamma$, i.e., $w = \gamma - \|\hat{x}_p\|$, and use it as a weighting vector. Therefore, it is obvious that we assign higher weights to locations where the abovementioned difference is small. Hence, we have \footnote{For example, calculating $p(S)$ for coefficients 1,2 being active, where the others are not, is $p(S) = p_1 p_2 \prod_{k \neq 1, 2}^N (1 - p_k)$ while in i.i.d. this is $p(S) = \rho^2 (1 - \rho)^{N-2}$.}

$$p(S) = \prod_{i=1}^{N} p_i \quad \text{for all } i = 1, 2, \ldots, N$$

(2.13)

where the probability $p_i = \rho e^{-w(i)}$. Note that, $p_i$'s are normalized such that the maximum value is 1.

By this modification, we increment the probability of those elements of $c$ where $\hat{x}_p$ is close to the threshold $\gamma$. Note that, $\rho$ represents the probability of an occurrence of a non-zero value at a location in $c$ which in our case translates to the probability of a clipping occurrence.

It remains to evaluate the likelihood $p(\mathbf{Y}|S)$. Since $\mathbf{Y}$ composes addition of two factors: a Gaussian noise vector and unknown vector in the subspace spanned by the columns of $\Phi_S$, it is very difficult to determined direct evaluation of the likelihood. To go around this, we eliminate the unknown component by projecting $\mathbf{Y}$ onto the orthogonal complement space of $\Phi_S$. This is done by multiplying $\mathbf{Y}$ by the projection matrix $P_S^\perp = \mathbf{I} - P_S = \mathbf{I} - \Phi_S \Phi_S^H \Phi_S \Phi_S^H \downarrow -1$. This leaves us with $P_S^\perp \mathbf{Y} = P_S^\perp \mathbf{Z}$. 


which is Gaussian with a zero mean and covariance

\[ K = \mathbb{E}[(P_S^\dagger \bar{Z})(P_S^\dagger \bar{Z})^H] \]
\[ = P_S^\dagger \mathbb{E}[\bar{Z} \bar{Z}^H] P_S^\dagger = P_S^\dagger \sigma_n^2 P_S^\dagger \]
\[ = \sigma_n^2 P_S^\dagger. \]  (2.14)

Thus we have,

\[ p(\bar{Y}|S) \simeq \frac{1}{\sqrt{(2\pi \sigma_n^2)^M}} \exp \left( -\frac{1}{2} (P_S^\dagger \bar{Y})^H K^{-1} (P_S^\dagger \bar{Y}) \right). \]  (2.15)

Simplifying and dropping the pre-exponential factor yields,

\[ p(\bar{Y}|S) \simeq \exp \left( -\frac{1}{2\sigma_n^2} \left\| P_S^\dagger \bar{Y} \right\|^2 \right). \]  (2.16)

Substituting (2.13) and (2.16) into (2.12) finally yields an expression for the posterior probability. In this way, we have all the ingredients to compute the sum in (2.11). Computing this sum is a challenging task when \( N \) is large because the number of support sets can be extremely large and the computational complexity can become unrealistic. To have a computationally feasible solution, this sum can be computed over a few support sets corresponding to significant posteriors. Let \( S_d \) be the set of supports for which the posteriors are significant. Hence, we arrive at an approximation to the MMSE estimate given by,

\[ \hat{c}_{\text{ammse}} = \mathbb{E}[c|\bar{Y}] = \sum_{S \in S_d} p(S|\bar{Y}) \mathbb{E}[c|\bar{Y}, S] \]  (2.17)

We follow the method mentioned in [2, 3] where they devised a greedy algorithm to find a subset of the dominant support \( S_d \). Noteworthy, the weighted version of \( p(S) \) in (2.13) also helps to find the dominant support faster than the unweighed version.
Table 1 Signal recovery algorithm

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>estimate $\hat{x}_p = F^H A \hat{y}$</td>
</tr>
<tr>
<td>2.</td>
<td>$\hat{\gamma} = \max(\hat{x}_p)$</td>
</tr>
<tr>
<td>3.</td>
<td>$\hat{\sigma}_n^2 = \text{var}(\hat{Y})$</td>
</tr>
<tr>
<td>4.</td>
<td>$w = \hat{\gamma} -</td>
</tr>
<tr>
<td>5.</td>
<td>$\hat{\rho}_o = Q\left(\frac{\hat{x}_p - \mu}{\sigma}\right)$, an initial estimate, where $\mu$ and $\sigma$ are the mean and standard deviation of $\hat{x}_p$, respectively.</td>
</tr>
<tr>
<td>6.</td>
<td>$i = 0$, repeat</td>
</tr>
<tr>
<td>7.</td>
<td>$p_k = \hat{\rho}_i e^{-w(k)}$, $k = 1, 2, ..., N.$</td>
</tr>
<tr>
<td>8.</td>
<td>Compute $\hat{c}<em>{\text{ammse}}$ and $\hat{\rho}</em>{i+1}$ using the technique discussed in [3]</td>
</tr>
<tr>
<td>9.</td>
<td>until $(</td>
</tr>
<tr>
<td>10.</td>
<td>$\hat{c} =</td>
</tr>
<tr>
<td>11.</td>
<td>$\hat{x} = \hat{x}_p - \hat{c}$</td>
</tr>
</tbody>
</table>

Note that, the Bayesian approach, discussed above, requires information about the sparsity rate ($\rho$), the noise variance ($\sigma_n^2$) and the threshold ($\gamma$), to recover $c$. However, exact values of these parameters may not be available at the receiver. Therefore, there should be a way to recover $c$ perfectly even in the presence of rough estimates. We start with initial estimates of required parameters and estimate $c$. This estimate is in turn used to refine the abovementioned parameters which are then used to get a better estimate of $c$ again. This process is repeated a number of times. Specifically, the refinement process is continued until the percentage change in the two consecutive estimates of sparsity rate ($\hat{\rho}$) becomes less than 2%. An algorithmic description of the recovery process that we follow is provided in Table 1.

