Nonlinear Dynamics of Memristor Based 2\textsuperscript{nd} and 3\textsuperscript{rd} Order Oscillators

THESIS BY

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Exceptional behaviours of Memristor are illustrated in Memristor based second order (Wien oscillator) and third order (phase shift oscillator) oscillator systems in this Thesis. Conventional concepts about sustained oscillation have been argued by demonstrating the possibility of sustained oscillation with oscillating resistance and dynamic poles. Mathematical models are also proposed for analysis and simulations have been presented to support the surprising characteristics of the Memristor based oscillator systems.

This thesis also describes a comparative study among the Wien family oscillators with one Memristor. In case of phase shift oscillator, one Memristor and three Memristors systems are illustrated and compared to generalize the nonlinear dynamics observed for both 2nd order and 3rd order system. Detail explanations are provided with analytical models to simplify the unconventional properties of Memristor based oscillatory systems.
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<tr>
<td>CMOS</td>
<td>Complementary Metal-Oxide-Semiconductor</td>
</tr>
<tr>
<td>CVD</td>
<td>Chemical Vapor Deposition</td>
</tr>
<tr>
<td>PVD</td>
<td>Physical Vapor Deposition</td>
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<tr>
<td>LPCVD</td>
<td>Low Pressure Chemical Vapor Deposition</td>
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<tr>
<td>PECVD</td>
<td>Plasma Enhanced Chemical Vapor Deposition</td>
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<tr>
<td>BIBO</td>
<td>Bounded Input Bounded Output</td>
</tr>
<tr>
<td>RAM</td>
<td>Random Access Memory</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field-Programmable Gate Array</td>
</tr>
<tr>
<td>OPAMP</td>
<td>Operational Amplifier</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>H</td>
<td>Magnetic Field</td>
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I INTRODUCTION

Resistor, capacitor, and inductor-the three basic elements of circuit have recently encountered the missing fourth element- ‘Memristor’, that relates between the flux-linkage ($\phi$) and the charge ($q$). HP lab first reported an experimental version of Memristor in 2008 [1]. That two terminal physical model achieves a resistance depending on the history of current. Memristor is modeled as a thin semiconductor film (TiO$_2$) sandwiched between two metal contacts. The semiconductor film has a region with low resistance $R_{on}$ (high concentration of dopants), and the remainder has higher resistance $R_{off}$ (low concentration of dopants). The total resistance of the Memristor, $R_M$, is the weighted summation of the resistances of the two regions depending on their width.

Before this establishment of the passive Memristor Leon O. Chua first envisioned its existence [2]. He envisaged the passivity by designing Memristor with active elements. During the long thirty years the theoretical Memristor was investigated independently or along with the other three passive components (R, L, and C). After the successful physical model implemented by HP, Memristor has enraptured the research world for its unprecedented behavior. Researchers have demonstrated its potential for being used as a memory switch which offers the replacement of conventional transistor. The nonvolatile memory applications of Memristor lead to more dense architecture where this nanoscale element is being implemented in crossbar arrays to perform either as a latch or logic module. Apart from the digital domain Memristor is shown to have pragmatic solution for chaotic system. But the resistive behavior of Memristor in usual
Oscillators are the widely used signal generators that can originate sine waves of various frequencies. There various oscillators considering application, stability, and frequency of oscillation: Wien oscillator, phase shift oscillator, crystal oscillator, Colpitts oscillator, etc. Of the various types, the simplest are Wien oscillator (2\textsuperscript{nd} order) and phase shift oscillator (3\textsuperscript{rd} order). They can generate a large range of frequencies. Both the oscillators are comprised of resistors and capacitors. Oscillators basically operate without external input and using feedback to drive them into an instability which begets the system to oscillate. The network formed with resistors and capacitors defines the condition of oscillation as well as the frequency of oscillation. Till now it is well established that: 1)
All the poles of the oscillatory system will be fixed in $s$ plane to give sustained oscillation, 2) all the resistors of the sustained oscillatory system must present constant value, 3) sustained oscillation will have a single frequency component in the Fourier spectra, and 4) the final amplitude of the sustained oscillation can be controlled after the start of oscillation. The increasing demands of Memristor thus demand more attention to investigate the conventional oscillator systems when Memristor is used.

1.1 Motivation

As a fourth basic circuit element Memristor has a lot to offer in physics and engineering. Researchers have demonstrated its capability experimentally though majority of its function still limited to theoretical approach. Even the theory of Memristor and memristive systems needs concentration as in future Memristor will play a vital role to re-engineer the existing appliances or renovate new implementations. State of the art researches include mostly: hybrid CMOS-Memristor design for memory, Memristive synapse, and chaotic systems. But few works incorporate the investigation of Memristor’s behavior in traditional analog circuitry. Thus this thesis targets conventional 2$^{nd}$ order and 3$^{rd}$ order oscillatory systems to reveal the unprecedented behaviors of Memristor. As oscillators are signal generating devices which act as the backbone for counters and timers, so Memristor based oscillators may be useful for future computers designed with Memristor or hybrid CMOS-Memristor. That is why this thesis aims to provide the details of the characteristics of Memristor based oscillators and the mathematical platform to design and analyze such systems.
1.2 Objectives

The main objectives of this thesis are given below:

1. To simulate the effects of Memristor when it replaces any resistor of the oscillatory systems.
2. To point out the unconventional properties with an argument to the traditional concepts of sustained oscillation.
3. To elucidate the properties of the circuits when one or more Memristors are used.
4. To mathematically explain the behaviors of the several newly proposed memristive system for oscillation.
5. To propose analytical models and approach in order to design such systems.
6. To compare several Memristor based oscillatory systems.

1.3 Contributions

The prime contributions of this thesis are:

- **Analyzing of Memristor based 2\textsuperscript{nd} order oscillator**: This thesis thoroughly investigates the effects of Memristor replacing resistor of Wien oscillator. It also reports some unconventional properties (oscillating resistance, dynamic poles, and operational frequency range) of the system which intuitively question the conventional concepts of oscillation.

- **Analyzing of Memristor based 3\textsuperscript{rd} order oscillator**: This thesis also reports the unorthodox behaviors (oscillating resistance, dynamic poles, and operational frequency range) of phase shift oscillator when Memristors are used.
Developing generalized analytical models: Generalized model for oscillating resistance for both 2\textsuperscript{nd} and 3\textsuperscript{rd} order systems are proposed in this thesis. Also analytical models for frequency of oscillation and gain are suggested for each system.

Publications: One conference paper is published in IEEE conference of ICM’2011 [3] which covers the 2\textsuperscript{nd} order system. Another paper on the 2\textsuperscript{nd} order system is submitted in Microelectronics Journal [4]. Three more journals are in preparation for submission to report the results of 3\textsuperscript{rd} order system and gain controlling Memristor feedback scheme.

1.4 Organization of Thesis

This thesis includes five chapters and the outline is given below:

- **Chapter 1**: outlines the introductory description of this thesis with contributions and an overview of what is being conducted and the results.
- **Chapter 2**: gives the summarized literature reviews of Memristor, modeling of Memristor, SPICE simulation of Memristors, and five major applications.
- **Chapter 3**: describes the design of Memristor based Wien family oscillators with some novel results supported by analytical models and simulation.
- **Chapter 4**: illustrates Memristor based phase shift oscillator with unprecedented results again supported by analytical models and simulation.
- **Chapter 5**: concludes the thesis with suggesting further researches in future.
II MEMRISTOR-THE 4th BASIC CIRCUIT ELEMENT

2.1 Theoretical Background

L. O. Chua was the first to give the hypothesis of Memristor in 1971 [2], 40 years ago he claimed that the missing element which relates between electrical charge ($q$) and magnetic flux ($\phi$) should exist along with resistor, capacitor, and inductor. He named that element as Memristor. In that paper he elucidated the scientific and logical foundation of the existence of Memristor. He not only foresaw but also predicted the behavior of Memristor with active implementation. The basic electrical quantities are: voltage ($v$), current ($i$), charge ($q$), and flux ($\phi$). Till then three passive electrical devices were existed which relates between $i$-$v$ by resistor ($R$), $q$-$v$ by capacitor ($C$), or $\phi$-$i$ by inductor ($L$).

![Diagram showing electrical quantities](image)

**Fig. 2.1:** Relation between electrical quantities with passive devices
Logically there should be another passive device which will relate $\varphi$-$q$ and eventually Chua proposed it as Memristor (all the relations are shown in Fig. 2.1). He mimicked the passive model of Memristor by active elements but did present the passivity property to emphasize its physical realization. Using Maxwell’s equations he also presented the electromagnetic field interpretation of Memristor.

### 2.1.1 Basic realization of Memristor

Chua proposed a hypothetical $\varphi$-$q$ curve realized by two types of $M$-$R$ mutator [5] which transformed the $i$-$v$ curve of nonlinear resistor into $\varphi$-$q$ curve of Memristor. These two types of $M$-$R$ mutator were based on either current controlled or voltage controlled sources. Using that active Memristor he then showed unusual waveforms of $v$ and $i$ proposing the circuit theory of the Memristor. For a charge controlled Memristor, the voltage across a Memristor is given by

$$v(t) = M(q(t))i(t) \quad (2.1)$$

As the unit of $M(q)$ is $\Omega$, so he called it incremental *memristance* which is defined by

$$M(q) = \frac{d\varphi(q)}{dq} \quad (2.2)$$

In case of a flux-controlled Memristor, the current is given as

$$i(t) = W(\varphi(t))v(t) \quad (2.3)$$

Where $W(\varphi)$ is named as *memductance* and it is defined as
From these equations it is evident that Memristor acts like resistor whose resistance completely depends on the previous values of the current through it. Chua postulated the passivity criterion for the Memristor characterized by \( q-\phi \) curve (monotonically increasing) which can possess passive form without any internal power supply. By solving Maxwell’s equations it was shown then that both the first order fields of curl \( \mathbf{E} \) and curl \( \mathbf{H} \) are not negligible which eventually characterize Memristor. He also demonstrated some novel applications of this theoretical Memristor in modeling amorphous ‘ovonic’ threshold switch and electrolytic cell. Though Memristor had no passive model, the circuit theoretic and electromagnetic analyses articulated by Chua in [2] was the plausible existence of what completes the missing link in basic circuit element.

2.1.2 Memristive Systems

In another article [6], Chua generalized Memristor to several nonlinear dynamic systems which he called ‘memristive systems’. These systems retained memory and endowed with peculiar hysteresis between input and output with zero phase shift. The most salient property of these systems is the zero crossing Lissajous figure which varies with the excitation frequency. He defined the memristive system by

\[
\frac{dx}{dt} = f(x, u, t) \\
y = g(x, u, t)u
\] (2.5)
Where $u$ is the input and $y$ is the output of the system. Function $f$ and $g$ are continuous vector and scalar function correspondingly. For a current-controlled memristive system (one port) can be expressed as

\[
\frac{dx}{dt} = f(x, i, t) \tag{2.6}
\]
\[v = R(x, i, t)i\]

For a voltage-controlled memristive system is presented as

\[
\frac{dx}{dt} = f(x, v, t) \tag{2.7}
\]
\[i = G(x, v, t)v\]

If $f$ and $g$ in (2.5) are not a function of time i.e. time invariant then relating (2.6) and (2.7), then the system can be presented as

\[
\frac{dx}{dt} = f(x, i), \text{or } \frac{dx}{dt} = f(x, v) \tag{2.8}
\]
\[v = R(x)i, \text{or } i = G(x)v\]

It is to be mentioned that for (2.8) to be valid $R$ cannot be a function of $i$ and $G$ cannot be a function of $v$. Using these models, Chua intuitively identified the memristive system in thermistor, ionic system, and discharge tubes. He then derived more generic properties which will identify memristive system from other nonlinear dynamical systems. Those properties are mentioned here:

a. The memristive system should have a dc characteristic curve passing through origin.
b. For any periodic excitation (with a zero mean) the $v-i$ Lissajous figure should pass through origin.

c. As the excitation frequency increases toward infinity the one port Memristor behaves as a linear resistor.

d. The small signal impedance of a memristive system can be resistive, capacitive, or inductive depending on the operating bias point.