2.4 Simulation Results and Discussion

For numerical implementation $N = 512$ subcarriers were used, where 20% of them were reserved randomly\(^5\) for clipping signal recovery ($M = 20\%$ of $N$). Data are generated from a 64-QAM constellation ($L = 64$). The SNR at the receiver is kept

\(^5\)It has been shown in [33,36] a small number of random measurements can recover a sparse signal with high accuracy from a fixed incoherent basis.
at 30dB and the noise is i.i.d. additive white Gaussian noise (AWGN). We consider a frequency-selective Rayleigh fading channel, with 64-taps, which is assumed to be known at the receiver. The two performance metrics considered are bit error rate (BER) and average run-time. An average of 200 independent realizations are used for all plots.

We used two Bayesian matching pursuit algorithms for sparse signal recovery, the fast Bayesian matching pursuit (FBMP) [2] and the support agnostic Bayesian matching pursuit (SABMP) [3]. In addition, we also estimated $c$ using $\ell_1$-optimization using CVX, a package for specifying and solving convex programs [1]. All these techniques are used to estimate the sparse signal from the system of equations given in (2.6). We compared the performance of these methods with the performance of our enhanced version i.e., weighted and phase augmented WPA-SABMP which is able to utilize the phase and clipping probability to solve for $c$. We also compare the performance of these methods to oracle-LS (as a benchmark) where the receiver knows the actual support of $c$.

2.4.1 Experiment 1: exact parameters

In this experiment, we study the performance of the abovementioned algorithms with respect to the varying clipping threshold $\gamma$. Specifically, we plot BER versus $\gamma$. In this experiment, exact values of the required parameters (i.e., sparsity rate $\rho$, noise variance and signal variance$^6$) were provided to the Bayesian estimation algorithms.

Fig. 2.2 shows the superior performance of WPA-SABMP over other algorithms. Please note that, in all of the figures as $\gamma$ increased from 7.8 to 11.3, sparsity rate decreased from 0.16 to 0.02. In a similar simulation setup, we found that the symbol error rate (SER) for no estimation of $c$ was in average 0.1242 while the SER of WPA-SABMP algorithm was reduced to 0.0339. Although, the Bayesian methods other than the

---

$^6$SABMP and WPA-SABMP do not require signal variance.
enhanced version (WPA-SABMP), are less effective than the $\ell_1$-optimization programming, they have the advantage of being fast especially for large size of OFDM signals ($N$).

![Graph](image)

Figure 2.2: BER versus $\gamma$ when using exact values of the recovery parameter

### 2.4.2 Experiment 2: estimated parameters

In this experiment, we study the performance of the algorithms when the exact parameter values are unknown. However, we used the information at the receiver to compute the initial estimates of the required parameters which are then provided to the Bayesian algorithms. The proposed WPA-SABMP algorithm is capable of refining these estimates in an iterative manner as mentioned in Table 1. We allow our algorithm to refine the initial estimates of the parameters and compare its performance to the two algorithms that performed best in Experiment 1. A plot of the BER versus the clipping threshold $\gamma$ is provided in Fig. 2.3.
As expected, WPA-SABMP (refined) performed better than its non-refined version. We also note, although, rough initial estimates were provided to non-refined WPA-SABMP, it still performed better than other algorithms (except the refined version). We would like also to mention that, as compared to $\ell_1$-optimization programming, our algorithm WPA-SABMP (refined) required much less time for estimation. It is also noteworthy to mention that by virtue of the weighted $p(S)$ the WPA-SABMP algorithm requires less time than plain SABMP as it is able to find the correct support quickly (see the discussion following (2.17)).
2.5 Conclusion

In this chapter, a robust Bayesian algorithm for clipping signal recovery to reduce PAPR in OFDM, is presented. This algorithm uses the reserved tones to sense clipping information. The proposed method requires no information at the receiver about the noise variance, clipping signal statistics nor the clipping threshold. It extracts estimates of all required parameters from the received signal. In addition, the algorithm makes it possible to utilize and exploit the prior information of clipping signal which can be obtained from the received signal. The estimation can be improved further by refining the estimating process.

However, the drawback of TR method is the data rate loss. A suggest of using channel estimation pilots to sense for clipping signal is possible. However, in the next chapter, data-aided method which allows recovery of clipping signal with no loss in data rate, is presented.
Chapter 3
PAPR Reduction by Bayesian Data-Aided (DA)

3.1 Motivation

The distortion of clipping affects all subcarriers. That means, both the free subcarriers and the data subcarriers are perturbed by clipping. In the previous chapter, we observe the distortion using the reserved subcarriers. This observation, actually, is the difference between original values at these tones (which are zeros) and the new ones (which are due distortion). Similarly, we can observe the deviation from the original data symbols. From this observation, we will be able to give good estimation of the sparse signal, yet without any tone reservation. However, this is true as long as the perturbation is not large enough to move the data across the original decision regions (maximum likelihood (ML) regions), which would otherwise produce a destructive information. In other words, the problem is how to know whether the received data symbols are laying within their original (true) constellation points or not. However, in [28] a reliability function is developed to select subset of tones which are most likely do not have this crossing phenomenon.

Specifically, if a received symbol is not at its original constellation point, then it is more probable to be deviated (due to clipping distortion and noise) from one of the nearest four neighbors. For example, in Fig. 3.1 we can see  \( \hat{X}(1) \) and  \( \hat{X}(2) \) lay in the ML region of point  \( X \), however,  \( \hat{X}(1) \) is more reliable than  \( \hat{X}(2) \).

Let us define the reliability function to be a ratio of: probability of deviation from
Figure 3.1: The nearest neighbours of $\mathcal{X}$. We can see two examples of received symbols deviated from their original points. A reliability function is required to select which subcarriers are most reliable.