An important aspect is pointed out in the frequency dependant Lissajous figure where it comprehends pinched hysteresis loop which adheres the Memristor a memory occupied device at low frequency but at higher frequency Memristor resembles conventional linear resistor. Although no passive model of Memristor was available at that time, the intuitive design of active implementation and logical theories of Memristor and memristive systems presented by Chua in [2, 6] were the only sources of insight for what people have waited for the missing 4th circuit element.

Before describing the memristive system L. Chua modeled a p-n junction diode by four elements including Memristor [7]. This model accurately simulated the diode’s dynamic behaviors under reverse, forward, and sinusoidal operating modes. Memristor was accounted for charge storage and conductivity modulation effects to successfully model the devices exhibiting delay and charge storage effects. That simple Memristor-diode circuit model attracted the computer simulation as it requires low memory. Previously he also demonstrated [8] the physical approach and black box approach for device modeling claiming that to account for second order effects both approach should be employed for simpler modeling of memristive device.
2.2 HP MEMRISTOR-The Passive Model

After long 36 years, the theoretical Memristor came into live when R. S. Williams of HP lab reported the first ever solid state version of Memristor in their famous Nature article [1]. This breakthrough did not come with a plan for passive model of Memristor rather somewhat prosperously. It all started when R. Williams’ team were looking into the aspect of defect tolerant computer architecture [9] which was urging for more dense CMOS integration opening up the opportunities for nanotechnology. This eventually led the team to crossbar array architecture (as in Fig. 2.2) which could possibly solve the problem of integration by switching huge amount of information. The most critical part of the crossbar array was the switch which was needed to be in nm range (2~3nm) (Fig. 2.3). The switch was expected to be resistive in nature with the capability of changing resistance correspond to the state of switching. Eventually they found out the switch is the Memristor but did not know how to engineer the passive model. They kept

Fig. 2.2: 3D view of crossbar array
Fig. 2.3: Device and circuit representation of TiO$_2$ based Memristor

(a) Pt-TiO$_{2-x}$-TiO$_2$-Pt switch under no bias
(b) Pt-TiO$_{2-x}$-TiO$_2$-Pt switch under +ve bias
(c) Pt-TiO$_{2-x}$-TiO$_2$-Pt switch under -ve bias
trying for years with platinum and switching molecules [10, 11]. Finally they came up with metal-insulator-metal (MIM) structure with Platinum (Pt) as Metal and TiO$_2$ as insulating layer. TiO$_2$ layer is comprised of oxygen enriched TiO2 layer and oxygen deficient TiO$_2$ layer (TiO$_{2-x}$ layer in Fig. 2.3(a)) resulting in metal-insulator transition. The oxygen deficient layer is almost metallic in nature and will have +2 charge dopants. These two TiO$_2$ layers can be represented as variable resistors with a movable slide connecting both the resistors. As a result when positive voltage is applied, the oxygen vacancies will move down (Fig. 2.3(b)) and analogously the slide will move below to change the resistance ratio of $\frac{R_{TiO_{2-x}}}{R_{TiO_2}}$. By applying negative voltage the oxygen vacancies will be pulled upwards and the slide will move up (Fig. 2.3(c)), again changing the ratio $\frac{R_{TiO_{2-x}}}{R_{TiO_2}}$. This observation led the team for a successful modeling of Memristor and its passive implementation. Memristor is modeled as a thin semiconductor film (TiO$_2$) interceded between two metal plates (Fig. 2.4). The semiconductor film has a region with low resistance $R_{ON}$ (high dopant concentration), and the remainder has higher resistance $R_{OFF}$ (low dopant concentration). The total resistance of the Memristor is the addition of the weighted resistances of the two regions based on the width of the regions. Depending on the polarity of applied voltage, drifting of charged dopants will cause the boundary between the two regions to move and so the width will change with time and essentially change the ratio $R_{ON}/R_{OFF}$. For positive bias, the width will increase and for negative bias the width will decrease.
For an applied bias $v(t)$, if the current through the Memristor is $i(t)$ then the memristance $M(q)$ is expressed as [1]

$$M(q) = \frac{v(t)}{i(t)} = \frac{R_{ON}w(t)}{D} + R_{OFF}(1 - \frac{w(t)}{D}) \quad (2.9)$$

As per Chua’s model for memristive system in (2.8), this HP model can be described by considering width ($w$) as the state

$$\frac{dw}{dt} = f(w, i) \quad (2.10)$$

$$v = R(w)i$$

which yields

$$\frac{dw(t)}{dt} = \frac{\mu_v R_{ON} i(t)}{D} \quad (2.11)$$
Here $\mu_v$ is the average ion mobility. As $q(t) = \int i(t)dt$, $w(t)$ can be obtained from (2.11) as

$$w(t) = \frac{\mu_v R_{ON} q(t)}{D} \quad (2.12)$$

Thus $M(q)$ can be represented using (2.12) in (2.9)

$$M(q) = R_{OFF}(1 - \frac{\mu_v R_{ON} q(t)}{D^2}) \quad (2.13)$$

Fig. 2.5: I-V characteristics of (a) typical Memristor, (b) coupled variable resistor model, and (c) experimental device (adapted from [1]).
Williams’ team named this model as ‘coupled variable resistor model’ which is valid \(0 \leq w \leq D\). Here to be mentioned that the model of memristance in (2.13) is derived from (2.11) considering linear drift of ion. But they figured out that, in nanoscale due to large electric field the ionic transport becomes nonlinear and so a window function \(f(x)\) is multiplied in the right hand side of (2.11) to incorporate the nonlinear drift of ions. \(f(x)\) is defined as

\[
f(x) = \frac{w(1-w)}{D^2}
\]  

(2.14)

The team then successfully showed the pinched hysteresis loop of Memristor (Fig. 2.5(a)) by both the theoretical model (Fig. 2.5 (b)) and experimental device (Fig. 2.5 (c)). The device was shown to remain ON for the whole period of a positive bias, but to turn it OFF, negative bias should be applied.

### 2.3 Modeling of Memristor

#### 2.3.1 Mathematical model

Chua in [12, 13] described thoroughly how to model Memristor and memristive devices. The basic formulation in (2.2) and (2.4) are applicable for any system but defining \(\varphi(q)\) or \(q(\varphi)\) depends on the system and the expected behavior of Memristor. As in [12] Chua characterized Memristor by ‘monotone-increasing’ and ‘piecewise-linear’ to model the nonlinearity of Memristor in one terminal circuit as follows

\[
\varphi(q) = bq + 0.5(a - b) + (|q + 1| - |q - 1|)
\]  

(2.15)
Or,

\[ q(\varphi) = d\varphi + 0.5(c - d) + (|\varphi + 1| - |\varphi - 1|) \]  \hspace{1cm} (2.16)

Here \( a, b, c, \) and \( d > 0 \). Now (2.2) and (2.4) are modified as

\[ M(q) = \frac{d\varphi(q)}{dq} = \begin{cases} a, & |q| < 1 \\ b, & |q| > 1 \end{cases} \]  \hspace{1cm} (2.17)

or

\[ W(\varphi) = \frac{dq(\varphi)}{d\varphi} = \begin{cases} c, & |\varphi| < 1 \\ d, & |\varphi| > 1 \end{cases} \]  \hspace{1cm} (2.18)

For a two terminal circuit the model of Memristor will need to be modified accordingly the component added with Memristor. As for example if a negative conductance (-\( G \)) is added in parallel with Memristor then (2.18) becomes

\[ \bar{W}(\varphi) = \frac{dq(\varphi)}{d\varphi} = \begin{cases} c - G, & |\varphi| < 1 \\ d - G, & |\varphi| > 1 \end{cases} \]  \hspace{1cm} (2.19)

The approach to model Memristor would be to constitute the KVL and KCL equations of the circuit defining the state variables and then transform \((i, v)\) to \((q, \varphi)\). The added state of Memristor can be obtained from (if single ended Memristor is used) either (2.17) or (2.18) correspond to charge or flux controlled Memristor. The model in [1] assumed Memristor to function linearly or at least piecewise linearly. Recently in another article [13] Chua has modeled Memristor considering memristive device and also the nonlinearity of ion drifting. He proposed the model as

\[ V_M = \beta(x^2 - 1)i_M \]  \hspace{1cm} (2.20)
\[ \frac{dx}{dt} = i_M - \alpha x - i_M x \]  

(2.21)

Here \( V_M \) and \( i_M \) is the voltage across and current through the Memristor and \( x \) is the internal state of Memristor. \( \alpha \) and \( \beta \) same coefficients as described in [13].

HP has modeled Memristor as described in (2.11)-(2.14) without concerning the boundary effects as the speed of the boundary between doped and undoped regions gets suppressed at either edge. Joglekar and Wolf accounted this suppression by proposing a new window function \( F(x) \) where \( x \) is the ratio of \( w \) and \( D \) and so they modified (2.11) as [14]

\[ \frac{dw(t)}{dt} = \eta \frac{\mu_r R_{ON} i(t)}{D} F\left(\frac{w}{D}\right) \]  

(2.22)

Here \( \eta=\pm1 \) which correspond to the polarity of Memristor. \( F(x) \) is symmetric about \( x=1/2 \) and \( F(0)=F(1)=0 \) (to restrict ion drifting at the edge). In the interval \( 0 \leq x \leq 1/2 \), \( F(x) \) is monotonically increasing. They defined a parameter \( p \) to constitute a family of window functions

\[ F_p(x) = 1 - (2x - 1)^{2p} \]  

(2.23)

As \( p \) approaches to infinity (2.22) becomes linear, for \( p=1 \) (2.22) reduces to the HP model. The symmetric nature of the window function at \( x=1/2 \) is shown for different values of \( p \) in Fig. 2.7. From this figure it is observed that at \( p=1 \), the window function will give nonlinear drift model with \( F_{p=1}(x) = 4x(1-x) \). But as \( p \) increases more than
20, the window function obtains constant value over a wide range of $x$ leading to linear drift model of Memristor. For the nonlinear drift mode ($p=1$), Joglekar suggested the state will have the following form (when $p=1$) [14]

$$W_{p=1}(q) = w_0 \frac{De^{4q(q_0)}}{D + w_0[e^{4q(q_0)} - 1]}$$ (2.24)

Here $w_0$ is the initial width of the doped region and $Q_0 = D^2/\mu VR_{ON}$ which yields $10^2$ C [1].