The nearest point (ML point\(^1\)) to sum of probabilities of nearest three neighbors. That is for the previous example

\[
\mathcal{R}(1) = \frac{\Pr(\hat{\mathcal{X}}(1) = \mathcal{X})}{\sum_{i=a,b,c,d} \Pr(\hat{\mathcal{X}}(1) = \mathcal{X}_i)}
\]

\[
\mathcal{R}(2) = \frac{\Pr(\hat{\mathcal{X}}(2) = \mathcal{X})}{\sum_{i=a,b,c,d} \Pr(\hat{\mathcal{X}}(2) = \mathcal{X}_i)}
\]

\(^1\)The nearest constellation point which the ML decoder demodulates the received symbol to.
This procedure is carried out for all subcarriers calculating $\mathcal{R}(1), \mathcal{R}(2), \ldots, \mathcal{R}(N)$. The next step is to order these reliabilities for maximum to minimum and choose the $M$ highest reliable subcarriers. These $M$ subcarriers denote our data-aided set of carriers for which deviation from the nearest data point is due to clipping distortion with high probability. This subset of the $M$ subcarriers will be used to span the subspace where we estimate the sparse signal$^2$. This will be demonstrated clearly in the next section.

3.1.1 Remark

Note that the reliability function in (3.1) takes into consideration the nearest neighbours only. One can also consider a reliability criterion that incorporate all other constellation points, but the performance improvement would be minimal at a much higher computational complexity.

Moreover, a simpler reliability functions can be adopted (e.g. the Euclidean distance of the point from the nearest neighbours constellation point). It is shown in [28] how the reliability defined in (3.1) reduces to squares and circles around the constellation points.

3.2 Data-aided Based Recovery

As mentioned before, the distortion which cause the deviation in the symbol constellation is due to two sources, the clipping and the additive Gaussian noise. The sum will be called the combined distortion.

Consider the received signal in the frequency domain after cycle prefix removal. As we saw in Chapter 2, the signal reads

$^2$Like the reserved subspace used in previous chapter
\[ \mathbf{Y} = \Lambda \mathbf{X}_p + \mathbf{Z} \]  

(3.3)

A simple estimate for clipped signal is obtained by equalizing the channel:

\[
\hat{\mathbf{X}}_p = \Lambda^{-1} \mathbf{Y} \\
= \mathbf{X} + \mathbf{C} + \Lambda^{-1} \mathbf{Z} \\
= \mathbf{X} + \mathbf{F}^H \mathbf{c} + \Lambda^{-1} \mathbf{Z}
\]

or

\[
\hat{\mathbf{X}}_p = \mathbf{X} + \mathbf{D} \tag{3.4}
\]

where \( \mathbf{D} \) is defined by

\[
\mathbf{D} = \mathbf{F} \mathbf{c} + \Lambda^{-1} \mathbf{Z} \tag{3.5}
\]

is the combined distortion (clipping signal can be considered as additive impulse noise). A simple ML decoder will demodulate the received data symbols to nearest constellation point for each symbol (i.e. \( \mathbf{X}_p(k) \)).

There are two scenarios: either the combined distortion i.e., \( \mathbf{D}(k) \) is large enough to move the symbol \( \mathbf{X}_p(k) \) from its original decision region to cross the boundary to another constellation point’s region. The other case is when the deviation is within the true decision region. At the receiver side we have no information whether the in-coming data symbols are of the first case or the other one. As discussed in the
previous section, we assign a reliability measure to each subcarrier given by

$$
R(k) = \frac{\Pr(D(k) = \hat{X}_p(k) - \langle \hat{X}_p(k) \rangle)}{\sum_{i=a,b,c,d} \Pr(D(k) = \hat{X}_p(k) - X_i)}
$$

(3.6)

where the numerator $\Pr(D(k) = \hat{X}_p(k) - \langle \hat{X}_p(k) \rangle)$ is the probability that the distance comes from the nearest constellation point to $\hat{X}_p(k)$, which we denote by $\langle \hat{X}_p(k) \rangle$. The denominator is the probability that the distance was caused by the nearest neighbours to $\langle \hat{X}_p(k) \rangle$ (i.e., $X_a, X_b, X_c$ and $X_d$).

It remains to decide on the distribution of $D$. Note from (3.5) that $D$ is composed of Gaussian noise and the frequency transform of the clipping noise. Therefore, the assumption that $D$ is Gaussian distributed with variance $\sigma_D^2$ that can be calculated from the noise variance and the clipping threshold. Therefore, the probability density function (PDF) of $D(k)$ is (drop the $k$ index)

$$
f_D(X) = \frac{-1}{\pi \sigma_D^2} \exp(\frac{1}{\sigma_D^2} |X|^2)
$$

(3.7)

A prior information about clipping location can be incorporated in calculating $\sigma_D^2$ for each subcarrier. Since, the clipping noise and the Gaussian noise are independent, we can evaluate each one separately. In the time domain of (3.5), the combined distortion does not always have the clipping noise part. Therefore, we obtain the variance of time domain $c$ only when it is most probably active and zero otherwise. To select probable locations of active elements of $c$, we do the following. From estimated $\hat{\rho}$ (sparsity rate), choose $(\hat{\rho}N)$ locations of $\hat{x}_p$ that are closest to clipping threshold $\gamma$ to be a rough estimate of the support. Thus, these information is transformed to frequency domain to obtain $\sigma_c^2$ for all subcarriers. Refer to expressions derived in [28] to evaluate $\sigma_c^2$.

Once the reliability in (3.6) is calculated for all carriers, we order the subcarriers and choose the $M$ ones with highest reliability.
Let the subspace of reliable data symbols be defined by the selection matrix $S_r$ that has 1’s only at locations determined according to the highest $M$ reliable subcarriers from (3.6). The complement subspace (where the data is unreliable) is defined by $\bar{S}_r$. Thus, we have

$$C = S_r(X_p - \langle X_p \rangle) + \bar{S}_r(X_p - X)$$  \hspace{1cm} (3.8)$$

In other words, at $S_r$, the ML decoder will demodulate $\langle X_p \rangle$ to $X$. That means, if we ignore other sources of perturbation, demodulation of clipped signal at $S_r$ will make no error. Fortunately, in practical, the reliable subspace (i.e. $S_r$) has the major part of tones space$^3$.

Recall from (3.4) that by taking the difference between the estimated clipped signal $\hat{X}_p$ and its ML decoded estimate $\langle \hat{X}_p \rangle$, we have

$$\hat{X}_p - \langle \hat{X}_p \rangle = X - \langle \hat{X}_p \rangle + D$$  \hspace{1cm} (3.9)$$

By projecting the above equation onto the reliable subspace, we have

$$S_r^T(\hat{X}_p - \langle \hat{X}_p \rangle) = S_r^T(X - \langle \hat{X}_p \rangle) + S_r^T D$$

Now we know that at the reliable carriers (defined by $S_r$) $S_r^T(X - \langle \hat{X}_p \rangle) = 0$ (see the discussion following (3.8)). So we have

$$S_r^T(\hat{X}_p - \langle \hat{X}_p \rangle) = S_r^T D$$

Using the fact that $D = Fc + \Lambda^{-1}Z$, we can write

$$S_r^T(\hat{X}_p - \langle \hat{X}_p \rangle) = S_r^T Fc + S_r^T \Lambda^{-1}Z$$

$^3$This can be seen from the results of the previous chapter. The average SER for the case where no estimation of $c$ is around $10^{-1}$. That means, 90% of symbols were in their true decision regions.
Alternative, we have:
\[ \mathcal{Y}_d = \Psi_d \cdot c + \mathcal{Z}_d \]  
(3.10)

where \( \mathcal{Y}_d = S_r^T(\hat{\mathcal{X}}_p - \langle \hat{\mathcal{X}}_p \rangle) \), \( \Psi_d = S_r^T \cdot F \), \( \mathcal{Z}_d = S_r^T \Lambda^{-1} \mathcal{Z} \) and \( c \) is the sparse clipping signal in the time domain.

Similar to our approach in Chapter 2, we augment the phase of \( c \), which can be estimated from the received signal to the observation matrix, that is \( \Phi_d = \Psi_d \Theta_c \). In addition, we can split the complex equation into real and imaginary parts which results in a system of \( 2M \) equations (double the number in (3.10)), as follows (\( c \) is the amplitude of \( c \))

\[
\begin{bmatrix}
\Re(\mathcal{Y}_d) \\
\Im(\mathcal{Y}_d)
\end{bmatrix} =
\begin{bmatrix}
\Re(\Phi_d) \\
\Im(\Phi_d)
\end{bmatrix} c +
\begin{bmatrix}
\Re(\mathcal{Z}'_d) \\
\Im(\mathcal{Z}'_d)
\end{bmatrix}, \text{ or}
\]

\[ \tilde{\mathcal{Y}} = \Phi \bar{c} + \tilde{\mathcal{Z}}. \]  
(3.11)

### 3.3 Simulation Results and Discussion

Performance of data aided method is verified by comparing BER of different recovery algorithms. The simulation system used 512 subcarriers (\( N=512 \)) and data were generated from 64-dimensions QAM constellation (\( L=64 \)), unless otherwise mentioned. The desired SNR was kept at 25dB and the noise is i.i.d. Gaussian. A frequency-selective channel was assumed to be known at the receiver. We used the highest (\( M = 30\% \) of \( N \)) reliable carriers to define the reliable subspace.

In Fig. 3.2, the BER is plotted versus the threshold. We can see the superiority of the WPA-SABMP over other algorithms. Plain Bayesian approaches (FBMP and SABMP [2, 3]) almost performed the same. However, our proposed technique WPA-SABMP outperforms the \( \ell_1 \)-optimization programming. In this experiment, sparsity rate decreased from 0.13 to 0.05 as threshold increased from 9.25 to 12.
Figure 3.2: Data aided: BER versus the threshold $\gamma$
In Fig. 3.3, the run time is plotted versus the threshold for large $N$ and $L$. We can see the high complexity of optimization programming when compared to Bayesian algorithms. The WPA-SABMP was the fastest algorithm in this experiment.

Fig. 3.4 shows the BER when the number of reliable carriers is increased. In this experiment, the sparsity is kept in average at $\rho = 0.0335$. In this Figure we can see the performance improves while more number of observations are used. However, Bayesian approaches are more robust against unreliability of the measurements when compared to $\ell_1$-optimization.
Figure 3.4: Data aided: BER versus the number of reliable carriers $M$
3.4 Conclusion

In this chapter, we discuss recovery of clipping signals via data-aided method. This method allows full use of spectrum for data transmission. At the receiver side, reliable data carriers are selected from which the clipping signal can be reconstructed. The Bayesian approach is applied to this method. Simulation results shows the superiority of enhanced Bayesian approach in both BER and computational complexity. However, it remains to utilize some features of a practical OFDM system in order to improve the recovery of clipped signals. The next Chapter is dedicated for this topic.
Chapter 4
Oversampling and Multiple Receivers

The purpose of this chapter is to investigate some features of practical wireless communications that would improve the clipping mitigation. Specifically, the focus of Chapter 4 is on a more practical approach by introducing oversampling and the presence of multiple receivers in OFDM systems. In particular, these practical features of OFDM systems can be exploited to improve the recovery of clipping signal.

Oversampling is an important part in wireless communication systems since the transmitted signal is obviously analog. A sufficient oversampling factor is required to convert a signal to the analog domain. A practical implementation of OFDM systems would perform clipping on analog signals. However, oversampling an OFDM signal results in an increased number of high peaks that appear in groups. Therefore, when an oversampled signal is clipped, a block sparse clipping signal is generated. Section 4.1 discusses how to modify our approach to utilize block sparse clipping signals.