### 2.3.2 SPICE model

Several SPICE models are proposed to simulate Memristor and memristive systems [15-18]. A. Rak in [17] suggested a macro model of SPICE which accounts the boundary conditions of Memristor. The main achievement of this model is that it is fast and can
simulate 100 Memristors by an average computer configuration. The problem lies in the wrong simulation because of the initial values of the parameters if be chosen by the model itself. D. Batas also proposed a SPICE model [15] for both charge-controlled and flux-controlled Memristor. This model has many parameters to simulate different physical Memristor devices. But this model is actually based on linear drift model which simulate HP Memristor to some extent but the non linearity issues may result in skeptical simulations. Though Benderli accounted the nonlinearity of the Memristor in his SPICE model [18] but is limited to TiO$_2$ memristor only. Z. Biolek on the other hand proposed a useful SPICE model [16] which uses the window function in (2.23). The model is implemented as a SPICE sub circuit with the following parameters: the initial $R_{\text{init}}$ resistance, the $R_{\text{off}}$ and $R_{\text{on}}$ resistances, width of the thin film $D$, the migration coefficient $\mu_v$ and the exponent $p$ of the window function. This SPICE model consists of many parameters which give freedom to simulate many aspects of Memristor and memristive system. Because of this model incorporating non linear drift model so the simulation results are expected to be more realistic. To account for the boundary conditions the model has an alternative window function which considers the boundary speeds of the approaching and receding from the thin film edge. The window function is given as

$$f(x) = 1 - (x - \text{stp}(-i))^{2p}$$ (2.25)

Here $i$ is the Memristor current and

$$\text{stp}(i) = \begin{cases} 1 & \text{pro } i \geq 0 \\ 0 & \text{pro } i \leq 0 \end{cases}$$ (2.26)
However in this thesis, the SPICE model of Biolek has been used with (2.23) as the window function. The parameters are taken as (unless otherwise stated): \( R_{\text{off}} = 16 \Omega \) and \( R_{\text{on}} = 100 \Omega \), \( D = 10^{-14} \) m, and \( p = 10 \) of the window function which match the HP-Memristor [1]. Using the spice model and window function as in (2.25), the transient response of Memristor’s voltage and current is shown in Fig. 2.7. Here we can see that Memristor does not behave as resistor (zero phase-shift between voltage and current), inductor/capacitor (90° phase-shift between voltage and current). In this case Memristor was connected to a sinusoidal voltage source of 1.5V (peak) and 1Hz frequency. The parameters were: \( R_{\text{off}} = 38 \Omega \) and \( R_{\text{on}} = 100 \Omega \), \( R_{\text{init}} = 2 \) KΩ, \( D = 10^{-14} \) m, and \( p = 10 \). The I-V curve of Memristor for similar condition is plotted in Fig. 2.8 where the hysteresis loop can be seen. That loop passes through origin which is one of the characteristics of Memristor as mentioned in section 2.1.2. In Fig. 2.9, the characteristics curves of Memristor are shown for different frequencies (1Hz, 10Hz, and 60Hz). With higher

![Transient response of Memristor for sinusoidal signal of 10Hz frequency and 1.5V (peak) when \( R_{\text{off}} = 38 \Omega \), \( R_{\text{on}} = 100 \Omega \), and \( R_{\text{init}} = 2 \) KΩ with \( p = 10 \).]
Fig. 2.8: $I-V$ curve of Memristor

frequency the width of the hysteresis loop decreases and eventually Memristor behaves as linear resistor for infinite frequency. For this SPICE model Memristor acts as linear resistor above 100Hz with linear relation between $I$ and $V$.

Fig. 2.9: Memristor’s characteristics curve at different frequencies
2.4 Fabrication Techniques

Though HP has not revealed the exact method of fabricating Memristor and integrating with CMOS structure, but several articles and patents [19, 20] do give a strong hint of the fabrication technique. Generally HP has made Memristors by laying down metal nanowires, coated with a layer of TiO$_2$, in parallel onto a substrate and then a second layer of nanowires is placed perpendicular to the first layer. Memristor thus formed where the wires cross. Q. Zia and R. Williams fabricated Memristor cross-point arrays by a single step of nanoimprint lithography (NIL) which allows simultaneous patterning of electrodes and switching material [21]. Among various techniques, commonly used methods for fabricating Memristor nano-imprint lithography (NIL) [22, 23] and atomic-layer deposition (ALD) [24, 25]. Fig. 2.10 shows an array of 1×17 nano-crosspoint devices where 50nm thick TiO$_2$ layer fabricated by ALD is sandwiched by platinum (Pt).

Fig. 2.10: An AFM image of 1×17 nano-crosspoint devices (adapted from [25])
nanowires which are NIL fabricated. The most critical part is to form the transition metal oxide film which acts as Memristor. Any Metal Insulator Metal (MIM) structure having a bipolar switching characteristic can be a candidate for Memristor. It means that TiO$_2$ is not the only material to fabricate Memristor. Zirconium oxide (ZrO$_2$), Niobium oxide (Nb$_2$O$_5$), Nickel oxide (NiO), and Zinc oxide (ZnO) are the suitable alternatives for transition metal oxide film. Conventional fabrication methods for these oxides are: sputtering, evaporation, Sol-gel, CVD and PVD. PVD and sputtering methods give less oxygen vacancies whereas CVD is vulnerable to pollution and Sol-gel gives highly porous material. More importantly these methods do not have good control over the distribution of oxygen vacancies inside the film, especially when few nanometers of film thickness is desired. Tang and Xiao have pointed those issues and patented ion implantation method to fabricate the transition metal oxide which provides better control over distribution and adjustment of oxygen vacancies [20]. They formed a transition metal (or non-metal such as usually silicon or carbon can also be formed) on a substrate following by implantation of atoms of electron rich element. Then another element should be chosen to implant on that layer so that the new element makes a volatile compound with the electron rich element and thus can be expelled away with short period of annealing. As a result the desired vacancies can be formed inside the transition metal (or non-metal) and electron rich element layer. Apart from TiO$_2$, researchers have shown some attractive switching medium for memristor. Jim Tour demonstrated resistive switches and memories in SiO$_x$ by observing memristive behavior in silicon-oxide dielectric spacers placed on graphene chip [26]. Lu et al. used amorphous silicon as the switching medium to fabricate CMOS compatible Memristor [27]. Amorphous silicon
layer was deposited with PECVD and LPCVD. Stewart in [11] demonstrated an organic monolayer as the switching medium to realize a Memristor by depositing titanium followed by the organic Langmuir-Blodgett (LB) monolayer. Recently a method of fabricating Memristor on a flexible polymer is reported using titanium isopropoxide [28].

2.5 Applications

After HP invented the passive model of Memristor researchers from all over the world have started significant experiments to demonstrate the plausible applications of Memristor. The prime applications include neuromorphic system, chaotic system, nonvolatile memory, nonlinear analog circuit etc.

2.5.1 Neuromorphic system

Neuromorphic system is a mixed mode analog-digital system mimicking neural architecture to pattern neurons by real time computation and simulation and emulate nervous system. But to simulate neural networks in electronic regime neurons and synapses (connection between neurons) are needed to implement with very low power consumption. Electronic synapses are more difficult to engineer as they require being flexible as well as dynamic with memory capability. People had simulated brains of small animals (cat, rat, and spider) [29-32] but associating computer memory more than Terra bytes (e.g. Blue Gene/P of IBM). Memristor thus play a significant role to perform as a synapse with negligible power thrust [33, 34]. Y. V. Pershin in [34] has made
Memristor emulator which shows associative memory function when three electronic neurons connected by two Memristor-emulator synapses. S. H. Jo made Memristor with Ag and Si active layer forming highly conductive Ag-rich region and less conductive Ag-poor region (Fig. 2.11). This hybrid system is capable of ‘spike timing dependent plasticity (STDP)’ [35, 36] which is an important synaptic function. If the synapse update rate is 1Hz, then this system can continue synaptic operation for around 5 years. Though the basic idea of STDP in memristive devices was proposed before [37] and a practical implementation of circuit learning was demonstrated by patterning the learning of amoeba like cell into memristive system [32].
2.5.2 Chaotic system

Because of the random nature of chaotic system it is well applicable for encryption and random number generation. Memristor makes it possible for better control and simpler version of chaotic system. Chua modeled the Memristor as (2.15)-(2.19) to produce chaotic attractor with Memristor, negative conductance and capacitor [12]. Though it was merely simulation based but the simplicity and functionality initiated Memristor based chaotic system. Recently Muthuswamy and Chua demonstrated a simplest chaotic oscillator [13] where they used inductor-capacitor-memristor series circuit. Though Memristor was actively realized (in Fig. 2.12) and modeled as (2.20)-(2.21), the pinched hysteresis loop was shown by both experimentally and theoretical simulation. The nonlinearity of Memristor adds up to the third state variables along with inductor and capacitor and the simplest system is also BIBO stable. Around the same time, Muthuswamy has shown another simpler practical implementation [38] of Memristor in generating chaos. The difference between the two circuits is that in [38] Memristor is flux controlled whereas in [13] it is charge controlled but both realizations look similar. On the other hand B. C. Cheng has demonstrated Memristor oscillator which gives periodic orbits of chaos from a 2 scroll transient chaos [39]. In his another paper [40] similar transition is observed but with more complicated dynamical behavior of Memristor where the initial condition plays the major role to generate periodic chaos. The effect of initial condition on chaotic behavior is well studied in [41] where both the piecewise-linear model and cubic model of Memristor are shown capable of periodic orbits somewhat similar to Hopf bifurcation. The theoretical study of generating chaos has also appeared in [42] with a cubic model of flux controlled Memristor.
2.5.3 Non-volatile memory

Memristor essentially shows resistive switching behavior as it has metal-insulator-metal configuration. Before the physical evolution of Memristor, people have demonstrated highly density memory applications of resistive switching [43-48] where the insulating layer works as a storage medium. Though the memristive characteristics were not realized that time, the results hold Memristor a promising candidate as non volatile memory. Chen assumed [49] Pt/MgZnO/Pt device as Memristor and showed its resistive switching characteristics which are reversible and steady, leading towards non volatile memory. Recently as a non volatile memory the density of Memristor is reported 100Gbits/cm² in [50] which requires very low energy compared to existing flash
memory. HP lab experimentally demonstrated the non volatility of Memristor which is CMOS compatible, fast in response, and requires very low power [25, 51, 52]. The non volatile Memristor latch in [52] is shown to have high endurance of $10^4$ write cycles. In [25], 1×17 cross point arrays of Pt/TiO$_2$/Pt Memristor was fabricated to show the non volatility where the oxygen vacancies were engineered for controlling polarity and resistance of switching. The mathematical explanation of resistive switching of Pt/TiO$_2$/Pt Memristor revealed that with higher applied current the switching time reduces sharply to decrease the input energy exponentially [51]. A comprehensive mathematical illustration of Memristor as nonvolatile memory has been reported in [53] which will help to design memristive system for memory application. The non volatile memory capability of Memristor will turn on the computers without rebooting them and hopefully in future no physical RAM will be required separately.

### 2.5.4 Logic implementation

Q. Xia fabricated novel hybrid structure of Memristor crossbar and CMOS to demonstrate FPGA functionality [23]. This hybrid architecture was configured to perform NOT, OR, AND, NAND, NOR, and D flip-flop operation. Non volatility of Memristor adds up to the reconfigurable logic with high retention time. Borghetti and HP lab integrated two 21×21 Memristor crossbars with Si-FETs to realize Boolean operation (sum of product) with a demonstration of self programming capability of the logic circuit [54]. Borghetti in his another work [55] shows how Memristor can be used for both memory and logic implementation using 1×17 crossbar arrays of Memristor which is fast to operate consuming low power. Lehtonen also demonstrated the stateful logic theoretically to determine the exact number of Memristors to perform certain logic
operation [56] and also realized all Boolean functions with only two Memristors [logic3]. Though crossbar architecture seems complex, it is not necessary to realize FPGA functionality as one transistor-one Memristor/ two transistor-one Memristor/ two transistor-two Memristor structure [57] is capable of 2D or 3D FPGA implementation for more dense, lower power, and faster logic circuitry [58].