Another important part is multiple receivers. Multiple receivers can be seen in widely accepted S/MIMO technology. Section 4.2 shows an efficient way to utilize the presence of multiple receivers for the recovery of clipping sparse signals.
4.1 Oversampling and Block Sparsity

The oversampling process is an essential part in digital communications systems. This means sampling a signal at a rate higher than Nyquist rate. This process is needed to remove aliasing (spectrum overlapping) by very sharp digital filters. Moreover, oversampling adds more gain to the signal-to-quantization noise ratio (SQNR), hence improve the resolution of quantization process. In multi-rate digital systems, oversampling plays a central role by the decimation and interpolation filters. These advantages and others make oversampling an important part in the practical digital communications.

As mentioned previously, the time domain OFDM signal (which suffers from high PAPR) is a combination of many independent modulated sinusoids. Oversampling in OFDM requires larger IFFT operations. As a result, the new time domain signal has even higher amplitude fluctuations that produce a larger PAPR. It can be noticed that high peaks of an oversampled OFDM signal appear in groups. Therefore, clipping oversampled signals produces block sparse clipping signals. Block sparse signals are signals that have a structure where the few active elements appear in groups.

The next section introduces the oversampled OFDM model.

4.1.1 Oversampling in OFDM

Recall from Section A, OFDM signal in frequency domain for a certain block can be represented as $\mathbf{X} = [\mathcal{X}(0), \mathcal{X}(1), \ldots \mathcal{X}(i), \ldots \mathcal{X}(N - 1)]^T$. Oversampling the OFDM signal is carried out by zero-padding the frequency domain signal. The added zeros are inserted in the middle of the data. This is to guarantee that, the zeros are in the high frequencies. Oversampling OFDM signal by a factor of $J$ is equivalent to zero-padding the data by $((J - 1)N)$ zeros. Thus, the oversampled version of $\mathbf{X}$ is

$$\mathbf{X}_J = [\mathcal{X}(0), \mathcal{X}(1), \ldots, \underbrace{\mathcal{X}(\frac{N}{2}), \ldots, 0}_{((J-1)N)\text{zeros}}, \mathcal{X}(\frac{N}{2} + 1), \ldots \mathcal{X}(N - 1)]^T$$ (4.1)
Figure 4.1: Part of the clipping signal when the OFDM time domain signal is over-sampled by a factor of $J=4$, shows the block sparsity structure.
Consequently, the channel would have the same dispersion duration with new in-between taps. The new channel length is $JD$. This can be explained as follows

$$h_j = [h(0), h(0 + \frac{1}{J}), h(0 + \frac{2}{J}), \ldots, h(0 + \frac{J-1}{J}), h(1), h(1 + \frac{1}{J}), \ldots, h(D + \frac{J-1}{J})]^T$$

where $[h(0), h(1), h(2), \ldots, h(D-1)]$ are taps of the critically sampled case ($J = 1$). Similarly, the cyclic prefix length is increased to $JG$. The oversampled signal can be considered as a regular signal with size $JN$ (with $((J-1)N)$ data symbols being zeros). The remaining parts of the OFDM system will not change.

### 4.1.2 Oversampling in Data-aided

In this section, we investigate how the oversampling would help to recover the sparse signal $c$. Specifically, improvement is achieved from two features: $i$) oversampling produces a block sparse clipping signal which is easier to detect its support, and $ii$) one may use the subcarriers corresponding to padded zeros (Section 4.1.1) as additional observations along with the reliable subcarriers, thus increasing the number of observations.

As mentioned previously, oversampling OFDM signal by a factor of $J$, is performed by inserting $(J-1)N$ zeros in the middle of data symbols. The oversampled time domain OFDM signal has two differences from a critically-sampled one: $i$) it has higher peaks, and $ii$) peaks appear in groups.

Consider the frequency domain signal in (4.1), the time domain signal $x$ is constructed by applying IFFT. Recall from Chapter 2, an amplitude limiter reduces the PAPR by clipping the signal $x$ and the remaining clipped signal is written as

$$x_p = F^H X + c$$

(4.2)

The clipping signal $c$ for oversampling case, can be modeled as $c = c_V \odot c_B$, where the vector $c_V$ is comprised of values of clipping signal. The vector $c_B$ is a block-
structured binary vector with maximum block size equals to $J$ (the oversampling factor).

At the receiver, the frequency domain signal is

$$\mathbf{Y} = \mathbf{\Lambda X}_p + \mathbf{Z},$$  \hspace{1cm} (4.3)

where $\mathbf{Z} = \mathbf{Fz}$ and $\mathbf{z}$ is a zero-mean i.i.d. AWGN and $\mathbf{\Lambda} = \text{diag}(\mathbf{H})$, where $\mathbf{H}$ is the new channel impulse response in frequency domain.

The reliable carriers are selected by following same procedure in the previous chapter. To exploit oversampling OFDM signals, the subcarriers of the padded zeros are added to the reliable subspace. Let $\mathbf{S}_t$ be defined by an $JN \times JN$ binary selection matrix with 1’s at $M$ locations along the diagonal determined according to both: the reliable subcarriers (subspace) and the $(J - 1)N$ subcarriers where zeros are padded.

Similar to the previous chapter, by taking the difference between the estimated clipped signal and its ML constellation points, we have

$$\hat{\mathbf{X}}_p - \langle \hat{\mathbf{X}}_p \rangle = \mathbf{X} - \langle \hat{\mathbf{X}}_p \rangle + \mathbf{D}$$  \hspace{1cm} (4.4)

where $\mathbf{D} = \mathbf{Fc} + \mathbf{\Lambda}^{-1}\mathbf{Z}$.