2.5.5 Analog operation

Not much experimental work in analog domain has been reported for Memristor. But the theoretical approaches report promising outcomes of Memristor in analog circuits. Pershin has reported [59] Memristor based analog circuits which can program gain, threshold, and frequency to perform as digital potentiometer. Memristor can tune its resistance with high resolution and thus gain and frequency can be programmed in analog circuit [60]. This ability of Memristor can be applied to receive ultra low power for ultra wide band signal [61]. Apart from those Memristor can be used to design adaptive filter with other passive elements (inductor, capacitor, and resistor) [62]. As the drift of oxygen vacancy is purely frequency dependant so this nonlinearity of Memristor can be applied to generate amplitude modulation [18] and possibly frequency modulation.
III MEMRISTOR BASED 2ND ORDER OSCILLATOR

The Wien bridge oscillator is one of the various types of electronic oscillators that can generate sine waves. It can generate a large range of frequencies. The main components of this oscillator are four resistors and two capacitors. It does not require external input to oscillate rather uses positive and negative feedback to drive the op amp into instability and thus oscillation starts. The derived equations for the frequency of oscillation and the condition of oscillation are demonstrated later. The schematic of the basic Wien oscillator is shown in Fig. 3.1.

![Schematic of Wien oscillator with Memristor](image-url)
Considering the circuit shown in Fig. 3.1 the characteristics equation of this system can be derived in the following form:

\[ s^2 + bs + d = 0 \]  \hspace{1cm} (3.1)

\[ b = \frac{1}{R_2 C_2} + \frac{1}{R_1 C_1} - \frac{R_3}{R_1 R_4 C_2}, \quad d = \frac{1}{R_1 R_2 C_1 C_2} \]  \hspace{1cm} (3.2)

The condition of sustained oscillation is given by

\[ \frac{C_2}{C_1} + \frac{R_1}{R_2} = \frac{R_3}{R_4} \]  \hspace{1cm} (3.3)

The frequency of oscillation is expressed as:

\[ f = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}} \]  \hspace{1cm} (3.4)

For sustain oscillation \( b \) must be equal to zero in (3.1). The location of the poles in this system can be found by solving the quadratic equation in (3.1). When \( b=0 \), there will be two conjugate poles lying on the imaginary axis. If \( b \) is a non-zero value then those two poles will be located in either the right half or left half of the s-plane. For oscillations to be sustained, the poles of the system must lie on the imaginary axis. Shifting of poles from the imaginary axis will result in a damped oscillation, which is not a sustained oscillation.
3.1 Wien Oscillator with Memristor

Referred to Fig. 3.1, $R_1$ is replaced with Memristor and both $C_1$ and $C_2$ are changed to same valued capacitance $C$ as shown in Fig. 3.2 (case 1). The resistance of the Memristor will be designated as $R_M$ and the voltage across the Memristor will be designated as $V_M$. From the simulation results $R_M$ is found to be oscillating. So $R_M$ will have an average value which can be denoted as $R_{avg}$. These changes will certainly modify equations (3.1)-(3.4). The frequency of oscillation of the new system, $f_M$ can be written as:

$$f_M = \frac{1}{2\pi C \sqrt{R_{avg} R_2}} \quad (3.5)$$

![Fig. 3.2: Schematics of conventional Wien oscillator (Type ‘A’) (R1 and R2 will be replaced by R_M to illustrate two cases).](image)
The condition of oscillation is modified as:

\[
1 + \frac{R_{avg}}{R_2} = \frac{R_3}{R_4} \equiv gain
\]  

(3.6)

To refresh the memory, Dmitri B. Strukov et al. first proposed a physical model to realize the Memristor as a two terminal device [1] considering TiO\textsubscript{2} film sandwiched between two Pt contacts. The film has a region with a higher dopants’ concentration to have low resistance (R\textsubscript{on}), and the remainder has higher resistance (R\textsubscript{off}) due to low dopant concentration. The total resistance of the Memristor, R\textsubscript{M}, is a sum of the weighted (depending on the width of the doped and undoped regions) resistances of the doped and undoped regions. The SPICE model proposed by Zdeněk Biolek et al. [16] is used to simulate the effect of Memristor in various circuits. The model is implemented as a SPICE sub circuit with the following parameters: the initial R\textsubscript{init} resistance, the R\textsubscript{off} and R\textsubscript{on} resistances, the width of the thin film D, the dopant mobility \(\mu_v\), and the exponent p of the window function. To counterpart the effect of the nonlinear dopant drift they have proposed a window function with p as the parameter to set the nonlinearity. The differences between models with linear and nonlinear drifts decrease if p increases. Recent mathematical modeling of Memristor in case of periodic signals is proposed in [63, 64].

For the simulations, R\textsubscript{2}=5k, C\textsubscript{1}=C\textsubscript{2}=C=3.2\mu F, R\textsubscript{4}=100k, R\textsubscript{1} (in Fig. 3.1) is replaced with Memristor (Fig. 3.2). In the SPICE model only R\textsubscript{init} is changed and the value of p is given 10. p=10 is good enough for linear approximation. The other parameters were kept as it is in the model file. R\textsubscript{init} was changed from 4.1k to 5.9k for simulation and also for mathematical modeling.
3.1.1 Oscillating resistance of Memristor

Sustained oscillations are found for each $R_{\text{init}}$ value of $R_M$. In Fig. 3.3 the simulated values of $V_M$ (voltage across the Memristor) is shown for every value of $R_{\text{init}}$. From this figure it is easily observed that $V_M$ has linear relation with $R_{\text{init}}$. So $V_M$ (the peak voltage of $V_M$) can be modeled with straight line approximation. $V_M$ can be expressed as a function of $R_{\text{init}}$:

$$ V_M = 0.3764 + 0.06R_{\text{init}} $$

(3.7)

The calculated values of $V_M$ using (3.7) give similar results as simulation and the error in this case is less than 0.22%. Thus $V_M$ can be approximated by two methods: the first method is by using (3.7) where $V_M$ can be calculated for every $R_{\text{init}}$. The second method would be to use conventional Wien oscillator (Fig. 3.1). As the average values of $V_M$ are almost constant whether a Memristor is used or not if similar values of $R_1$ and $R_{\text{init}}$ are

![Fig. 3.3: Voltage across the Memristor ($V_M$) from simulation](image)
used so $V_1$ can be taken from conventional Wien oscillator to be used as $V_M$ in the proposed formula described later. In Fig. 3.4 the simulation result is shown for $R_M$ and $V_{out}$. $V_{out}$ is the final output voltage. For this simulation, $R_{init}=5k$. Similar results are found for other values of $R_{init}$. The sustained oscillation is clearly observed in both the final output $V_{out}$ and $R_M$ (Fig. 3.4). $R_3$ has to be changed to accomplish this oscillation. The values of $R_3$ are found equal to the calculated values derived from (3.6). In this case $R_{init}$ is taken as $R_{avg}$. This approximation does not alter the result as $R_{avg}$ (from the simulation) is very close to $R_{init}$ with an error less than 0.02%. The most interesting observation is that the output oscillation is sustained as shown in Fig. 3.4. Though from the SPICE model $R_M$ can take values like $R_{on}=0.1k$, $R_{off}=16k$, and $R_{init}$ but from the simulation it is found that $R_M$ is oscillating in a different range across $R_{avg}$. So Memristor

![Fig. 3.4: Simulation result for $R_M$ and $V_{out}$ of the final output oscillation.](image)
resistance, \( R_M \) can be expressed as:

\[
R_M = R_{avg} + \Delta R_M \sin(\omega t \pm \varphi)
\]  (3.8)

Where, \( \Delta R_M \) is the sinusoidal amplitude. If only the magnitude peaks of \( R_M \) is considered then (3.8) can be written as:

\[
R_{\text{max, min}} = R_{avg} \pm \Delta R_M
\]  (3.9)

As \( R_M \) is oscillating, it will have a maximum (\( R_{\text{max}} \)) and a minimum (\( R_{\text{min}} \)) values. Using the concept described in [63, 64] and reusing (3.5) and (3.6) the maximum and minimum values of \( R_M \) can be modeled as:

\[
R_{max}^2 = R_{min}^2 \pm \left[ \frac{2V_M k (R_{off} - R_{on})}{\pi f_M} \right]
\]  (3.10)

Where, \( R_{on} \) and \( R_{off} \) are described previously. \( k \) is the same factor as described in [3] and it is given as \( k = (\mu_v R_{on})/D^2 \). Using the binomial expansion, equation (3.10) can be approximated as following:

\[
|R_{\text{max/min}} - R_{\text{init}}| \approx \frac{V_M k (R_{off} - R_{on})}{2 \pi R_{\text{init}} f_{avg}} = \Delta R_M
\]  (3.11)

For any value of \( R_{\text{init}} \), the peaks \( R_{\text{max}} \) and \( R_{\text{min}} \) can be easily calculated using (3.11). In Fig. 3.5 the calculated range of the Memristor resistance \( R_M \) using (3.11) is shown for different values of \( R_{\text{init}} \). The inset figure in Fig. 3.5 shows the percentage error between simulation and calculated values for every range of \( R_M \). From Fig. 3.5 it is seen that the maximum error is 2.2%
Fig. 3.5: Range of calculated values of $R_M$; Range of percentage errors between simulated and calculated $R_M$ (inset).

### 3.1.2 Poles as a function of time: “Oscillating poles”

It is well known that for any oscillation to be stable, the poles of that system must lie on imaginary axis. In other words, there should not be any real part in the value of the pole. If the poles of the system fall in right half of the $s$ plane, then the system will have a over damped oscillation, which will be increasing by time, and as a result, the oscillation will saturate. If the poles of the system fall in the left half of $s$ plane, the system will have a damped oscillation, which dies out eventually. Using (3.2) and (3.6) the expression for $b$ and $d$ can be written as:

$$b = \frac{1}{CR_M R_2} [R_M - R_{avg}] \approx \frac{\Delta R_M}{C R_M R_2}, \quad d = \frac{1}{C^2 R_M R_2}$$ (3.12)
The solution for $s$ can be written as:

$$s = \sigma \pm j\omega = \frac{b}{2} \pm j \left[ \frac{\sqrt{4R_M R_2 - (\Delta R_M)^2}}{2C R_M R_2} \right]$$

(3.13)

Conventional stable oscillation can only be achieved if $b=0$ as in case of the Wien oscillator presented in Fig. 3.1 where the oscillation is sustained by adjusting the values of $R_1$, $R_2$, $C_1$, $C_2$, $R_3$, and $R_4$. But in the case where a Memristor is used (refer to (3.12)), $b$ is found directly proportional to $\Delta R_M$. So a small value of $\Delta R_M$ can give a significant value of $b$. If $\Delta R_M=0$, poles will be on $\pm j\omega$ axis. These poles can be named as “fixed poles”. In Fig. 3.4 we can see the resistance of Memristor, $R_M$ is oscillating and so definitely $\Delta R_M\neq0$ and so $b$ cannot be zero. As a result the poles will not be fixed. But still sustained oscillation is found and it is clear in the simulation result of $V_{out}$ (in Fig. 3.4). These poles can be named as “oscillating poles”. In Fig. 3.6, the oscillating nature of poles is shown. In Fig. 3.6(a), $R_M$ is plotted using (3.8) and (3.11). This result is almost similar to the simulation result of $R_M$ shown in Fig. 3.4. As $R_M$ is oscillating so $R_M$ can be replaced using (3.8) in (3.12) to plot the real part of pole in (3.13). The plot is shown for $R_{init}=5k$ and $R_{avg}=5.045k$. In the same way the imaginary part of pole is plotted in Fig. 3.6(c). For both cases the oscillating behavior of pole is found. Fig. 3.6(d) shows the complex pole of the system when $R_M$ is changing from its minimum to maximum value. In this figure $+j\omega$ axis is shown only. The mirror image is found in $-j\omega$ axis. When $R_M$ is at its maximum value the poles will be at the lowest point of the straight line (in Fig. 3.6(d)). When $R_M$ is at its minimum value the poles will be at the highest point of the line (in Fig. 3.6(d)). The pole will oscillate between these two points along the line for the whole period of oscillation. The pole for which this line crosses $+j\omega$ axis corresponds
Fig. 3.6: (a) Oscillating resistance $R_M$, (b) Oscillating real part of pole, (c) Oscillating imaginary part of pole, and (d) Pole of the described system when $R_M$ is changing from its minimum to maximum value.