Now by projecting above equation onto the extended reliable subspace (defined by $\mathbf{S}_t$) we can write a linear system with $U = M + (J - 1)N$ equations as

$$\mathbf{Y}_U = \mathbf{\Psi}_U \mathbf{c} + \mathbf{Z}_U$$  \hspace{1cm} (4.5)

where $\mathbf{Y}_U = \mathbf{S}_t \mathbf{\Lambda}(\hat{\mathbf{X}}_p - \langle \hat{\mathbf{X}}_p \rangle)$, and $\mathbf{\Psi}_U = \mathbf{S}_t \mathbf{\Lambda} \mathbf{F}$, and $\mathbf{Z}_U = \mathbf{S}_t \mathbf{Z}$.

Similar to Chapter 3, phase augmentation is utilized ($\mathbf{\Phi}_U = \mathbf{\Psi}_U \Theta_c$). Therefore

\footnote{Due to the clipping scheme, the clipping signal may have some blocks with size less than $J$}
the complex-split system is constructed as follows (c is only the amplitude of c)

$$
\begin{bmatrix}
\text{Re}(\mathbf{y}'_U) \\
\text{Im}(\mathbf{y}'_U)
\end{bmatrix} =
\begin{bmatrix}
\text{Re}(\mathbf{\Phi}_U) \\
\text{Im}(\mathbf{\Phi}_U)
\end{bmatrix}c +
\begin{bmatrix}
\text{Re}(\mathbf{z}'_U) \\
\text{Im}(\mathbf{z}'_U)
\end{bmatrix},
$$
or

\[ \hat{\mathbf{y}} = \hat{\mathbf{\Phi}}c + \mathbf{\bar{z}}. \] (4.6)

which looks similar to (2.9) and (3.11) but with 2U equations which is much larger than 2M.

Similar recovery procedure used in previous chapters can be carried out to evaluate the AMMSE estimate of c via the Bayesian approach. In addition, the algorithm utilizes the block sparsity property of the desired signal. It searches the dominant support by finding locations of high posteriors. Then it expands the size of these locations to form a block support. This expansion process is evaluated by a metric function that allows better picking of a block location and size. A more details can be found in [31].

### 4.1.3 Simulation Results and Discussion

For numerical implementation \( N = 256 \) subcarriers were used. Data are generated from a 64-QAM constellation \( (L = 64) \). The signal is oversampled by a factor of \( 4^2 \) \( (J = 4) \). The SNR at the receiver is kept at 30dB and the noise is i.i.d. AWGN. We consider a frequency-selective Rayleigh fading channel, with 64-taps, which is assumed to be known at the receiver. The data-aided method is used in this simulation with \( M = 20\% \) of \( JN \). An average of 200 independent realizations are used for the plot.

Fig. 4.2 depicts the performance of the data-aided based Bayesian recovery of an oversampled OFDM signal. Specifically, it compares the Block version of SABMP (i.e. Block-SABMP [31]) with non-block version. We can see the improvement acquired by utilizing the block sparsity of clipping signal. Moreover, the average run-time of

\[ ^{2} \text{A factor of oversampling } J \geq 4 \text{ is enough to capture the peaks of continuous OFDM time domain signal [14, 30, 47].} \]
both algorithms shows that the Block-SABMP is also faster than the plain SABMP by a ratio of 5.4. The sparsity rate decreased from 0.136 to 0.046 as the threshold increased from 4.51 to 5.63.

4.2 Multiple Receivers

Multiple transmit and receive antennas (i.e. MIMO systems) exploit the advantage of spatial diversity obtained by separated antennas, especially in dense propagation scattering environments. Different types of MIMO techniques are implemented either: to maximize spatial diversity which improves power efficiency, to increase the capacity of the system or to increase the data rate. The MIMO-OFDM technology is already part of many modern wireless communications systems e.g. IEEE 802.11 (Wi-Fi),
IEEE 802.16 (WiMax) and 4G cellular communications. However, since MIMO-OFDM is based-on OFDM, it inherits same PAPR problem.

As illustrated in Section A, OFDM system in frequency domain can be seen as a parallel transmission process. We know that clipping signals is convoluted with channel coefficients and corrupted by the AWGN noise. However, ideally we would expect $\mathbf{c}$ to be the same in all antennas. However, for practical cases and with a sufficiently high SNR, we expect support of sparse signals at each receiver to remain the same. In other words, different sparse signals will be received at different antenna, yet these sparse signals share the same support but different values at active elements.

![Figure 4.3: Clipping signal has same support in different receivers.](image)

Let the receiver has $R_x$ number of antennas. The received signal at each antenna
Above systems of equations can be concatenated to form one combined system as follows

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_{R_x}
\end{bmatrix} =
\begin{bmatrix}
\Psi_1 \\
\Psi_2 \\
\vdots \\
\Psi_{R_x}
\end{bmatrix}
\begin{bmatrix}
c \\
c \\
\vdots \\
c
\end{bmatrix}
+ 
\begin{bmatrix}
Z_1 \\
Z_2 \\
\vdots \\
Z_{R_x}
\end{bmatrix}, \text{ or (4.7)}
\]

\[
Y_c = \Psi_c c + Z_c \quad \text{(4.8)}
\]

Usually the system has \(M\) observations at each receiver. In contrast, the system in (4.8) has \(MR_x\) observations. This will result in a significant improvements as we can see in the simulations results. Moreover, multiple receivers can also help to improve estimating the clipped signal \(x_p\) and estimating other required parameters for the recovery of \(c\) (i.e. \(\rho, \gamma, \text{ and } \sigma_n^2\)).

### 4.2.1 Simulation Results and Discussion

A Matlab experiment is used to simulate the data aided method in multiple receivers environments. 512 subcarriers are used \((N=512)\) and data are generated from 64-dimensions QAM constellation \((L=32)\). The number of antenna is \(R_x = 4\). The desired SNR is kept at 30dB and the noise is i.i.d. Gaussian. Frequency-selective channels with 64-taps are assumed independent and known at each receiver. We used the highest \((M = 30\%N)\) reliable carriers.
In Fig. 4.4, the BER is plotted versus \( \gamma \). This figure shows the superiority of concatenated system. Obviously, averaging the estimated sparse signals from all receivers performs better than estimating the signal from one receiver. An improvement of approximately 5.8dB from averaging is archived by using concatenated system.