...Thus the poles of this system will continue to oscillate with time.

3.1.3 Frequency of oscillation

As $R_M$ is oscillating, the frequency of oscillation $f_M$ can be calculated by replacing $R_M$ with $R_{avg} \pm \Delta R_M$ in (3.13). It is found that the frequency of oscillation will have a maximum and a minimum value. As the poles of this system are oscillating so the oscillation cannot have a single frequency component in Fourier spectra like in the case of the conventional Wien oscillator. This logical interpretation is well supported by the simulation result shown in Fig. 3.7. Here the Fourier spectrum is shown for the same
transient response of $V_{out}$ shown in Fig. 3.4. The center frequency, $f_{M,avg} = 9.90$Hz corresponds to the purely imaginary pole and the corresponding value of $R_M$ is $R_{init}$. The calculated values of the maximum and minimum value of $f_M$ are found 9.98Hz and 9.78 Hz correspondingly and these values have very close match with $f_{M,max}$ and $f_{M,min}$ in Fig. 3.7. Other frequencies are due to the shifting of poles from the imaginary axis. This triangular shape of FFT validates the range of frequency of oscillation. It is to be mentioned that for any sustained oscillation the FFT of the sinusoidal oscillation is an impulse. Surprisingly when Memristor is used with Wien oscillator the FFT changes to triangular shape which means that the frequency of oscillation will not be single valued rather it will have a range. But the width of the triangle is very small to justify the claim properly. As a result, it will be more logical to say that the frequency of oscillation tries to have a range but settles to a certain frequency which is explained in the next section.

Fig. 3.7: FFT of output voltage
3.1.4 Frequency response of Memristor:

From (3.13), the imaginary part of $s$ gives the expression for the frequency of oscillation. $R_M$ can be replaced as $(R_{avg} \pm \Delta R_M)$, so that the frequency of oscillation is modeled as:

$$f_M = \sqrt{\frac{4R_2(R_{avg} \pm \Delta R_M) - (\Delta R_M)^2}{16\pi^2C^2(R_{avg} \pm \Delta R_M)R_2}}$$  (3.14)

Conventional stable oscillation can only be achieved if the poles are located on the imaginary axis. The first instance of oscillation in time-dependant poles is presented in [3], where type ‘A’ Wien oscillator is discussed. Since $R_M$ is oscillating with time, the poles of the characteristic equation of the Memristor-based oscillators become time-dependent and oscillating with time-dependent frequency as described in (3.14). From

Fig. 3.7: Frequency response of $R_M$ for type ‘A’ (for $R_{init}$=5kΩ). Operational frequency range of this system is shown in the zoomed view.
(3.11), it is observed that when \( f_M \) increases \( R_M \) approaches to \( R_{init} \). In Fig. 3.7 the solid line shows the behavior of \( R_M \) with \( f_M \) obtained from (3.11). Here we can see that for \( R_{init}=5\Omega\), both \( R_{max} \) (16K\( \Omega \)) and \( R_{min} \) (0.1K\( \Omega \)) tend to merge with \( R_{init} \) at higher frequency. If we now plot the frequency of oscillation from (3.5) by replacing \( (R_{avg} \pm \Delta R_M) \) with \( R_M \) on the same graph (dotted curve in Fig. 3.7), two intersection points will be found. These points will give the valid frequency range where Memristor can be expected to give sustained oscillation. When oscillation is sustained the system chooses the frequency where the dotted curves coincide with \( R_{init} \) line which is \( f_{M,avg} \) and it is due to the purely imaginary pole. For instance, with \( R_{init}=5\Omega\), \( f_{M,avg} \) is found in Fig. 3.7 as 9.94Hz. From simulation \( f_{M,avg}=9.90\text{Hz} \). This negligible difference is due to the fact that \( R_{avg} \) and \( R_{init} \) differ by 0.043\( \Omega \). If \( R_{init} \) is increased the operational frequency range \((f_2-f_1)\) will decrease as shown in Table 3.1 and thus the frequency of oscillation has smaller range to settle down providing the oscillatory system better chance for sustained oscillation.

**TABLE 3.1**

<table>
<thead>
<tr>
<th>( R_{init} ) (K( \Omega ))</th>
<th>( f_1 ) (Hz)</th>
<th>( f_2 ) (Hz)</th>
<th>( f_2-f_1 )</th>
<th>( f_{M,avg} ) (Hz)</th>
<th>( f_M ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.4</td>
<td>9.853</td>
<td>11.56</td>
<td>1.707</td>
<td>10.604</td>
<td>10.549</td>
</tr>
<tr>
<td>4.7</td>
<td>9.581</td>
<td>11.11</td>
<td>1.529</td>
<td>10.26</td>
<td>10.21</td>
</tr>
<tr>
<td>5.0</td>
<td>9.321</td>
<td>10.71</td>
<td>1.389</td>
<td>9.94</td>
<td>9.90</td>
</tr>
<tr>
<td>5.4</td>
<td>9.004</td>
<td>10.24</td>
<td>1.236</td>
<td>9.572</td>
<td>9.53</td>
</tr>
<tr>
<td>5.7</td>
<td>8.8</td>
<td>9.924</td>
<td>1.124</td>
<td>9.316</td>
<td>9.28</td>
</tr>
</tbody>
</table>
3.1.5 Output voltage control

When simulating with SPICE the initial voltages of $C_1$ and $C_2$ set the oscillating voltage. The initial value of $C_1$ is set to 0.1 and for $C_2$ it is changed to see the effect on $V_1$ (voltage across $R_1$ in Fig. 3.1 for type ‘A’). Then the initial value of $C_1$ is changed by keeping the initial value of $C_2$ at 0.1. Linear relation is found between the initial condition and $V_1$ for both cases which are shown in Fig. 3.8. Therefore, estimating the output oscillating voltage ($V_{out}$), the effect of initial voltage is accounted for in Fig. 3.8. When $R_1$ is replaced with $R_M$, $V_M$ is almost equal to $V_1$. Consequently $V_M$ can be approximated by $V_1$. For type ‘A’ Wien oscillator the relation between the output oscillation $V_{out}$ and $R_M$ can be derived as:

$$\frac{V_{out}}{V_M} = \left(\frac{1}{1 + (\omega R_2 C)^2}\right) \left[1 + \frac{R_2}{R_M} + (\omega R_2 C)^2 - j \left(\frac{1 + \omega R_2 C^2 + C^2 (\omega R_2)^2}{\omega R_M C}\right)\right]$$  \hspace{1cm} (3.15)

![Fig. 3.8: Voltage across $R_1$ vs initial condition (IC)](image-url)
Here, \( \omega = 2\pi f_M \) and \( f_M \) can be estimated using (3.14). This expression can then be used to establish the amplitude of oscillation. Table 3.2 summarizes the values of \( f_M \), \( V_M \), and \( V_{out} \) for both the calculated and simulation results are compared. In Fig. 3.9, the errors

<table>
<thead>
<tr>
<th>( R_{init} ) (k( \Omega ))</th>
<th>( f_M^a ) (Hz)</th>
<th>( f_M^b ) (Hz)</th>
<th>( V_M^a ) (Volt)</th>
<th>( V_M^b ) (Volt)</th>
<th>( V_{out}^a ) (Volt)</th>
<th>( V_{out}^b ) (Volt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textbf{4.1}</td>
<td>10.985</td>
<td>10.92</td>
<td>0.6224</td>
<td>0.621</td>
<td>1.5826</td>
<td>1.5761</td>
</tr>
<tr>
<td>\textbf{4.4}</td>
<td>10.604</td>
<td>10.549</td>
<td>0.6404</td>
<td>0.64</td>
<td>1.567</td>
<td>1.5633</td>
</tr>
<tr>
<td>\textbf{4.7}</td>
<td>10.26</td>
<td>10.21</td>
<td>0.6584</td>
<td>0.659</td>
<td>1.5556</td>
<td>1.555</td>
</tr>
<tr>
<td>\textbf{4.9}</td>
<td>10.05</td>
<td>10</td>
<td>0.6704</td>
<td>0.672</td>
<td>1.5499</td>
<td>1.512</td>
</tr>
<tr>
<td>\textbf{5}</td>
<td>9.94</td>
<td>9.9</td>
<td>0.6764</td>
<td>0.678</td>
<td>1.5477</td>
<td>1.549</td>
</tr>
<tr>
<td>\textbf{5.1}</td>
<td>9.849</td>
<td>9.8</td>
<td>0.6824</td>
<td>0.684</td>
<td>1.5457</td>
<td>1.5468</td>
</tr>
<tr>
<td>\textbf{5.4}</td>
<td>9.572</td>
<td>9.53</td>
<td>0.7004</td>
<td>0.702</td>
<td>1.5415</td>
<td>1.5415</td>
</tr>
<tr>
<td>\textbf{5.7}</td>
<td>9.316</td>
<td>9.28</td>
<td>0.7184</td>
<td>0.72</td>
<td>1.5397</td>
<td>1.54</td>
</tr>
<tr>
<td>\textbf{5.9}</td>
<td>9.1571</td>
<td>9.12</td>
<td>0.7304</td>
<td>0.732</td>
<td>1.5395</td>
<td>1.54</td>
</tr>
</tbody>
</table>

*a* Represents calculated values and *b* represents simulated values.

Fig. 3.9: Percentage of error between simulated value and calculated value for \( f_M \) and \( V_{out} \)
between the simulation results and calculations are shown for both \( f_M \) and \( V_{out} \) for different \( R_{init} \). We can see that as \( R_{init} \) increases the percentage of error decreases. It has been established that if \( R_{init} \) is increased the operational frequency range of this system (Table 3.1) will shorten, allowing increased possibility of sustained oscillation. So higher \( R_{init} \) reduces the error for estimating \( f_M \) and consequently, approximating \( V_{out} \) using (3.15) gets us closer to the simulated value. Nevertheless, the nonlinear Memristor behaves according to the suggested analytical expressions for any \( R_{init} \) as the errors are very low for both \( f_M \) and \( V_{out} \). Previously it was not possible to calculate the amplitude of oscillation before the oscillation had begun, and it was necessary to adjust it later by changing the gain of the circuit. Therefore (3.15) can be used to estimate the amplitude of the final oscillation which possibly helps to start the oscillation with predetermined amplitude. In Fig. 3.10 the simulation results of the relation between output oscillations

![Graph showing the relation between \( R_M \) and \( V_{out} \) for \( R_{init}=5k\Omega \)](image)

**Fig. 3.10:** Simulation result for \( R_M \) and \( V_{out} \) (for \( R_{init}=5k\Omega \))
$V_{\text{out}}$ and $R_M$ are shown. Here the shape of the graph is elliptical. Without any external input, this elliptical shape (Fig. 3.10) confirms that both $V_{\text{out}}$ and $R_M$ are periodic.

**Case 2: $R_2$ as a Memristor**

For this case similar simulation and analytical calculation has been followed by taking $R_1$ equals to 5KΩ, and $R_2$ is replaced with Memristor. The proposed formulas in the previous case are in perfect agreement with the simulation. The maximum error for $V_{\text{out}}$ is 1.5%, for $R_M$ is 3.1%, and for $f_M$ is 1.8%. Only difference is that now there is a range of gain for which the output oscillation will be fully sustained, for instance, when $R_{\text{init}}=5k\Omega$ sustained oscillation is found for any gain in the range of 1.97-1.98.