N=512, 64-QAM, size of reliable subspace M=154

\[ \begin{align*} &\text{No Estimation for } c \\ &\text{SABMP (one antenna)} \\ &\text{SABMP (averaging)} \\ &\text{SABMP (concat.)} \\ &\text{Oracle-LS} \end{align*} \]

\[
\begin{array}{c}
\text{BER} \\
10^{-1} \\
10^{0} \\
\gamma \\
8.5 \quad 9 \quad 9.5 \quad 10 \quad 10.5 \quad 11 \quad 11.5 \quad 12
\end{array}
\]

Figure 4.4: BER versus \( \gamma \) for data-aided with \( R_x = 4 \)

### 4.3 Conclusion

In this chapter, we discuss how to utilize some feature of practical wireless systems. Specifically, it has been shown that, oversampling an OFDM signal produces block sparsity to the clipping signal. Therefore, suitable Bayesian technique can be adopted to recover the block sparse signal. Moreover, multiple receivers allow us to build a larger system by concatenating all received signals in one system which results in a considerable improvement in estimating the clipping signal.
Chapter 5

Conclusion & Future Work

5.1 Concluding Remarks

In this thesis we have studied the Bayesian recovery of OFDM clipped signals. PAPR reduction by clipping was considered in literature as a distorting method. However, this is because there was no mitigation of the clipping distortion at the receiver side. In this thesis, it was shown that clipped signals can be mitigated at the receiver side by two methods. A relatively few tones are reserved at the transmitter as used at the receiver as sensing carriers. Specifically, these reserved tones are used as pilots to collect information about clipping signals. However, the tone reservation method reduces the data rate, therefore the data-aided method was proposed as a second method. The data-aided method allows us to sense clipping information from transmitted data by developing a function that selects reliable subcarriers (tones).

In addition, the recovery of the clipping signal in both sensing methods is carried out via a Bayesian approach. This low-complexity approach is robust against uncertain information about clipping signal. Therefore, the proposed approach tries to rely only on received information. Moreover, the recovery process can be enhanced by utilizing many pieces of prior information. Clipping probability was modified according to the amplitude of the clipped signal. The phase of clipping signal was evaluated from the phase of the clipped signal. Moreover, estimates of sparsity rate and noise variance were extracted from the received information. Clipping locations were incorporated in evaluating the reliability function of all carriers.
A more practical approach was proposed to exploit some features of a typical OFDM system. Specifically, oversampled OFDM signals were taken into consideration. The performance of the recovery was improved by virtue of block sparsity in clipping signal. A more practical challenge was added to the problem by considering multiple receivers. Therefore, this thesis recommends that clipping and recovery is a promising approach to reduce high PAPR of OFDM signals especially for practical systems.

5.2 Ideas for Future Work

The study of the thesis can be expanded to some potential directions. Listed below are some topics for future research.

1. Further study on effects of clipping on estimating channel coefficients is needed. We have already seen that clipping generates additive impulse noise (clipping noise). Therefore, estimating the channel by pilots may not be accurate since the pilots are contaminated by both clipping noise and the additive Gaussian noise. However, when estimating the channel, some prior information about the clipping signal can be incorporated to cancel the effects of clipping. More advanced approaches may propose iterative cancellation of clipping effects on channel estimation. This is a more practical problem which is worthy of consideration.

2. We have already seen how to select the most reliable subcarriers that are used to sense clipping information. A more intelligent account to look for the reliable carriers that not only utilize the clipping locations but also the phase at these locations. Hence, the reliability function will be more accurate.

3. This thesis consider clipping mitigation at the base station for a single user. It would interesting to consider PAPR reduction and clipping mitigation in
OFDMA systems, then the clipping will spill in the frequency domain to all users bands. Therefore, the base station needs to jointly recover the clipping distortion for all users.
REFERENCES


A Discrete representation of OFDM

The incoming information is mapped into \( N \) split symbols. The data is to be transmitted by block of size \( N \), 
\[
X_k = [X_k(0), X_k(1), \ldots, X_k(i), \ldots, X_k(N-1)]^T,
\]
where the index \( k \) is the block number (to be dropped later for compactness), and \( i \) is the subcarrier index. Then, the time domain \( J \)-times oversampled\(^1\) baseband signal is constructed by performing the Inverse Fast Fourier Transform (IFFT) to \( X \), that is
\[
x(n) = \frac{1}{\sqrt{JN}} \sum_{k=0}^{N-1} X(k)e^{j2\pi \frac{nk}{JN}}
\]
This can be rewritten in a matrix form as \( x = F^H X \), where \( F \) is the FFT matrix which is square unitary \((FF^H = F^H F = I)\) and its Hermition is the IFFT matrix \([48]\).

\[
F = \frac{1}{\sqrt{N}} \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & e^{-j2\pi/N} & e^{-j2\pi(2)/N} & \cdots & e^{-j2\pi(N-1)/N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & e^{-j2\pi(N-2)/N} & e^{-j4\pi(N-2)/N} & \cdots & e^{-j2\pi(N-1)(N-2)/N} \\
1 & e^{-j2\pi(N-1)/N} & e^{-j4\pi(N-1)/N} & \cdots & e^{-j2\pi(N-1)(N-1)/N}
\end{bmatrix}
\]

In order to circularize the multipath convolution (channel effects) and therefore avoid ISI, a cyclic prefix is inserted in the front of the signal \( x \). The cyclic prefix header at the transmitter is generated by a \((N + G) \times N\) binary matrix \( C_T \) where \( G \)

\(^1\)For simplicity we assume \( J=1 \), in chapter 4 we discuss the case for oversampling.
is the length of the cyclic prefix. The structure of $C_T$ is

$$C_T = \begin{bmatrix} O_{G \times (N-G)} & I_G \\ I_N & 0 \end{bmatrix}$$

(A.2)

where $O_{G \times (N-G)}$ is zero-matrix and $I_G$ is the identity matrix with size $G \times G$, similarly for $I_N$.