### 3.2 Other Members of Wien Family Oscillator with Memristor

In Fig. 3.11, the three other members of Wien oscillator are shown. In these three types either $(R_1-C_1)$ series branch or $(R_2-C_2)$ parallel branch has been interchanged with $R_3$ or $R_4$. These changes in configuration give the characteristics equation in a slightly different form, but they do not alter the frequency or conditions of oscillation; instead they share the same expressions as (3.3) and (3.4). For these three cases, $R_1$ is replaced with $R_M$ and $R_{\text{init}}$ is changed from 4.4KΩ to 5.7KΩ. Surprisingly, for all types, sustained oscillations are found in spite of having the $R_M$ oscillating across $R_{\text{avg}}$. From simulations it is observed that type ‘C’ and ‘D’ present similar behavior in $R_M$ and $V_{\text{out}}$. It may be due to the configuration of the series and parallel branch. If we look from $V_{\text{out}}$ point in Fig. 3.11, $R_1$-$C_1$ series branch and $R_2$-$C_2$ parallel branch in type ‘C’ and ‘D’ are found to be grounded, connecting by either $R_3$ or $R_4$. But type ‘B’ has these two branches connected
Referring to Fig. 3.12 we see that in type ‘B’, $R_M$ is oscillating from 4.69KΩ to 5.2KΩ with $R_{avg}=4.945KΩ$ for $R_{init}=5KΩ$. For type ‘C’ and ‘D’ (when $R_{init}=5KΩ$), $R_M$ is found to be oscillating from 4.842KΩ to 5.254KΩ across $R_{avg}=5.048KΩ$. For other $R_{init}$ values similar oscillating behaviors of $R_M$ and $V_{out}$ are observed for these three types. For different $R_{init}$, $R_M$ is calculated using (3.11) for these three types of Wien oscillator, and the maximum errors between simulations and calculations are found to be 3.35% (for type ‘B’) and 0.73% (for type ‘C’ and ‘D’). Although $R_M$ is oscillating, all these Wien oscillators are found to settle down at a certain frequency when the dynamic poles of
these systems are at the purely imaginary axis. Again the frequency of oscillation is calculated for each of the Wien family using (3.14) and the maximum error obtained for type ‘B’ is 0.9% and for type ‘C’ and ‘D’, 1.09%. As $R_M$ is oscillating for all types, poles of these systems eventually oscillate, yet make the output oscillation sustained. The gains to obtain sustained oscillations from these three types are precisely defined by $R_{avg}$ and the purely imaginary pole. The gain is governed by the following equation

$$1 + \frac{R_{avg} \pm \Delta R_M}{R_2} \equiv gain$$  (3.16)
The gains for types ‘B’, ‘C’, and ‘D’ are found from simulations as 1.99, 2.01, and 2.01 respectively when \( R_{\text{init}} = 5K\Omega \). When comparing (3.16) with simulations, the maximum errors are obtained as 0.4%, 0.2%, and 0.2%.

### 3.3 Comparative Study among Memristor based Wien Family Oscillators

In previous sections, the Memristive effects in Wien oscillators and the analytical models are summarized. Based on those criteria, the four types of Memristor-based Wien oscillators (case 1 for type ‘A’) will be compared here. First of all, in each type sustained oscillation is observed in spite of having the Memristor’s resistance \( (R_{\text{M}}) \) oscillating. Fig. 3.12 compares the simulated \( R_{\text{M}} \) and \( V_{\text{out}} \) for each type when \( R_{\text{init}} = 5K\Omega \). In case of \( R_{\text{M}} \), for type ‘A’ \( R_{\text{M}} \) oscillates from 4.69\( K\Omega \) to 5.396\( K\Omega \) with \( R_{\text{avg}} = 5.04K\Omega \). \( R_{\text{avg}} \) gets shifted up by 0.043\( K\Omega \) from \( R_{\text{init}} \) for any values of \( R_{\text{init}} \). For type ‘B’ \( R_{\text{avg}} \) is observed to shift down from \( R_{\text{init}} \) by 0.055\( K\Omega \). For type ‘C’ and ‘D’ \( R_{\text{avg}} \) is found to shift up by 0.05\( K\Omega \) for all \( R_{\text{init}} \). Even with the shifting of \( R_{\text{avg}} \), oscillating \( R_{\text{M}} \) results in the sustained oscillation at the output which is evident from \( V_{\text{out}} \) in Fig. 3.12. For other \( R_{\text{init}} \) values similar oscillations are observed. When calculating \( R_{\text{M}} \), a generalized model is proposed in (3.11) which is used to estimate \( R_{\text{M}} \) for any type. The maximum errors in calculating \( R_{\text{M}} \) for the four types are shown in Table 3.3. Note that \( V_{\text{M}} \) in (3.11) can be approximated from \( V_{1} \) which is the voltage across \( R_{1} \) in the conventional circuit. In Fig. 3.13 the
TABLE 3.3
Comparisons of the four members in Wien family

<table>
<thead>
<tr>
<th>Results</th>
<th>Wien A</th>
<th>Wien B</th>
<th>Wien C</th>
<th>Wien D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillating Real part of Pole</td>
<td>From $-1.952$ to $2.374$</td>
<td>From $-1.463$ to $1.683$</td>
<td>From $-1.207$ to $1.3357$</td>
<td>From $-1.207$ to $1.3357$</td>
</tr>
<tr>
<td>Oscillating Imaginary part of Pole</td>
<td>From $60.18$ to $64.47$</td>
<td>From $61.31$ to $64.46$</td>
<td>From $60.94$ to $63.49$</td>
<td>From $60.94$ to $63.49$</td>
</tr>
<tr>
<td>Maximum error (%) in $R_M$</td>
<td>2.85</td>
<td>3.35</td>
<td>0.73</td>
<td>0.73</td>
</tr>
<tr>
<td>(using (3))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum error (%) in $f_M$</td>
<td>1.1</td>
<td>0.9</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td>(using (5))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum error (%) in gain</td>
<td>0.5</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>(using (7))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

simulated results for $V_{out}$, $R_M$, $f_M$, and gain are plotted for different values of $R_{init}$ ($4.4K\Omega$ to $5.7K\Omega$). Type ‘C’ and ‘D’ show exactly the same responses. All the types have a similar trend with $R_{init}$. When $R_{init}$ is increased the amplitude swing of $R_M$ tends to be constant, resulting in constant magnitude of output oscillation ($V_{out}$) but with decreased frequency of oscillation ($f_M$). The gain of each type needs to be increased to obtain sustained oscillation when $R_{init}$ is increased. For the purpose of calculating $f_M$ and gain of the circuit for any type of Wien family, generalized analytical models are suggested in (3.14) and (3.16). Table 3.3 summarizes the comparative results using the proposed models in (3.11), (3.14), and (3.16). The negligible errors affirm that these models are compatible to analyze any Wien family oscillator when Memristor replaces one of the resistors. Because of oscillating $R_M$ one may expect $f_M$ to have a range, rather than to
Fig. 3.13: Simulated results of $V_{out}$, range of $R_M$, $f_M$, and gain for the Wien family when $R_{init}$ is changed from 4.4KΩ to 5.7KΩ.

have a single value $f_M$ indeed has a valid operational range, as shown in Fig. 3.14, for all types of Wien family (when $R_{init}$=5KΩ). Type ‘A’ has the largest range among all types. The solid black line is obtained from (3.14) as all the types use (3.14) as generalized expressions of $f_M$. But $V_M$ varies for different types of Wien oscillator; as a result the frequency response using (3.11) will be different for the four types, as shown in different colors for the four types of Wien oscillator in Fig. 3.14. Sustained oscillation can only be obtained in that region of operation. In Table 3.2, it is noted that if $R_{init}$ is increased this range will decrease in type ‘A’ but this is also true for the other three types. With the
Fig. 3.14: Frequency response of $R_M$ for types ‘A’, ‘B’, ‘C’, and ‘D’

during the operational range the error in estimating $f_M$ by (3.14) becomes less for types ‘B’, ‘C’, and ‘D’. Although $f_M$ has a range, all the oscillatory systems choose a certain frequency at which the black solid line intersects the $R_{\text{init}}$ line. The chief reason for this is that when $R_M=R_{\text{init}}$ (more accurately $R_{\text{avg}}$) the pole of this second order system falls on the imaginary axis in the $s$ plane. This purely imaginary pole defines $f_M$ eventually. In Fig. 3.15, the oscillating behavior of one of the conjugate poles of Type ‘A’ is shown in $s$ plane when $R_{\text{init}}=5K\Omega$. The pole will be at point $A$ when $R_M=R_{\text{max}}$ and it will be at $C$ when $R_M=R_{\text{min}}$. Thus the pole will move from $A$ to $C$ then $C$ to $A$ and continue oscillating for oscillating $R_M$. When $R_M=R_{\text{init}}$, that pole will be at $B$ which is on $jw$ axis. When the dynamic pole is at $B$, it settles the frequency of oscillation. This Perhaps could be the prime reason for the oscillatory system to have a single frequency of oscillation, bearing
Fig. 3.15: Dynamic pole in $s$ plane for type ‘A’

in mind that $R_M$ is oscillating. Similar behaviors of pole are also observed in types ‘B’, ‘C’, and ‘D’. In order to calculate $V_{out}$ for all the types, a similar approach can be taken, as described in section 3.1.5. Because of the circuit configuration (3.15) will be applicable only for type ‘A’, but $V_{out}$ certainly can be expressed as a function of $R_M$ for types ‘B’, ‘C’, and ‘D’. The method for estimating $V_{out}$ is shown below:

<table>
<thead>
<tr>
<th>Simulate conventional Wien oscillator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get $V_1$ across $R_1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Set desired $R_{init} = R_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get $R_M$ from (3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Express $V_{out} = f(R_M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plug in $R_M$ and $f_M$</td>
</tr>
</tbody>
</table>


3.4 State space modeling of Memristor based Wien Oscillator

The general representation of the circuit shown in Fig 3.2 can be expressed in state equations where the state variables will be $V_1$ (voltage across $C_1$) and $V_2$ (voltage across $C_2$). $V_1$ and $V_2$ can be expressed by a $2\times2$ matrix as shown in (3.17). The nonlinearity of OPAMP can be accounted by the saturation condition [65] in (3.18).

$$\begin{pmatrix}
\frac{dv_1}{dt} \\
\frac{dv_2}{dt}
\end{pmatrix} = \begin{pmatrix}
\frac{-1}{R_1C_1} & \frac{a-1}{R_1C_1} \\
\frac{1}{R_1C_2} & \frac{a-1}{R_1C_2} - \frac{1}{R_MC_1}
\end{pmatrix} \begin{pmatrix}
v_1 \\
v_2
\end{pmatrix} + \begin{pmatrix}
b \\
\frac{b}{R_1C_1}
\end{pmatrix}$$ (3.17)

$$\begin{cases}
(0, V_{sat}) \\
(\beta, 0) \\
(0, -V_{sat})
\end{cases}, \quad \begin{cases}
\beta V_{c1} \geq V_{sat} \\
-V_{sat} < \beta V_{c1} < V_{sat} \\
-V_{sat} \geq \beta V_{c1}
\end{cases}, \quad \beta = 1 + \frac{R_3}{R_4}$$ (3.18)

Based on HP model in section 2.3.2 and considering $p=2$ (to incorporate non linear drift model), the Memristor’s resistance $R_M$ can be derived as

![State models for Memristor based Wien ‘A’ oscillator](image)

Fig. 3.16: State models for Memristor based Wien ‘A’ oscillator
The Runge-Kutta 4th order method is implemented in MATLAB to solve the differential equations in (3.17) using (3.18) and (3.19) as well. Fig. 3.17 shows the oscillating behaviors in the MATLAB simulation for \( R_{\text{init}} = 5 \text{K}\Omega \) (other parameters are taken similar to PSPICE simulation). To compare the result, PSPICE simulation was run with \( p = 1 \) (as it gives the non-linear drift model), similar results were obtained as MATLAB with the exception in the gain of 1.48 (in PSPICE the gain was found 1.95).

![Fig. 3.17: MATLAB output of states (\( V_1 \) and \( V_2 \)) and \( R_M \)]
IV MEMRISTOR BASED 3\textsuperscript{RD} ORDER OSCILLATOR

Phase shift oscillator is a simple kind of sine wave oscillator comprised with three resistors and three capacitors. For the simplest form of this oscillator it uses an opamp with another feedback resistor to set up the close loop system. The three resistors and capacitors constitute a looped network which as a whole works as filter circuit and gives the necessary condition for sustained oscillation. Each R-C section gives phase shift of 60 degrees. So the whole section produces phase shift of 180 degrees. It shifts the output of inverting amplifier by 180 degrees to fulfill the Barkhausen stability criteria.

Fig. 4.1: Schematics of conventional phase shift oscillator
is a third order oscillator as it has three R-C sections. Fig. 4.1 shows the schematic diagram of conventional phase shift oscillator. To simplify the mathematical modeling of this third order system, all equal valued capacitance C is considered. The characteristics equation for this oscillator then can be expressed as:

\[ as^3 + bs^2 + cs + d = 0 \]  \hspace{1cm} (4.1)

\[
\begin{align*}
    a &= R_1R_2R_3C^3(1 + k), \\
    b &= 3R_1R_2C^2 + 2R_1R_3C^2 + R_3R_2C^2, \\
    c &= 2R_1C + 2R_2C + R_3C, \\
    d &= 1
\end{align*}
\]  \hspace{1cm} (4.2)

For sustained oscillation, the frequency of oscillation \( f \) and the condition for the gain \( k \) can be derived as:

\[
\omega = 2\pi f = \frac{1}{C\sqrt{3R_1R_2 + 2R_1R_3 + R_3R_2}}
\]  \hspace{1cm} (4.3)

\[
\begin{align*}
    k &= 8 + \frac{R_2}{R_3} + 6 \frac{R_1}{R_3} + 4 \frac{R_1}{R_2} + 2 \frac{R_2}{R_1} + 2 \frac{R_3}{R_2} + \frac{R_3}{R_1}
\end{align*}
\]  \hspace{1cm} (4.4)

For the conventional case (4.3) and (4.4) are simplified by taking \( R_1=R_2=R_3=R \) which gets the value of \( k \) as 29 and \( f \) as:

\[
\begin{align*}
    f &= \frac{1}{2\pi RC\sqrt{6}}
\end{align*}
\]  \hspace{1cm} (4.5)

The characteristics equation for this system has three roots, two of those are complex conjugate roots and the other is a real root. Traditionally it is well believed that those roots must be fixed to their distinct locations for the whole oscillation period. In addition
to that the resistors must show constant value as any change in the value will make the oscillation unstable.

4.1 Three Memristors based phase shift oscillator

This section will detail about the Memristor based 3\textsuperscript{rd} order system where all the three resistors (in Fig. 4.2) are changed with Memristors. C\textsubscript{1}, C\textsubscript{2}, and C\textsubscript{3} are taken equal valued C (= 1\textmu F). The parameters of SPICE model is kept same only \( R_{on} \) is changed to 0.5K\Omega (for Wien oscillator it was 0.1K\Omega). The different value of \( R_{on} \) for the two oscillators does not alter any result. Only the factor \( k \) gets changed to 50000 (for Wien oscillator it is 10000). \( R_{init} \) is the only variable parameter and it is changed from 6.1k to 6.9k for simulation. It is to be noted that \( R_{init} \) is taken equal for all three replaced Memristors.

4.1.1 Oscillation in \( R_M \)

From the simulation result, sustained oscillation is found even with oscillating behavior of \( R_M \). As all the \( R_M \)'s are found to be oscillating then \( R_M \) can be represented as (3.8):

\[
R_{Mi} = R_{avgi} \pm \Delta R_M \sin(2\pi f_M t), \ i = 1,2,3
\]  

It is indeed not possible to get sustained oscillation in conventional phase shift oscillator if one of the resistors of the three RC sections changes its value. Though \( R_{on} \) (0.5K\Omega) and \( R_{off} \) (16K\Omega) are supposed to be the most likely values for \( R_M \) but \( R_M \) itself shows sustained oscillation with an average value of \( R_{avg} \). In Fig. 4.3, the simulation result of \( R_{M1} \), \( R_{M2} \), and \( R_{M3} \) are shown when \( R_{init}=6.5\text{K}\Omega \). It is observed that \( R_{M1} \) oscillates from 6.05K\Omega to 6.95K\Omega with \( R_{avg1} \) exactly falls on \( R_{init}=6.5\text{K}\Omega \). It is to be noted that \( V_{M1} \)
(voltage across $R_{M1}$) is approximated as $V_1$ (voltage across $R_1$) to compute the values of $R_{M1}$. From the simulation (for Fig. 4.1), it is observed that $V_1=234\,\text{mV}$ (when $R_1=6.5\,\text{K}\Omega$) and $V_{M1}$ is found as $233\,\text{mV}$ for $R_{init}=6.5\,\text{K}\Omega$. For other values of $R_{M1}$, $V_{M1}$ can be well estimated as $V_1$ (for different $R_1$ but in correspondence to $R_{M1}$) with a maximum error of less than 1%. When $R_{init}$ is changed from $6.1\,\text{K}\Omega$ to $6.9\,\text{K}\Omega$, the maximum error between simulation and calculation is calculated to be 1.22%. It is found that $R_{M2}$ is also oscillating with a maximum amplitude of $6.55\,\text{K}\Omega$ and minimum amplitude of $6.28\,\text{K}\Omega$. $R_{avg2}$ is found to be $6.415\,\text{K}\Omega$ which is shifted down from $R_{init}$ by $0.085\,\text{K}\Omega$. The amplitude swing of $R_{M2}$ ($\Delta R_{M2}$) from simulation is $0.27\,\text{K}\Omega$ and if we use (3.11) then the swing is calculated to be $0.27\,\text{K}\Omega$. From the simulation it is found that $V_2$ (voltage across $R_2$) $=69.7\,\text{mV}$ (when $R_2=6.5\,\text{K}\Omega$) and $V_{M2}$ (voltage across $R_{M2}$) is $69.4\,\text{mV}$ for
Fig. 4.3: Simulation result of $R_{M1}$, $R_{M2}$, and $R_{M3}$ for $R_{\text{init}} = 6.5\,\text{K}\Omega$

$R_{\text{init}} = 6.5\,\text{K}\Omega$. Maximum error in calculating $R_{M2}$ using (3.11) is then found as 6.3%. In the case of $R_{M3}$, it oscillates in different magnitude because of $V_{M3}$ has different amplitude. $V_{M3}$ (voltage across $R_{M3}$) in this case is very close to $V_3$ (voltage across $R_3$). For $R_{\text{init}} = 6.5\,\text{K}\Omega$, $V_{M3}$ is 25mV and for $R_3 = 6.5\,\text{K}\Omega$, $V_3$ is 26mV. For other values of $R_{\text{init}}$, both $V_{M3}$ and $V_3$ are approximately equal with a maximum difference of 1.03mV (when $R_{\text{init}} = 6.9\,\text{K}\Omega$). Thus $V_3$ can be a good estimation of $V_{M3}$ to calculate $R_{M3}$ (when $R_{\text{init}} = 6.5\,\text{K}\Omega$). $R_{M3}$ is found to be oscillating from 6.37KΩ to 6.47KΩ. $R_{\text{avg3}}$ gets shifted up by 0.08KΩ. The simulated values of $R_{M3}$ and calculated values using (3.11) are well matched with a maximum error of 5%. Fig. 4.4 compares the errors in calculating $R_{M1}$, $R_{M2}$, and $R_{M3}$ for different $R_{\text{init}}$. The error for $R_{M1}$ is less than the errors in case of $R_{M2}$ and $R_{M3}$ due to the reason that $R_{\text{avg2}}$ and $R_{\text{avg3}}$ get shifted from $R_{\text{init}}$. Table 4.1 compares the result for $\Delta R_M$ for phase shift oscillator with Wien Oscillator. The negligible
difference between the calculation and simulation confirms (3.11) as a generalized model for calculating $R_M$ in both second order and third order oscillatory systems.

**TABLE 4.1**
Comparison between Wien and Phase shift oscillator

<table>
<thead>
<tr>
<th>Replaced Resistor with $R_M$</th>
<th>Wien Oscillator $[R_{init}=5K\Omega]$</th>
<th>Phase shift Oscillator $[R_{init}=6.5K\Omega]$</th>
<th>$(R_{max} - R_{min})$ using (3.11)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculation</td>
<td>Simulation</td>
<td>Calculation</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.70</td>
<td>0.728</td>
<td></td>
</tr>
<tr>
<td>$R_2$</td>
<td>NA</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>$R_3$</td>
<td>NA</td>
<td>0.10</td>
<td></td>
</tr>
</tbody>
</table>
4.1.2 Time dependent oscillating poles

In section 3, Wien oscillator is shown to have oscillating poles which are time dependent. Even for phase shift oscillator similar oscillations of poles are found. If we solve the third order equation in (4.1), three roots will be found having one purely real root and the other two be complex conjugates. These roots are defined in terms of the coefficients \(a, b, c,\) and \(d\). The coefficients in (4.2) can be derived as:

\[
\begin{align*}
a &= [R_{avg1}R_{avg2}R_{avg3} + \{R_{avg1}R_{avg2}\Delta R_{M3} + R_{avg1}R_{avg3}\Delta R_{M2} + R_{avg3}R_{avg2}\Delta R_{M1}\}]C^3(1 + k), \\
b &= 3R_{M1}R_{M2}C^2 + 2R_{M1}R_{M3}C^2 + R_{M3}R_{M2}C^2, \\
c &= [(2R_{avg1} + 2R_{avg2} + R_{avg3}) + (2\Delta R_{M1} + 2\Delta R_{M2} + \Delta R_{M3})]C, \\
d &= 1,
\end{align*}
\]

From (4.7), it is found that except \(d\), the other three coefficients are completely dependent on \(R_M\). As poles of such system are dependent on \(R_M\) and \(C\), and because of oscillating \(R_M\) with constant \(C\), the poles will follow time dependent oscillation as \(R_M\). The real pole is found to have little effect for this case as it is located far from the imaginary axis. But this real pole shows oscillating behavior and also time dependent. The two conjugate poles are also oscillating with time. This is surprising to have all the poles of this third order system to be oscillating with time and yet sustained oscillation is achieved in the output. In Fig. 4.5, the time dependent oscillating poles are shown for the three cases. One of the complex conjugate poles for this system is displayed in Fig. 4.5(a). Here we can see that the pole is not fixed but oscillating in different planes with different
amplitude for the three cases. Mirror image is found for the conjugate pole. Fig. 4.5(b) represents the oscillating real pole of the system.