Multiplying $x$ by $C_T$ is equivalent to insert a copy of the last $G$ samples of $x$ to be in the front.

$$x' = C_T \cdot x = \begin{bmatrix} x(N-G), \ldots, x(N-1), x(0), \ldots, x(N-1) \end{bmatrix}^T$$

(A.3)

For example, for $N = 5, G = 2$

$$\begin{bmatrix} 4 \\ 5 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} O_{3 \times 2} & I_2 \\ I_5 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

(A.4)
After the digital-to-analog convertor (DAC), the signal is sent through the channel. The channel can be represented by a discrete model as a FIR filter of order $D$ (assume $D = G$) with channel impulse response $h = [h(0), h(1), \cdots, h(D - 1)]^T$. Hence, the received signal is

$$y(n) = \sum_{i=0}^{D-1} h(i)x'(n - i) + z(n), \quad n = 0, 1, \cdots, N - 1$$

where $z \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_N)$ is independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN). The convolution operation in above equation can be represented in matrix form with channel matrix $H \in \mathbb{C}^{(N+D) \times (N+D)}$ as follows:

$$H = \begin{bmatrix}
  h(0) & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
  h(1) & h(0) & 0 & 0 & \cdots & \vdots & \vdots & 0 \\
  \vdots & h(1) & h(0) & 0 & 0 & \vdots & \vdots & \vdots \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  h(D-1) & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  0 & h(D-1) & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  0 & \cdots & 0 & h(D-1) & \cdots & h(2) & h(1) & h(0) & \end{bmatrix} \in \mathbb{C}^{(N+D) \times (N+D)}$$
Therefore, the received time domain signal is written as:

\[ y = Hx' + z = HC_T x + z \]  \hspace{1cm} (A.5)

At the receiver side, the cyclic prefix removal can be obtained by multiplying the received signal by a \( N \times (N + G) \) matrix, i.e.

\[ G_R = [O_{N \times G} \ I_N] \]

Thus, we can write the signal after cyclic prefix removal as

\[ y' = C_R HC_T x + C_R z \]  \hspace{1cm} (A.6)

The term \( C_R HC_T \) changes the channel matrix to be circulant, this is equivalent to transform the linear convolution into a circular convolution. The following example demonstrates this transformation:

\[
C_R HC_T = \begin{bmatrix}
O_{N \times G} & I_N
\end{bmatrix}
\begin{bmatrix}
O_{G \times (N-G)} & I_G \\
& I_N
\end{bmatrix}
\begin{bmatrix}
h(0) & 0 & \cdots & h(G-1) & \cdots & h(2) & h(1) \\
h(1) & h(0) & 0 & 0 & \cdots & \vdots & \vdots & h(2) \\
\vdots & h(1) & h(0) & 0 & 0 & \vdots & \vdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\
h(G-1) & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\
0 & h(G-1) & \ddots & \ddots & \ddots & \vdots & \vdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & h(1) & h(0) \\
0 & \cdots & 0 & h(G-1) & \cdots & h(2) & h(1) & h(0)
\end{bmatrix}
\]

\[ \begin{bmatrix}
h(0) & 0 & \cdots & h(G-1) & \cdots & h(2) & h(1) \\
h(1) & h(0) & 0 & 0 & \cdots & \vdots & \vdots & h(2) \\
\vdots & h(1) & h(0) & 0 & 0 & \vdots & \vdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\
h(G-1) & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\
0 & h(G-1) & \ddots & \ddots & \ddots & \vdots & \vdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & h(1) & h(0) \\
0 & \cdots & 0 & h(G-1) & \cdots & h(2) & h(1) & h(0)
\end{bmatrix}
\]

\[ N \times N \]
The frequency domain of (A.6) is obtained by performing FFT operation as follows (with substituting $x = F^H X$):

$$Y = FC_R HC_T F^H X + FC_R z$$

(A.7)

The circulant channel matrix can be decomposed to be diagonal matrix using the DFT and IDFT matrices. Specifically, the circulant channel matrix can be written as $C_R HC_T = F^H \Lambda F$. Thus, the equivalent channel matrix becomes $\Lambda = FC_R HC_T F^H$ which appears in (A.7). Let, $H' = C_R HC_T$, we have

$$\Lambda = F H' F^H = \begin{bmatrix}
H'(0) & 0 & \ldots & \ldots & 0 \\
0 & H'(1) & 0 & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \ldots & 0 & H'(N-2) & 0 \\
0 & \ldots & \ldots & 0 & H'(N-1)
\end{bmatrix}_{N \times N}$$

Note that, it is a diagonal matrix containing eigenvalues of $H'$ and the eigenvalues are obtained by taking DFT of first column of $H'$.

Figure A.2: The transmission process is parallel in frequency domain.

With the above definition and by letting $Z' = FC_R z$, the received frequency
domain signal (A.7) can be rewritten as

$$\mathbf{y} = \mathbf{\Lambda} \mathbf{x} + \mathbf{z}'$$

(A.8)

where $\mathbf{z}'$ still holds the same statistics of $\mathbf{z}$.

It can be seen from (A.8), the relationship between the two ends of the OFDM system in frequency domain is a parallel transmission process. Therefore, the complexity of channel equalizer is reduced to 1-tap per subcarrier in the frequency domain. This explains why OFDM is an efficient communication scheme against multipath propagations which occurs in typical urban environments.
B Papers Submitted and Under Preparation

- Al-Rabah, Abdullatif R. , Mudassir, M., and Al-Naffouri, Tareq Y., Award of the best poster at graduate student posters competition (MS students category) in WEP’13, at KAUST, Thuwal.