### 4.1.3 Frequency of oscillation

The frequency of oscillation for this system, \( f_M \) can be modified from (4.3) as:

\[
 f_M = \frac{1}{2\pi C \sqrt{(3R_{M1}R_{M2} + 2R_{M1}R_{M3} + R_{M3}R_{M2})}} \quad (4.8)
\]

As all three \( R_M \)'s are oscillating, one may expect that the frequency of oscillation will have a range rather be single valued as in conventional case. But from the simulation the output is found oscillating presenting an impulse on FFT. Though the width of the impulse is little broader than the impulse attained from the traditional case. If we plot
Fig. 4.6: Frequency response of $R_{M1}$, $R_{M2}$, and $R_{M3}$

$R_M$’s using (3.11) versus $f_M$ (in Fig. 4.6 for $R_{init}=6.5\,\text{K\Omega}$), $R_M$’s will be seen to merge on $R_{init}$ at higher frequency. This in fact the case with Memristor as at higher frequency Memristor works as a linear resistor. Now assuming that $R_{avg1}$, $R_{avg2}$, and $R_{avg3}$ are equal to $R_{init}$ (though $R_{avg2}$ and $R_{avg3}$ are shifted from $R_{init}$ by $0.085\,\text{K\Omega}$ and $0.08\,\text{K\Omega}$ respectively), then replacing $R_M$ by $R_{init}\pm\Delta R_M$ in (4.8) we can again plot $R_{init}$ vs $f_M$ as shown in Fig. 4.6 (bold black line). This curve will intersect each $R_M$’s curves (zoomed in Fig. 4.6) at two frequency points which will give the valid frequency region for this Memristor based third order system to oscillate. As there are three frequency ranges, the oscillatory system chooses the common region of operation. Though the system has a frequency range to operate, sustained oscillation can only be possible at that frequency.
which is obtained from the intersection point of $R_{\text{init}}$ vs $f_M$ curve with $R_{\text{init}}$ line. For $R_{\text{init}}=6.5\,\text{K}\Omega$, $f_M$ from Fig. 4.6 is found 10.03Hz whereas from simulation $f_M$ is observed to be 10Hz. It is necessary to point out here that all the poles of this system are found to be oscillating in $s$ plane. But the dominating conjugate pole crosses the imaginary axis when $R_M=R_{\text{init}} \approx R_{\text{avg}}$. This might be the crucial reason for the system to settle down at one intersecting frequency to give sustained oscillation. Nevertheless the range of frequency of oscillation gives the hint of possible region of operation of this system for sustained oscillation. Apart from the graphical method of estimating $f_M$, using (4.8) $f_M$ can be analytically expressed as:

$$f_M = \frac{1}{2\pi C \sqrt{(6R_{\text{init}}^2 \pm 5(\Delta R_{M1} + 4\Delta R_{M2} + 3\Delta R_{M3})}}}$$  \hspace{1cm} (4.9)$$

Using (4.9), $f_M$ can be calculated for this system. Fig. 4.7 compares the simulated and the

![Comparison between simulated and calculated $f_M$ for different $R_{\text{init}}$. Percentage error in $f_M$ (inset)](image-url)
calculated values of $f_M$ for different $R_{\text{init}}$. The maximum error in approximating $f_M$ is 0.67% which is very negligible.

### 4.1.4 Gain ($k$) adjustment

For the traditional phase shift oscillator the necessary condition for sustained oscillation is to have $k=29$. For this Memristor based third order system, gain $k$ can be derived as:

$$k = 8 + \frac{6R_{M2} + 6R_{M1}}{R_{M3}} + \frac{4R_{M1} + 2R_{M3}}{R_{M2}} + \frac{2R_{M2} + R_{M3}}{R_{M1}} \quad (4.10)$$

As $R_M$ is oscillating so $k$ is expected to behave as sinusoid which is modeled in (4.10). From simulation it is observed that sustained oscillation is possible only for a certain value of $k$. When $R_{\text{init}}=6.5\,\Omega$, the simulated value of $k$ for this case is found as 28.75.

To mathematically calculate the value of $k$, three different methods are taken: (i) As $R_{\text{init}}$...
is very close to $R_{\text{avg}}$ so $R_{\text{init}}$ can be directly put in place of $R_M$ in (4.10). $k$ is found to be exactly 29 in this straightforward method. (ii) Another way can be, if we replace the sinusoidal $R_M$ in (4.10) using (4.6), it will give $k$ as a function of $t$ which is plotted in Fig. 4.8. The average value of this curve can be computed by integrating the curve for the half cycle of $R_M$ and dividing it by the half time period of $R_M$. The average value of $k$ becomes as 29.0862. (iii) The third method is that, $k$ can be plotted against $R_M$ using (4.10) for different values of $R_{\text{init}}$ (for this $R_{\text{init}}$ is changed from 6.1KΩ to 6.9KΩ). Fig. 4.9 shows $k$ as a function of $R_M$. Again integrating this curve and averaging by range of the limit of integration, the calculated value of $k$ can be found as 29.087. All of these three methods give a very close value of $k$ with the simulated one. Even when $R_{\text{init}}$ is changed to any value (from 6.1KΩ to 6.9KΩ) all these methods are well applicable with a maximum error of 1.03%, 0.99%, and 1.01% respectively for the three methods.
4.2 One Memristor based phase shift oscillator

Fig. 4.10 shows the schematic diagram of the proposed phase shift oscillator. $R_M$ is the resistance of Memristor which replaces the conventional resistor one by one. The same SPICE model is used to simulate the effect of Memristor on phase shift oscillator. The parameters are kept as mentioned in section 4.1.

4.2.1 Oscillating $R_M$

From the simulation result, sustained oscillation is found even in this case with oscillating behaviour of $R_M$. As mentioned previously, no sustained oscillation is attainable conventionally if one of the resistors of any RC sections changes its value. Though $R_{on}$ (0.5KΩ) and $R_{off}$ (16KΩ) are supposed to be the most likely values for $R_M$ but $R_M$ itself

Fig. 4.10: Schematics of one Memristor based phase shift oscillator
shows sustained oscillation with an average value of $R_{avg}$. Fig. 4.11 shows both the sustained output oscillation and the oscillating $R_M$ (for $R_{init}=6.5\,\text{K}\Omega$) when $R_1$ is replaced with $R_M$ keeping $R_2$ and $R_3$ as $6.5\,\text{K}\Omega$. In this figure it is clearly observed that both $R_2$ and $R_3$ are constant at $6.5\,\text{K}\Omega$ whereas $R_M$ oscillates within a definite range from $6.05\,\text{K}\Omega$ to $6.95\,\text{K}\Omega$. When $R_2$ or $R_3$ is replaced with $R_M$, similar oscillation is found but with different amplitude swing. In case of $R_2$, $R_M$ oscillates from $6.28\,\text{K}\Omega$ to $6.55\,\text{K}\Omega$ (when $R_{init}=6.5\,\text{K}\Omega$). For $R_3$, the amplitude swing of $R_M$ is observed from $6.37\,\text{K}\Omega$ to $6.47\,\text{K}\Omega$. If we compare these results with section 4.1.1, it is found that the value of any $R_M$ is similar to the corresponding $R_M$ of the three Memristors case. As for example for any $R_{init}$, $R_{M1}$ in one Memristor case is exactly equal to when three Memristors are used and it is

![Figure 4.11: Simulation results of $V_{out}$, $R_2$, $R_3$, and $R_M$ for $R_{init}=6.5\,\text{K}\Omega$ when $R_1$ is replaced with $R_M$](image)
evident in $R_{M2}$ and $R_{M3}$ also. The reason may be the voltage across the Memristor defines the value of $R_M$ and the linear dependence of $R_M$ on $V_M$ is obvious in (3.11).

### 4.2.2 Oscillating poles

The characteristic equation of this system can be expressed as:

$$as^3 + bs^2 + cs + d = 0$$  \hfill (4.11)

$$a = R_M R^2 C (1 + m), d = 1$$  \hfill (4.12)

$$b = 4R_M RC^2 + 2R^2 C^2, c = 2R_M C + 3RC,$$

Where $m=R_M/R_1$ (in Fig. 4.1). There will be three roots which will be defined by the coefficients $a$, $b$, $c$, and $d$. Two of the roots are complex conjugate and the other one is real. These roots/poles determine the stability of the system where conventionally it is stable.

![Diagram](image-url)

Fig. 4.12: (a) Oscillating pole with time; (b) $s$ plane representation of oscillating pole
believed that all the poles will be fixed in the left half of the s plane. But surprisingly in this system poles are found oscillating. From (4.12), it is obvious that \( a, b, \) and \( c \) are dependent on \( R_M \) and as \( R_M \) is sinusoidally oscillating so the poles are expected to be dynamic. In Fig. 4.12 (a) (when \( R_1 \) is replaced by \( R_M \)), the oscillating behavior of one of the conjugate poles is plotted. Here both the real and imaginary part of complex pole are oscillating with time that means in the s plane that pole will not be fixed rather it will oscillate from the left half s plane to right half s plane which is shown in Fig. 4.12(b) (red circle symbolizes pole). When the pole shifts to the right half plane then the oscillating \( R_M \) fetches the pole from the unstable region to the stable region of left half plane but then \( R_M \) changes its value which tends to pull back the poles towards right half plane and this periodic incident continues. These oscillations in all three poles are observed for every \( R_{\text{init}} \) as well as when \( R_2, \) and \( R_3 \) are also replaced by \( R_M \).

4.2.3 Frequency of oscillation, \( f_M \)

Due to the sinusoidally oscillating \( R_M, f_M \) can be modeled as:

\[
f_M = \frac{1}{2\pi C \sqrt{R (R + 5(R_{\text{avg}} \pm \Delta R_M))}}
\]  

(4.13)

Observing (4.13), \( f_M \) will have a range of values from maximum, \( f_{M,\text{max}} \) (when \( R_M=R_{\text{min}} \)) to minimum, \( f_{M,\text{min}} \) (when \( R_M=R_{\text{max}} \)). \( f_M \) will have a range of frequencies for oscillation but the output oscillation set itself to that frequency when \( R_M \) is equal to \( R_{\text{avg}} \) as described in section 4.1.3. Using (3.11) \( R_M \) can be approximated to be used in (4.13) to calculate \( f_M \) for corresponding \( R_{\text{init}} \). The maximum percentage of error in estimating \( f_M \) is 0.4% when
$R_1$ is replaced by $R_M$. In case of $R_2$ and $R_3$ replaced by Memristor the maximum error in calculating $f_M$ is 0.45% and 0.1% correspondingly.


V CONCLUSION AND FUTURE WORK

6.1 Conclusion

This thesis reports the unprecedented characteristics of Memristor in conventional Wien oscillator and phase shift oscillator. The effect of using Memristor in place of resistor has been described mathematically which is verified by the simulation results. The model for \( R_M \) is generalized for Memristor based both second and third order oscillatory system. It is now definite that sustained oscillation can be achieved in despite of the presence of the oscillating resistance of Memristor and time dependant oscillating poles of the system. The unconventional properties of Memristor are not only limited to second order oscillator (Wien oscillator) but also for third order oscillator (phase shift oscillator) similar behavior of Memristor is observed. The mathematical models for characterizing the oscillation in these two schemes match very closely with the simulation results. The new exciting results in oscillation will help to redefine the traditional concepts about sustained oscillation. The oscillating resistance, operational frequency range, and dynamic poles of the Memristor based third order system can surely help to redefine the conventional concept. Challenges remain with the availability of passive Memristor and nonlinearity of both Memristor and opamp. As the order of the system increases there are added complexities to the mathematical modeling. Simpler and more generalized model is required to describe the unconventional characteristics of Memristor in oscillation. The unprecedented results can have impact in designing neural network, oscillatory circuits, signal generation, sensitive control systems, programmable frequency counter, low
frequency oscillator, timers (e.g. heartbeat timers, and watchdog timers), and amplitude/frequency modulator, etc. As for now the proposed models and mathematical reasoning can surely help circuit designers to implement Memristor in various circuits.

6.2 Future Direction

✓ Closed form models are required to describe the frequency of oscillation for Memristor based phase shift oscillator and gain for both oscillatory systems.

✓ Modification in state space model to achieve the theoretical gain.

✓ Modeling of Memristor as a feedback element to control the gain.

✓ Capacitor less Wien and phase shift oscillator systems may be developed with the Memristor.

✓ Practical implementations of the described systems will be required for more pronounced realization of the systems.
REFERENCES


